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Discussion on the paper 'Threshold models in time series analysis — 30 years on,' by Professor Howell Tong

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I congratulate Professor Howell Tong for providing an impressive review of the development of threshold models in time-series analysis. Professor Tong has not only provided this review, but has proven to be one of the pivotal leaders in this field. If we are to use his quote of Confucius, 'Re-visiting the past can lead to new discoveries,' we must certainly study the works of Professor Tong himself, among others. This paper highlights the increasing need for modelling nonlinear time-series data using threshold models.

In my discussion, I shall refer to a new offspring of the open-loop TAR (TARX) model, namely the Generalized Threshold Model (see Samia et al. [3]; Samia and Chan [4]). Threshold models often assume Gaussian conditional responses whose mean response is a piecewise-linear stochastic regression function. In many applications, a new offspring of TARX is needed to model time-series counts (e.g. Poisson or binomially distributed data) or positive time-series data with a right-skewed distribution (e.g. gamma distribution) or other non-normal temporal data. Such data can be analyzed using the so-called generalized threshold model (GTM) which is an extension of the class of generalized linear models (Nelder and Wedderburn [2]; McCullagh and Nelder [1]), where both non-Gaussian response distributions and piecewise linearity are entertained.

The GTM, introduced and developed in Samia and Chan [4], generalizes the TARX model to account for non-Gaussian errors. Specifically, it is assumed that the conditional probability distribution of the response variable belongs to the exponential family, and the conditional mean response is linked to some piecewise-linear stochastic regression function through a known and invertible link function. Hence, the link function removes any inherent constraint on the range of the mean function due to non-Gaussianity, so that on the scale of the link function, the mean response is piecewise linear.

Maximum likelihood techniques are used to fit the GTM. Samia et al. [3] studied the large-sample properties of the particular case where the response variable equals zero in the lower regime and used it to model nonlinearity in serially correlated epizootic events of plague in the host reservoir; see Stenseth et al. [6]. Samia and Chan [4] provided rigorous proofs of the limiting properties of the GTM where the response variable is not identically zero in the lower regime. Then, this latter model was used to explain sporadic occurrences of bubonic plague in humans (Samia et al. [5]). Hence, to fully explain the cycle and transmission of bubonic plague from its host reservoir to humans required the development of the two separate threshold models, one related to the host reservoir and a second explaining the transmission to humans.

This offspring of the TARX model has demonstrated its usefulness in the field of epidemiology and infectious diseases. However, it can be adapted for use in many other fields, including natural sciences, marketing, economics, political science, and business.

It should be noted that Professor Tong has been one of the founders and pioneering leaders in the development of threshold models in time series. Without his groundbreaking work, the GTM likely would not have seen the light of day. It is to this avail that I admire and congratulate him on his past and continuing accomplishments.

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