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Discussion of 'Threshold models in time series analysis — 30 years on' by H. Tong

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It is a great pleasure to participate the discussion of Professor Tong's most authoritative paper. It develops an interesting and complete systematic treatment of the threshold idea, reporting also less well known results and fresh research findings.

In particular, the comprehensive form (3.1):

$$X_t = a_0^{(J_t)} + \sum_{i=1}^{p} a_i^{(J_t)} X_{t-i} + b^{(J_t)} \varepsilon_t$$

relating the model to a general indicator time series $\{J_t\}$ (univariate or multivariate, random or deterministic, observable or not) allows to put into the threshold framework a very large class of models, including hidden Markov models and many others that may be plausible and useful in specific applications. I found the suggestion of partially hidden switching models particularly stimulating, and think that it could be developed in various directions with interesting results.

I will comment briefly on one of the particularization of (3.1), which is also cited in the paper's concluding remarks as a possible advancement: non stationary – non linear models.

The threshold idea, though originated by cycles, may be easily employed for modeling structural change in time, suggesting autoregressive models where the parameter values change both according to time, in two or more regimes, and according to the value of a previous observation. Essentially, one may think of a model of form (3.1) with the indicator time series given by:

$$J_t = f(t, X_t)$$

and the domain $\mathbb{R}^+ \times \mathbb{R}$ split into rectangular regions where the indicator value is one or two.

A model of such kind, based on a smooth threshold autoregressive form, was studied by [3] and an extension, allowing also for piecewise linear threshold form (where the autoregressive parameters inside each regime are not constant, but linearly dependent on the threshold variable) was recently presented in [2].

Identifying a model of that kind involves a search inside a large discrete space of solutions, trying to optimize a fitting criterion and without the help of any analytical tool. Therefore [2] suggests to employ a genetic algorithm.

Evolutionary computation methods, like genetic algorithms, have been applied increasingly often in several statistical problems for the last two decades, and I believe they will soon become a standard tool in Statistics [for an introduction, see 1].

One of the main (and perhaps exciting) problems with non stationary – non linear models, is that the two regime dimensions, time and delayed observations, often interact, and make their unfolding difficult and not univocal. This was indeed already pointed out by Professor Tong, who in [4] wonders about "unscrambling the omelette".

There are several other passages in the paper suggesting new and interesting details: for example the idea of m-step-ahead forecasting by refitting a SETAR model with a larger delay for approximately periodic data. And the discussion of the relationship between STAR and SETAR: I must admit that I have always thought of a SETAR model as the limit of a STAR as the smoothness parameter tends to zero. And this is true in theory, but I know now clearly, as Professor Tong explains, that the two model classes should be kept quite distinct in practice.

But the most intriguing part of the paper, in my view, is the small Section on threshold volatility, where the author reports about new unpublished research, citing new findings and a very important and surprising (at least to me) theorem.

This suggests that remarkable research results on threshold volatility are about to be published, and induces to foresee that, if in the last 30 years Professor Tong's work was fundamental for time series analysis, his next 30 years of contributions will be important as well. And so we wish him heartily.

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