Preface

A dozen years have elapsed since Besse's *Einstein Manifolds* first offered the mathematical public a comprehensive overview of this vital area of differential-geometric research. The book's unqualified success is witnessed not only by the hundreds of research articles which cite it, but also by the entire generation of geometers for whom the book served as a catechism. A concomitant measure of Besse's success, however, is the degree to which the book has consequently become outdated; for every research problem discussed there has inevitably been attacked by scores of geometers, sometimes to considerable effect. Thus, while Besse still remains the best all-around introduction to the subject, its utility as a self-contained guide for the aspiring researcher has palpably declined in recent years.

With this in mind, Professor S.-T. Yau suggested to us, in early 1997, that the time was ripe for a broad new survey of the field. When he went on to invite us to oversee the project, however, we could only feel thunder-struck: deeply honored, of course, but also staggered by the prospect. The monumental task of compiling a comprehensive replacement for Besse, with uniform notation, a unified bibliography, and a common style of exposition, simply seemed too much to contemplate. On reflection, though, we eventually came to realize that what the mathematical community actually needed was, in any case, something rather different.

Thus, our aim here is merely to offer a photo-mosaic of current research: a collage of snapshots, each capturing some major trend, each taken from the vantage point of a major contributor to the field. The participating authors were given free rein to adopt whatever style seemed best adapted to their topics; and whatever success the book may have must merely reflect the quality of their work. We are therefore deeply grateful to these friends and colleagues for the time and energy they have so willingly sacrificed in order to bring these essays into the world.

We have organized our montage into a triptych, the first panel of which depicts the Einstein manifolds of special holonomy. Since geometries of this sort are naturally classified according to the parallel tensor fields they entail, this part of the book necessarily runs parallel to portions of Besse, although our treatment focuses almost entirely on recent developments. Topics covered include Kähler-Einstein manifolds (essays by G. Tian and S. T. Yau), hyper-Kähler manifolds (essay by A. Dancer), manifolds with holonomy G_2 or Spin(7) (essay by D. Joyce), and quaternion-Kähler manifolds (essay by S. Salamon). We are also delighted to be able to offer an authoritative treatment (essay by C. Boyer and K. Galicki) of 3-Sasakian manifolds, the importance of which was quite unforeseen by Besse; for while these manifolds *do not* have special holonomy, their fundamental ties to the Kähler, hyper-Kähler, and quaternion-Kähler cases allow one to generate them in profusion, providing a dazzling new source of explicit compact Einstein manifolds.

The second panel of our triptych concerns techniques applicable to the study of general Einstein manifolds. Topics covered include Ricci flow (essay by B. Chow), convergence and compactness (essay by P. Petersen), and the use of symmetry or bundle constructions (essay by M. Wang). This section also surveys the 4-dimensional case (C. LeBrun) and the asymptotically hyperbolic case (O. Biquard) in essays which compare metrics of special holonomy with general Einstein metrics.

The last panel of our triptych harkens back to the physical inspirations of our subject. We are particularly happy to be able to feature two essays on the Lorentzian case, written for an audience of Riemannian geometers. The first of these, by K. P. Tod, offers a broad survey of recent research on General Relativity; and the second, by D. Christodoulou, focuses on the author's joint work with Klainermann on the stability of Minkowski space. Finally, this section includes an essay by D. Calderbank and H. Pedersen on the Riemannian analog of Hermann Weyl's unified theory of gravitation and electro-magnetism.

Obviously, we have not been able to cover every active area with ties to the theory of Einstein manifolds. Sometimes, as with the study of self-dual 4-manifolds or extremal Kähler metrics, we have chosen to omit a topic entirely because the main thrust of current work has little bearing on our present subject. In other cases, we have been forced to give only a limited treatment of a major recent development because the experts we approached felt disinclined to duplicate up-to-date surveys they had written for publication elsewhere. Thus, while LeBrun's essay includes a brief introduction to entropy and minimal volume, the interested reader is urged to read the article by Besson, Courtois and Gallot entitled *Minimal Entropy and Mostow's Rigidity*, in *Ergodic Theory and Dynamical Systems* **16** (1996) 623–649. Similarly, while the articles of Joyce and Salamon draw attention to the importance of harmonic and Killing spinors on certain Einstein manifolds, the reader should refer to J.-P. Bourguignon's forthcoming survey in the *Bulletin of the American Mathematical Society* for a systematic development of these ideas.

Despite the notable progress chronicled in this volume, the study of Einstein manifolds remains in its infancy. For example, we still haven't a clue as to whether there are obstructions to the existence of Einstein metrics on compact manifolds of dimension > 4. It is our fondest hope that this book may play an indirect rôle in settling fundamental questions of this kind by refocusing the attention of the mathematical community on the many open problems in this fascinating and important area of Riemannian geometry.

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