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Fibered knots with the same 0-surgery and the slice-ribbon conjecture

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Dedicated to Professors Taizo Kanenobu, Yasutaka Nakanishi, and Makoto Sakuma for their 60th birthday

Akbulut and Kirby conjectured that two knots with the same 0-surgery are concordant. In this paper, we prove that if the slice-ribbon conjecture is true, then the modified Akbulut-Kirby's conjecture is false. We also give a fibered potential counterexample to the slice-ribbon conjecture.

1. Introduction

The slice-ribbon conjecture asks whether any slice knot in S^3 bounds a ribbon disk in the standard 4-ball B^4 (see [18]). There are many studies on this conjecture (cf. [5, 6, 13, 23, 25, 27, 31–33]). On the other hand, until recently, few direct consequences of the slice-ribbon conjecture were known. This situation has been changed by Baker. He gave the following conjecture.

Conjecture 1.1 ([9, Conjecture 1]). *Let K_0 and K_1 be fibered knots in S^3 supporting the tight contact structure. If K_0 and K_1 are concordant, then $K_0 = K_1$.*

Baker proved a strong and direct consequence of the slice-ribbon conjecture as follows:

Theorem 1.2 ([9, Corollary 4]). *If the slice-ribbon conjecture is true, then Conjecture 1.1 is true.*

Originally, Conjecture 1.1 was motivated by Rudolph's old question [48] which asks whether the set of algebraic knots is linearly independent in the

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knot concordance group. Here we observe the following, which was implicit in [9].

Observation 1.3. *If Conjecture 1.1 is true, the set of prime fibered knots in S^3 supporting the tight contact structure is linearly independent in the knot concordance group (see Lemma 3.1). Moreover, the set of such knots contains algebraic knots (see Lemma 3.2). In this sense, Conjecture 1.1 is a generalization of Rudolph's question. Therefore Theorem 1.2 implies that if the slice-ribbon conjecture is true, then the set of algebraic knots is linearly independent in the knot concordance group –an affirmative answer of Rudolph's question–.*

Theorem 1.2 and Observation 1.3 make the slice-ribbon conjecture more important and fascinating.

In this paper, we give another consequence of the slice-ribbon conjecture. To state our main result, we recall Akbulut and Kirby's conjecture on knot concordance in the Kirby's problem list [29].

Conjecture 1.4 ([29, Problem 1.19]). *If* 0*-surgeries on two knots give the same 3-manifold, then the knots are concordant.*

Livingston [34] demonstrated a knot K such that it is not concordant to K^r, where K^r is the knot obtained from K by reversing the orientation. Therefore Conjecture 1.4 is false since 0-surgeries on K and K^r give the same 3-manifold, however, the following conjecture seems to be still open.

Conjecture 1.5. *If* 0*-surgeries on two knots give the same 3-manifold, then the knots with relevant orientations are concordant.*

Note that Cochran, Franklin, Hedden and Horn [14] obtained a closely related result to Conjecture 1.5. Indeed they gave a negative answer to the following question: "If 0-surgeries on two knots are integral homology cobordant, preserving the homology class of the positive meridians, are the knots concordant?"

Our main result is the following.

Theorem 1.6. *If the slice-ribbon conjecture is true, then Conjecture 1.5 is false*[1]*.*

[1] Recently, Kouichi Yasui [53] proved that there are infinitely many counterexamples of Conjecture 1.5.

In Section 2, we will prove Theorem 1.6. We outline the proof as follows: Let K_0 and K_1 be the unoriented knots depicted in Figure 1 and give arbitrary orientations on K_0 and K_1.

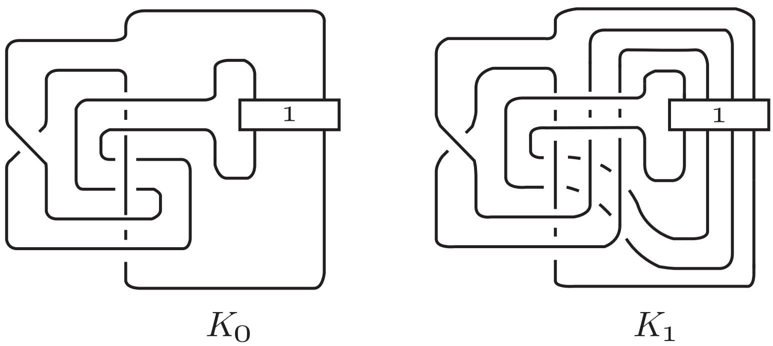

Figure 1: The definitions of K_0 and K_1. Each rectangle labeled 1 implies a full twist.

By using annular twisting techniques developed in [1, 2, 40, 51], we see that 0-surgeries on K_0 and K_1 give the same 3-manifold. On the other hand, by Miyazaki's result [36], we can prove that $K_0\#\overline{K_1}$ is not ribbon, where $K_0\#\overline{K_1}$ denotes the connected sum of K_0 and the mirror image of K_1. Suppose that the slice-ribbon conjecture is true. Then $K_0\#\overline{K_1}$ is not slice. Equivalently, K_0 and K_1 are not concordant. As a summary, 0-surgeries on K_0 and K_1 give the same 3-manifold, however, they are not concordant if the slice-ribbon conjecture is true, implying that Conjecture 1.5 is false.

Here we consider the following question.

Question 1.7. Are the knots K_0 and K_1 in Figure 1 concordant?

This question is interesting since the proof of Theorem 1.6 tells us the following:

1) If K_0 and K_1 are concordant, then $K_0\#\overline{K}_1$ is a counterexample to the slice-ribbon conjecture since $K_0\#\overline{K}_1$ is not ribbon.
2) If K_0 and K_1 are not concordant, then Conjecture 1.5 is false since 0-surgeries on K_0 and K_1 give the same 3-manifold.

This paper is organized as follows: In Section 2, we prove Theorem 1.6. In section 3, we consider consequences of Baker's result in [9]. In Appendix A, we give a short review for Miyazaki's in-depth study on ribbon fibered knots which is based on the theorem of Casson and Gordon [11]. In Appendix

B, we recall twistings, annulus twists and annulus presentations. Moreover, we define annulus presentations compatible with fiber surfaces and study a relation between annulus twists and fiberness of knots. Finally, we describe monodromies of the fibered knots obtained from 6_3 (with an annulus presentation) by annulus twists. Using these monodromies, we can distinguish these knots.

Notations. Throughout this paper, we will work in the smooth category. Unless otherwise stated, we suppose that all knots are oriented. Let K be a knot in S^3. We denote $M_K(0)$ the 3-manifold obtained from S^3 by 0-surgery on K in S^3, and by $[K]$ the concordance class of K. For an oriented compact surface F with a single boundary component and a diffeomorphism $f\colon F \to F$ fixing the boundary, we denote by $\widehat{F}$ the closed surface $F \cup D^2$ and by $\widehat{f}$ the extension $f \cup \mathrm{id}\colon \widehat{F} \to \widehat{F}$. We denote by t_C the right-handed Dehn twist along a simple closed curve C on F.

2. Proof of Theorem 1.6

In this section, we prove our main theorem. The main tools are Miyazaki's result [36, Theorem 5.5] and annular twisting techniques developed in [1, 2, 40, 51]. For the sake of completeness, we will review these results in Appendices A and B.

Proof of Theorem 1.6. Let K_0 and K_1 be the unoriented knots as in Figure 1 and give arbitrary orientations on K_0 and K_1. By [40, Theorem 2.3] (see Lemma 5.6), 0-surgeries on K_0 and K_1 give the same 3-manifold (for detail, see Appendix B. In this case, K_0 admits an annulus presentation as in Figure 4 and $K_1 = A(K_0)$).

On the other hand, $K_0\#\overline{K}_1$ is not a ribbon knot as follows: First, note that K_0 is the fibered knot 6_3 in Rolfsen's knot table, see KnotInfo [12]. By Gabai's theorem in [20], K_1 is also fibered since 0-surgeries on K_1 and K_2 give the same 3-manifold (see also Remark 5.9). Therefore $K_0\#\overline{K}_1$ is a fibered knot. Here we can see that K_0 and K_1 are different knots (for example, by calculating the Jones polynomials of K_0 and K_1). Also, we see that K_0 and K_1 have the same irreducible Alexander polynomial

$$\Delta_{K_0}(t) = \Delta_{K_1}(t) = 1 - 3t + 5t^2 - 3t^3 + t^4.$$

By Miyazaki's result [36, Theorem 5.5] (or Corollary 4.3), the knot $K_0\#\overline{K}_1$ is not ribbon.

Suppose that the slice-ribbon conjecture is true. Then $K_0 \# \overline{K_1}$ is not slice. Equivalently, K_0 and K_1 are not concordant. Therefore, if the slice-ribbon conjecture is true, then Conjecture 1.5 is false. $\square$

3. Observations on Baker's result

In this section, we consider consequences of Baker's result in [9].

First, we recall some definitions. A fibered knot in S^3 is called *tight* if it supports the tight contact structure (see [9]). A set of knots is *linearly independent* in the knot concordance group if it is linearly independent in the knot concordance group as a $\mathbb{Z}$-module. We observe the following.

Lemma 3.1. *If Conjecture 1.1 is true, then the set of prime tight fibered knots in S^3 is linearly independent in the knot concordance group.*

Proof. Let $K_1, K_2, \ldots, K_n$ be distinct prime tight fibered knots. Suppose that for some integers $a_1, \ldots, a_n$ we have

$$a_1[K_1] + \cdots + a_n[K_n] = 0.$$

We will prove that if Conjecture 1.1 is true, then $a_1 = a_2 = \cdots = a_n = 0$. When $a_1 \geq 0$, ..., $a_n \geq 0$, then

$$[(\#^{a_1} K_1) \# \cdots \# (\#^{a_n} K_n)] = 0.$$

Note that $(\#^{a_1} K_1) \# \cdots \# (\#^{a_n} K_n)$ is a tight fibered knot. If Conjecture 1.1 is true, then

$$(\#^{a_1} K_1) \# \cdots \# (\#^{a_n} K_n)$$

is the unknot. By the prime decomposition theorem of knots, we obtain

$$a_1 = a_2 = \cdots = a_n = 0.$$

When $a_1 \leq 0$, ..., $a_n \leq 0$, then

$$[(\#^{-a_1} K_1) \# \cdots \# (\#^{-a_n} K_n)] = 0.$$

By the same argument, we obtain

$$a_1 = a_2 = \cdots = a_n = 0.$$

For the other case, we may assume that $a_1 \geq 0$, ..., $a_m \geq 0$ and $a_{m+1} \leq 0$, ..., $a_n \leq 0$ by changing the order of the knots. Then we obtain

$$a_1[K_1] + \cdots + a_m[K_m] = (-a_{m+1})[K_{m+1}] + \cdots + (-a_n)[K_n].$$

Equivalently,

$$[(\#^{a_1} K_1)\# \cdots \#(\#^{a_m} K_m)] = [(\#^{-a_{m+1}} K_{m+1})\# \cdots \#(\#^{-a_n} K_n)].$$

Note that $(\#^{a_1} K_1)\# \cdots \#(\#^{a_m} K_m)$ and $(\#^{-a_{m+1}} K_{m+1})\# \cdots \#(\#^{-a_n} K_n)$ are tight fibered knots. If Conjecture 1.1 is true, then

$$(\#^{a_1} K_1)\# \cdots \#(\#^{a_m} K_m) = (\#^{-a_{m+1}} K_{m+1})\# \cdots \#(\#^{-a_n} K_n).$$

By the prime decomposition theorem of knots, we obtain

$$a_1 = a_2 = \cdots = a_n = 0.$$ □

Lemma 3.1 leads us to ask which knots are (prime) tight fibered. Recall that *algebraic knots* are links of isolated singularities of complex curves and *L-space knots* are those admitting positive Dehn surgeries to L-spaces[2].

Lemma 3.2. *We have the following.*

1) *A fibered knot is tight if and only if it is strongly quasipositive.*
2) *An algebraic knot is a prime tight fibered knot.*
3) *An L-space knot is a prime tight fibered knot.*
4) *A divide knot is a tight fibered knot.*
5) *A positive fibered knot is a tight fibered knot.*
6) *An almost positive fibered knot is a tight fibered knot.*

Proof. (1) This follows from [26, Proposition 2.1] (see also [8]).

(2) It is well known that any algebraic knot is fibered and strongly quasipositive. By (1), it is tight fibered. In fact, any algebraic knot is an iterated cable of a torus knot. This implies that it is prime. For the details on algebraic knots, see [16], [28], [52].

[2] In our definition, the left-handed trefoil is not an L-space knot. Note that some authors define *L-space knots* to be those admitting non-trivial Dehn surgeries to L-spaces. In this definition, the left-handed trefoil is an L-space knot.

(3) By [38, 39] (see also [21], [43]), an L-space knot is fibered. Hedden [26] proved that it is tight. It is also known that an L-space knot is prime, see [30].

(4) A'Campo [7] proved that a divide knot is fibered and its monodromy is a product of positive Dehn twists. Such a fibered knot is known to be tight, for example, see Remark 6.5 in [19]. For the details, see [41].

(5) Nakamura [37] and Rudolph [49] proved that any positive knot is strongly quasipositive. By (1), a positive fibered knot is tight.

(6) The authors [4] proved that any almost positive fibered knot is strongly quasipositive. By (1), an almost positive fibered knot is tight. □

Remark 3.3. By Theorem 1.2 and Lemmas 3.1 and 3.2, if the slice-ribbon conjecture is true, then the set of L-space knots in S^3 is also linearly independent in the knot concordance group.

The following conjecture may be manageable than Conjecture 1.1.

Conjecture 3.4. *The set of L-space knots in S^3 is linearly independent in the knot concordance group. In particular, if L-space knots K_0 and K_1 are concordant, then $K_0 = K_1$.*

4. Appendix A: Miyazaki's results on ribbon fibered knots

In this appendix, we recall Miyazaki's results [36] on non-simple ribbon fibered knots, in particular, on composite ribbon fibered knots.

Let K_i be a knot in a homology 3-sphere M_i for $i = 0, 1$. We write

$$(M_1, K_1) \geq (M_0, K_0) \text{ (or simply } K_1 \geq K_0)$$

if there exist a 4-manifold X with $H_*(X, \mathbb{Z}) \simeq H_*(S^3 \times I, \mathbb{Z})$ and an annulus A embedded into X such that

$$\begin{aligned}(\partial X, A \cap \partial X) &= (M_1, K_1) \sqcup (M_0^r, K_0^r),\\ \pi_1(M_1 \setminus K_1) &\to \pi_1(X \setminus A) \text{is surjective, and}\\ \pi_1(M_0 \setminus K_0) &\to \pi_1(X \setminus A) \text{is injective,}\end{aligned}$$

where (M_0^r, K_0^r) is (M_0, K_0) with reversed orientation. We say K_1 is *homotopically ribbon concordant* to K_0 if $K_1 \geq K_0$. Note that this is a generalization of the notion of "ribbon concordant", see in [24, Lemma 3.4]. A knot K in a homotopy 3-sphere M is *homotopically ribbon* if $K \geq U$, where U is the unknot in S^3. A typical example of a homotopically ribbon knot is a ribbon knot in S^3 (for detail see [36, p.3]).

Theorem 4.1 ([36, Theorem 5.5]). *For $i = 1, \ldots, n$, let K_i be a prime fibered knot in a homotopy 3-sphere M_i satisfying one of the following:*

- *K_i is minimal with respect to "$\geq$" among all fibered knots in homology spheres,*
- *there is no $f(t) \in \mathbf{Z}[t] \setminus \{\pm t^k\}_k$ such that $f(t)f(t^{-1})|\Delta_{K_i}(t)$.*

If $K_1 \# \cdots \# K_n$ is homotopically ribbon, then the set $\{1, \ldots, n\}$ can be paired into $\sqcup_{s=1}^m \{i_s, j_s\}$ such that $K_{i_s} = \overline{K_{j_s}}$, where $\overline{K_{j_s}}$ is $(M_{j_s}^r, K_{j_s}^r)$.

Remark 4.2. By a solution of the geometrization conjecture (see [44], [45] and [46]), each homotopy 3-sphere M_i is S^3 in the above theorem.

As a corollary, we obtain the following.

Corollary 4.3. *Let K_0 and K_1 be fibered knots in S^3 with irreducible Alexander polynomials. If $K_0 \# \overline{K_1}$ is ribbon, then $K_0 = K_1$.*

5. Appendix B: Twistings, Annulus twists and Annulus presentations

In this appendix, we recall two operations. One is *twisting*, and the other is *annulus twist* [1]. In a certain case, annulus twists are expressed in terms of twistings and preserve some properties of knots. Finally, we describe monodromies of the fibered knots obtained from 6_3 (with an annulus presentation) by annulus twists. We begin this appendix with recalling the definition of an open book decomposition of a 3-manifold.

5.1. Open book decompositions

Let F be an oriented surface with boundary and $f \colon F \to F$ a diffeomorphism on F fixing the boundary. Consider the pinched mapping torus

$$\widehat{M_f} = F \times [0, 1]/\sim,$$

where the equivalent relation $\sim$ is defined as follows:

1) $(x, 1) \sim (f(x), 0)$ for $x \in F$, and

2) $(x,t) \sim (x,t')$ for $x \in \partial F$ and $t,\, t' \in [0,1]$.

Here, we orient $[0,1]$ from 0 to 1 and we give an orientation of $\widehat{M}_f$ by the orientations of F and $[0,1]$. Let M be a closed oriented 3-manifold. If there exists an orientation-preserving diffeomorphism from $\widehat{M}_f$ to M, the pair (F,f) is called an *open book decomposition* of M. The map f is called the *monodromy* of (F,f). Note that we can regard F as a surface in M. The boundary of F in M, denoted by L, is called a *fibered link* in M, and F is called a *fiber surface* of L. The *monodromy of* L is defined by the monodromy f of the open book decomposition (F,f).

5.2. Twistings and annulus twists

Let M be a closed oriented 3-manifold, and (F,f) an open book decomposition of M. Let C be a simple closed curve on a fiber surface $F \subset M$. Then, *a twisting along C of order n* is defined as performing $(1/n)$-surgery along C with respect to the framing determined by F. Then we obtain the following.

Lemma 5.1 (Stallings). *The resulting manifold obtained from M by a twisting along C of order n is (orientation-preservingly) diffeomorphic to $\widehat{M}_{t_C^{-n}\circ f}$.*

For a proof of this lemma, see Figure 2 (see also [10], [41] and [50]). Lemma 5.1 implies that, by a twisting along C of order n, the fibered link with monodromy f is changed into the fibered link with monodromy $t_C^{-n} \circ f$.

Remark 5.2. Our definition on the pinched mapping torus differs from Bonahon's [10]. We glue $(x,1)$ and $(f(x),0)$ in the pinched mapping torus. On the other hand, $(x,0)$ and $(f(x),1)$ are glued in Bonahon's paper.

Hereafter we only deal with the 3-sphere S^3. Let $A \subset S^3$ be an embedded annulus and $\partial A = c_1 \cup c_2$. Note that A may be knotted and twisted. In Figure 3, we draw an unknotted and twisted annulus. An *n-fold annulus twist along A* is to apply $(+1/n)$-surgery along c_1 and $(-1/n)$-surgery along c_2 with respect to the framing determined by the annulus A. For simplicity, we call a 1-fold annulus twist along A an *annulus twist along A*.

Remark 5.3. An n-fold annulus twist does not change the ambient 3-manifold S^3, (see [6, Lemma 2.1] or [40, Theorem 2.1]). However, each

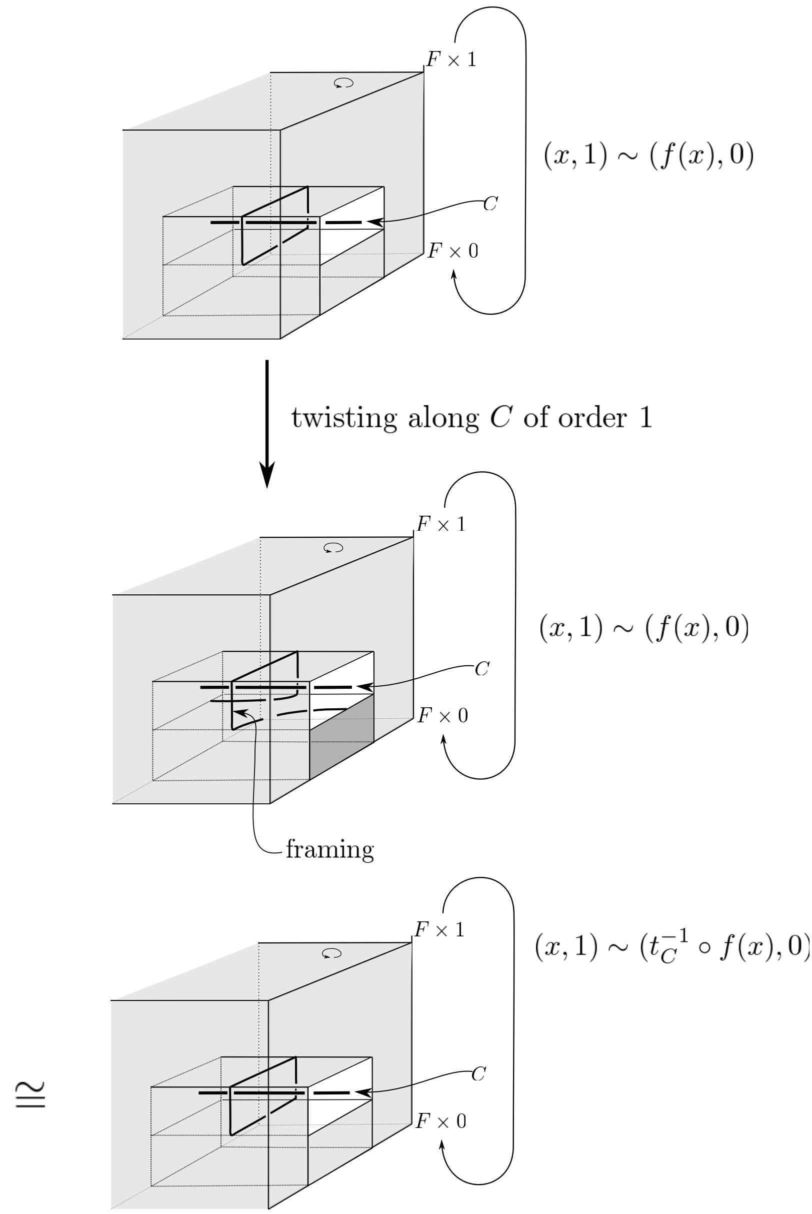

Figure 2: The top picture is $\widehat{M}_f$, the middle picture is the resulting manifold obtained from $\widehat{M}_f$ by a twisting along C of order 1 and the bottom picture is $\widehat{M}_{t_C^{-1} \circ f}$. In the pictures, we remove a tubular neighborhood of C. The last diffeomorphism is given by twisting the deep gray area (which is the solid torus below the neighborhood of C) in the middle picture to the left.

surgery along c_1 (resp. surgery along c_2) often changes the ambient 3-manifold S^3. For example, if A is an unknotted annulus with k full-twists, the n-fold annulus twist along A is to apply $(k + 1/n)$-surgery along c_1 and $(k - 1/n)$-surgery along c_2 with respect to the Seifert framings. Therefore each surgery along c_1 (resp. surgery along c_2) indeed changes the ambient 3-manifold S^3 frequently.

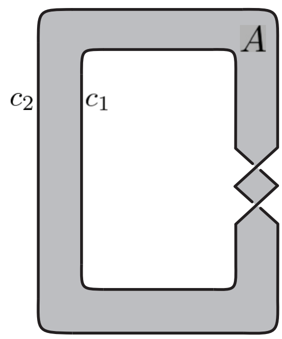

Figure 3: An unknotted annulus $A \subset S^3$ with a $+1$ full-twist.

5.3. Annulus presentations

The first author, Jong, Omae and Takeuchi [1] introduced the notion of an annulus presentation of a knot (in their paper it is called "band presentation"). Here, we extend the definition of annulus presentations of knots.

Let $A \subset S^3$ be an embedded annulus with $\partial A = c_1 \cup c_2$, which may be knotted and twisted. Take an embedding of a band $b\colon I \times I \to S^3$ such that

- $b(I \times I) \cap \partial A = b(\partial I \times I)$,
- $b(I \times I) \cap \operatorname{int} A$ consists of ribbon singularities, and
- $A \cup b(I \times I)$ is an immersion of an orientable surface,

where $I = [0, 1]$. If a knot K is isotopic to the knot $(\partial A \setminus b(\partial I \times I)) \cup b(I \times \partial I)$, then we say that K admits an *annulus presentation* (A, b).

Example 5.4. The knot 6_3 (with an arbitrary orientation) admits an annulus presentation (A, b), see Figure 4.

Let K be a knot admitting an annulus presentation (A, b). Then, by $A^n(K)$, we denote the knot obtained from K by n-fold annulus twist along

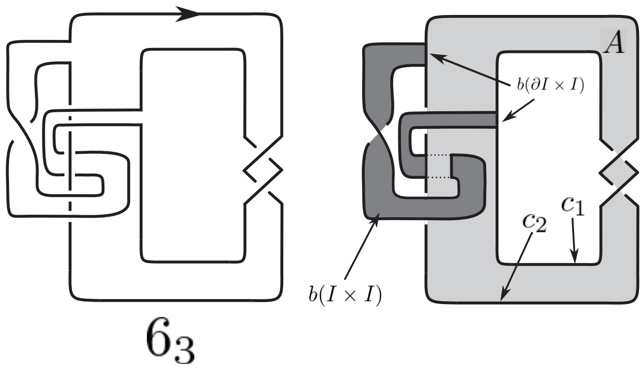

Figure 4: The definitions of the knot 6_3 (left) and its annulus presentation (right).

$\widetilde{A}$ with $\partial\widetilde{A} = \widetilde{c}_1 \cup \widetilde{c}_2$, where $\widetilde{A} \subset A$ is a shrunken annulus. Namely, $\overline{A \setminus \widetilde{A}}$ is a disjoint union of two annuli, each $\widetilde{c}_i$ is isotopic to c_i in $\overline{A \setminus \widetilde{A}}$ for $i = 1, 2$ and $A \setminus (\partial A \cup \widetilde{A})$ does not intersect $b(I \times I)$. For simplicity, we denote $A^1(K)$

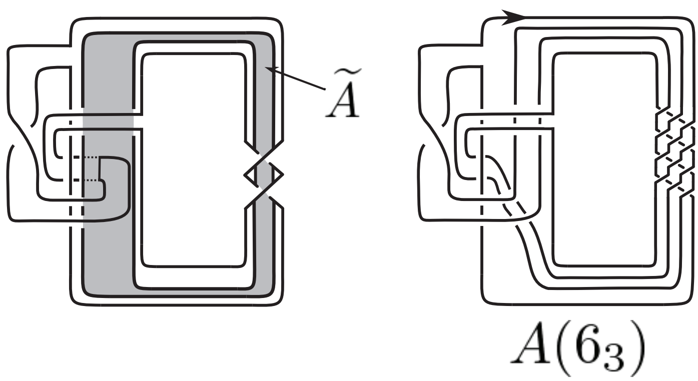

Figure 5: A shrunken annulus $\tilde{A}$ for the annulus presentation of 6_3 (left) and the knot $A(6_3)$ (right).

by $A(K)$.

Example 5.5. We consider the knot 6_3 with the annulus presentation (A, b) in Figure 4. Then $A(6_3)$ is the right picture in Figure 5.

The following important lemma is a special case of Osoinach's result [40, Theorem 2.3].

Lemma 5.6. *Let K be a knot admitting an annulus presentation (A,b). Then, the 3-manifold $M_{A^n(K)}(0)$ does not depend on $n \in \mathbf{Z}$.*

5.4. Compatible annulus presentations and twistings

Let $K \subset S^3$ be a fibered knot admitting an annulus presentation (A,b), and F a fiber surface of K. We say that (A,b) is *compatible with* F if there exist simple closed curves c_1' and c_2' on F such that

- $\partial\widetilde{A} = \widetilde{c}_1 \cup \widetilde{c}_2$ is isotopic to $c_1' \cup c_2'$ in $S^3 \setminus K$, where $\widetilde{A} \subset A$ is a shrunken annulus defined in Section 5.3, and
- each annular neighborhood of c_i' in F $(i = 1,2)$ is isotopic to A in S^3.

Let $\widetilde{c}_1 \cup \widetilde{c}_2$ be the framed link with framing $(1/n, -1/n)$ with respect to the framing determined by the annulus A, and $c_1' \cup c_2'$ the framed link with framing $(1/n, -1/n)$ with respect to the framing determined by the fiber surface F. Then, by the first compatible condition, $\widetilde{c}_1 \cup \widetilde{c}_2$ is equal to $c_1' \cup c_2'$ as links in $S^3 \setminus K$. Moreover, by the second compatible condition, their framings coincide. As a result, $\widetilde{c}_1 \cup \widetilde{c}_2$ is equal to $c_1' \cup c_2'$ as framed links in $S^3 \setminus K$. Hence, if K is a fibered knot with (A,b) which is compatible with the fiber surface F, then $A^n(K)$ is the knot obtained from K by twistings along c_1' and c_2' of order $+n$ and $-n$, respectively. In particular, $A^n(K)$ is a fibered knot and the monodromy of $A^n(K)$ is $t_{c_1'}^{-n} \circ t_{c_2'}^{n} \circ f$, where f is the monodromy of K. As a summary, we obtain the following.

Lemma 5.7. *Let $K \subset S^3$ be a fibered knot admitting a compatible annulus presentation (A,b). Then $A^n(K)$ is also fibered for any $n \in \mathbf{Z}$. Moreover, the monodromy of $A^n(K)$ is*

$$t_{c_1'}^{-n} \circ t_{c_2'}^{n} \circ f,$$

where f is the monodromy of K, and c_1' and c_2' are simple closed curves which give the compatibility of (A,b).

Example 5.8. We consider the knot 6_3 with the annulus presentation (A,b) in Figure 4. It is known that 6_3 is fibered. We choose a fiber surface as in the left picture in Figure 6, and denote it by F. In this case, the annulus presentation (A,b) is compatible with F. Indeed we define simple closed curves c_1' and c_2' on F by $\widetilde{c}_1$ and $\widetilde{c}_2$, where $\partial\widetilde{A} = \widetilde{c}_1 \cup \widetilde{c}_2$. Then $c_1' \cup c_2'$ clearly satisfies the compatible conditions.

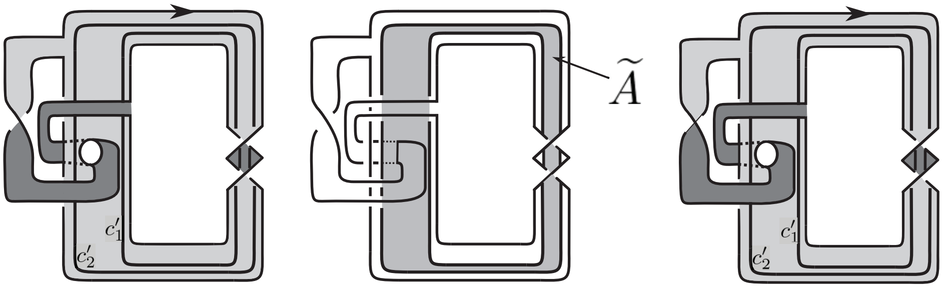

Figure 6: A fiber surface F of 6_3 (left) and a shrunken annulus $\widetilde{A}$ (center). The annulus presentation (A, b) of 6_3 is compatible with the fiber surface F (right).

Remark 5.9. Let K_1 and K_2 be knots which have the same 0-surgery. Gabai [20] proved that if K_1 is fibered, then K_2 is also fibered. Here let K be a fibered knot admitting an annulus presentation (A, b) (which may not be compatible with the fiber surface for K). Then, by Lemma 5.6 and the above fact (Gabai's theorem), $A^n(K)$ is also fibered.

5.5. The monodromy of $A^n(6_3)$

At first, we describe the monodromy of 6_3. We draw a fiber surface of 6_3 as a plumbing of some Hopf bands (see Figure 7). From Figures 7 and 9, the monodromy of 6_3 is given by $t_d^{-1} \circ t_b \circ t_c^{-1} \circ t_a$.

Now we describe the monodromy of $A^n(6_3)$. From Figures 8, 9, and Lemma 5.7, the monodromy f_n of $A^n(6_3)$ is given by $t_{c_1'}^{-n} \circ t_{c_2'}^{n} \circ t_d^{-1} \circ t_b \circ t_c^{-1} \circ t_a$.

Let K_n be the fibered knot $A^n(6_3)$. Then the closed monodromies $\widehat{f_n}$ are conjugate with each other. It follows from two facts:

1) 0-surgeries on K_n are the same 3-manifold (which is the surface bundle over S^1 with monodromy $\widehat{f_n}$ and whose first Betti number is one).

2) The monodromy of any surface bundle over S^1 with first Betti number one is unique up to conjugation.

Hence, the closed monodromies $\widehat{f_n}$ do not distinguish the knots K_n. On the other hand, we see that the monodromies f_n distinguish the knots K_n by Remark 5.10 below.

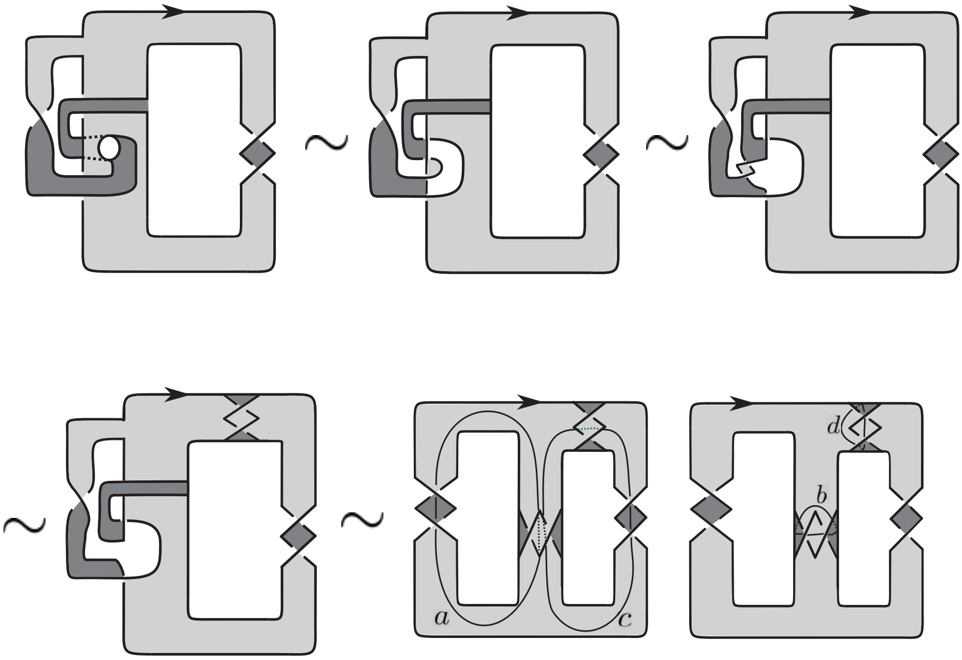

Figure 7: The bottom right pictures are fiber surfaces of 6_3 given by a plumbing of some Hopf bands. The loops a, b, c and d are core lines of these Hopf bands.

Remark 5.10. Let ξ_n be the contact structure on S^3 supported by the open book decomposition (F, f_n). Oba told us that

$$d_3(\xi_n) = -n^2 - n + \frac{3}{2},$$

where d_3 is the invariant of plane fields given by Gompf [22]. In order to compute $d_3(\xi_n)$, he used the formula for d_3 introduced in [15, 17], see [3] for the details. By this computation, if K_n and K_m are the same fibered knots, then $n = m$ or $n + m = -1$. Moreover if $n + m = -1$, we can check that K_n and K_m are the same fibered knots. As a result, we see that K_n and K_m are the same fibered knots if and only if $n = m$ or $n + m = -1$. In particular, knots K_n ($n \geq 0$) are mutually distinct.

For a knot K with an annulus presentation (A, b), in general, it is hard to distinguish $A^n(K)$ and $A^m(K)$. Indeed, they have the same Alexander modules. Osoinach [40] and Teragaito [51] used the hyperbolic structure of the complement of $A^n(K)$ to solve the problem (more precisely, they

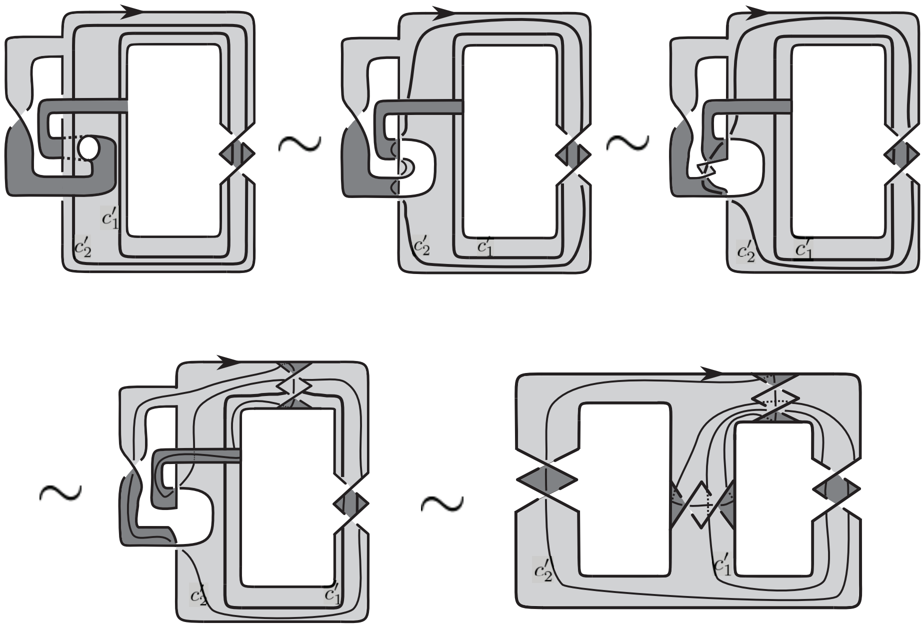

Figure 8: The simple closed curves c_1' and c_2' on the fiber surface of 6_3.

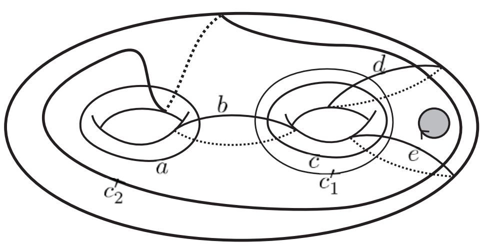

Figure 9: The monodromy f_n of $A^n(6_3)$ is $t^n_{c_2'} \circ t^{-n}_{c_1'} \circ t_d^{-1} \circ t_b \circ t_c^{-1} \circ t_a$. This is equal to $t_a^{-1} \circ t_b^{-1} \circ t_e \circ t_c^n \circ t_e^{-1} \circ t_b \circ t_a \circ t_c^{-n} \circ t_d^{-1} \circ t_b \circ t_c^{-1} \circ t_a$, where e is the circle depicted in this picture.

considered the hyperbolic volume of $A^n(K_n)$). On the other hand, in Oba's method, we consider contact structures.

Remark 5.11. In the proof of Theorem 1.6, we proved that $K_0\#\overline{K_1}$ is not ribbon. By the same discussion, if $K_n \neq K_m$, we also see that $K_n\#\overline{K_m}$ is not ribbon and it is a counterexample for either Conjecture 1.5 or the slice-ribbon conjecture. In particular, by Remark 5.10, we obtain infinitely many fibered potential counterexamples to the slice-ribbon conjecture by utilizing annulus twists.

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