

Preface

The present special volume of “Pure and Applied Mathematics Quarterly” is dedicated to the 75th birthday of Kyoji Saito. This volume arose from the interacting atmosphere of the “International Conference on Singularity Theory 2019”, which was held at the Tsinghua Sanya International Mathematics Forum (TSIMF) from December 9, 2019 till December 13, 2019.

Kyoji Saito’s research lies on the interplay among Lie theory and singularity theory. One of his most influential work was the discovery of the theory of primitive forms around early 1980s as a generalization of the elliptic period integral theory. It leads to first systematic examples of Frobenius manifold structure on the universal unfoldings of isolated singularities. The importance of this structure has blown up a decade later along with the explosive development of topological field theory and mirror symmetry. In fact, Kyoji Saito’s theory of primitive forms has been recognized as the mathematical theory of topological Landau-Ginzburg B-model, and has become the fundamental structure of any two dimensional topological field theory. This has influenced vast subjects from different fields in the last 30 years, including Gromov-Witten theory, quantum singularity theory, representation theory, category theory, topological string etc. In another deep direction on connecting the geometric theory of simple elliptic singularities and infinite dimensional Lie theory, Kyoji Saito introduced a general notion of extended affine root system beyond that of symmetrizable Kac-Moody Lie algebras; he and his collaborators started a systematic way to build up the corresponding infinite dimensional Lie algebras. Nowadays people recognized that the theory by Kyoji Saito has an intrinsic relation with the categorified realization of quantum algebras and Lie algebras arising from the tilting quiver representations and the coherent sheaves of weighted projective lines.

Beyond mathematics, Kyoji Saito has also made tremendous contributions in global collaborations. This special volume collects papers from leading experts in various fields who are influenced by Kyoji Saito’s mathematics and personality. It covers areas from Riemannian geometry, Lie theory (invariant theory, Coxeter group, primitive form), number theory (modular form, hyperelliptic integral), homotopy algebra (A-infinity algebra), representation theory (Kac-Moody algebra) and topology (moduli space, perverse sheave). The major contributions are devoted to the study related to Lie theory and singularity theory, in which Kyoji Saito has been building bridge in his whole career.

We would like to take this opportunity to celebrate Kyoji Saito's 75th birthday and to acknowledge his profound achievements in his life. We would like to thank all the people who have contributed to the success of the conference and to this special volume.