

# Non-squeezing property for holomorphic symplectic structures

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We prove a Gromov’s non-squeezing type theorem in holomorphic symplectic geometry. We further define holomorphic symplectic invariants on Grassmanian bundles and use them to give a more general version of the non-squeezing theorem.

## 1. Introduction

Most of the studies in holomorphic symplectic geometry have been focused on algebraic geometry, dealing with compact holomorphic symplectic manifolds. The construction of such manifolds is related to Beauville’s decomposition theorem asserting that compact holomorphic symplectic manifolds may be regarded as ”building blocks” of the Calabi-Yau manifolds (see [1]). Some recent studies have been inspired by real symplectic geometry, in view of Hamiltonian dynamics (see, for instance [7, 8]). Our paper focuses on some open holomorphic symplectic manifolds. The aim is to prove a holomorphic symplectic version of Gromov’s seminal work [6] for the non-squeezing property on the real symplectic invariant theory.

In [6], Gromov proved that if there exists a real symplectic embedding from the real  $(2n + 2)$ -dimensional ball  $D^{2n+2}(r)$  of radius  $r$  to the cylinder  $D^2(R) \times \mathbb{R}^{2n}$  for the standard real symplectic form, then  $r$  cannot exceed  $R$ . This result is called the non-squeezing theorem. There is no direct complex analogue of Gromov’s non-squeezing theorem due to the lack of volume invariance in complex geometry; a holomorphic bijective map may not preserve the area. Holomorphic symplectic structures are much more rigid structures than both real symplectic structures and complex structures. Theorem 1.1 is a quantitative holomorphic version of the real non-squeezing property that emphasizes the rigidity of such structures. For a positive integer  $N$  we denote by  $B^N(R)$  the ball centered at 0 in  $\mathbb{C}^N$ , with radius  $R > 0$ .

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We also denote by  $\sigma_0$  the standard holomorphic symplectic structure on the even dimensional complex Euclidean spaces. (See Section 3 for explicit definition.)

**Theorem 1.1.** *Let  $(M, \sigma)$  be a holomorphic symplectic manifold of dimension  $2n$ . Suppose that there exists a holomorphic symplectic map*

$$F : (B^{2n+2}(r), \sigma_0) \rightarrow (B^2(R) \times M, \sigma_0 \oplus \sigma).$$

*Let  $\pi : B^2(R) \times M \rightarrow B^2(R)$  be the natural projection. Then if  $\pi \circ F(0) = p \in B^2(R)$ , we have*

$$(1.1) \quad r \leq \frac{(R^2 - |p|^2)^{3/4}}{R^{1/2}}.$$

*In particular,  $r \leq R$  and if  $r = R$ , then  $\pi \circ F(0) = 0$ .*

We first point out that as in real symplectic geometry, the Darboux theorem holds in holomorphic symplectic geometry : for any point  $p$  in a  $2n$ -dimensional holomorphic symplectic manifold  $(M, \sigma)$ , there exists a neighborhood of  $p$  and a holomorphic symplectic embedding from that neighborhood into  $(\mathbb{C}^{2n}, \sigma_0)$ . Therefore, holomorphic symplectic manifolds do not have any local invariants.

In contrast with the real case, Theorem 1.1 provides a finer estimate on the radius of the embedded ball by the location of the image of the origin. This shows the strong rigidity of holomorphic symplectic structures.

Crucial ingredients in our approach are provided by intrinsic complex two-dimensional area invariants since complex two-dimensional submanifolds may be preserved by holomorphic symplectic maps. These area invariants are known as Carathéodory and Kobayashi 2-volumes. These are generalizations of intrinsic pseudo-distances defined on complex manifolds, known as the Carathéodory and the Kobayashi pseudodistances, see [2, 4, 10, 11]. Important properties of complex manifolds rely on the properties of these distances or volume forms (see for instance [3, 5, 9, 13–15] for some of these applications).

Our proof of the non-squeezing theorem is obtained by a comparison of these two-dimensional holomorphic invariants. Exploiting these invariant 2-forms, we also define in Section 4 some holomorphic symplectic invariants on the Grassmanian bundles of holomorphic symplectic manifolds, and obtain a slight generalization of Theorem 1.1.

We are indebted to professor Jun-Muk Hwang for bringing that question to our attention.

## 2. Carathéodory and Kobayashi $k$ -volumes

Let  $M$  be a complex manifold of dimension  $N$ . Let  $k$  be an integer satisfying  $1 \leq k \leq N$ . Denote by  $B^k$  the unit ball in  $\mathbb{C}^k$  and by  $\{e_1, \dots, e_k\}$  the standard complex basis of  $\mathbb{C}^k$ .

Let  $z \in M$  and let  $v_1, \dots, v_k$  be  $k$  vectors in the complex tangent space  $T_z M$  of  $M$  at  $z$ .

We have the following

**Definition 2.1.** (i) The *Carathéodory  $k$ -pseudovolume* of  $v_1 \wedge \dots \wedge v_k$  is defined by

$$c_M(z, v_1 \wedge \dots \wedge v_k) := \sup\{|\lambda|, \quad \lambda \in \mathbb{C},$$

such that there is a holomorphic map  $f : M \rightarrow B^k$  satisfying

$$f(z) = 0, \quad df_z(v_1) \wedge \dots \wedge df_z(v_k) = \lambda e_1 \wedge \dots \wedge e_k\}.$$

(ii) The *Kobayashi  $k$ -pseudovolume* is defined by

$$k_M(z, v_1 \wedge \dots \wedge v_k) := \inf\{|\lambda|, \quad \lambda \in \mathbb{C},$$

such that there is a holomorphic map  $f : B^k \rightarrow M$  satisfying

$$f(0) = z, \quad \lambda df_0(e_1) \wedge \dots \wedge df_0(e_k) = v_1 \wedge \dots \wedge v_k\}.$$

First notice that both  $c_M$  and  $k_M$  are well-defined. In case  $k = 1$ ,  $c_M$  and  $k_M$  are called the Carathéodory and the Kobayashi pseudometrics, respectively. In case  $k = N$ , they are called the Carathéodory-Eisenman and the Kobayashi-Eisenman pseudovolumes.

**Lemma 2.2.** *The  $k$ -volume forms  $c_M^k$  and  $k_M^k$  are  $U(k)$ -invariant on each  $k$ -complex dimensional space.*

*Proof.* Let  $v_1, \dots, v_k$  be  $k$  vectors on  $T_z M$ . Suppose that  $w_1, \dots, w_k$  are vectors such that

$$w_j = A_j^i v_i$$

for some  $(k \times k)$ -complex matrix  $A = (A_j^i)$ . Then it turns out that

$$c_M(z, w_1 \wedge \dots \wedge w_k) = |\det(A)| c_M(z, v_1 \wedge \dots \wedge v_k)$$

and

$$k_M(z, w_1 \wedge \dots \wedge w_k) = |\det(A)| k_M(z, v_1 \wedge \dots \wedge v_k).$$

In particular,  $c_M$  and  $k_M$  are  $U(k)$ -invariant on each fixed  $k$ -plane. □

**Remark 2.3.** Let  $G(k, M)$  denote the  $k$ -planes Grassmannian bundle, over  $M$ , of complex  $k$ -dimensional subspaces. For each  $p \in M$ , the fiber  $G_p(k, M)$  of  $G(k, M)$  consists of all  $k$ -dimensional subspaces of  $T_pM$ , the complex tangent space of vectors of type  $(1, 0)$ . We denote by  $L$  the tautological complex line bundle over  $G(k, M)$ , that is, for every  $p \in M$  and  $H \in G_p(k, M)$ , the fiber  $L_H$  of  $L$  at  $H$  is defined by

$$L_H = \{\lambda v_1 \wedge \cdots \wedge v_k : \lambda \in \mathbb{C}\}$$

where  $v_1, \dots, v_k$  form a basis for  $H$ . Then it turns out that  $c_M$  and  $k_M$  define (possibly degenerate) Hermitian structures on  $L$ . We call a Hermitian structure on  $L$  a *Finsler  $k$ -volume* on  $M$ .

Both  $c_M$  and  $k_M$  have the following important invariance property.

**Lemma 2.4.** *Let  $M$  and  $M'$  be complex manifolds and let  $F : M \rightarrow M'$  be a holomorphic mapping. Assume  $\dim_{\mathbb{C}} M$  and  $\dim_{\mathbb{C}} M'$  are greater than or equal to  $k \geq 1$ . Then :*

$$c_{M'}(F(p), dF_p(v_1) \wedge \cdots \wedge dF_p(v_k)) \leq c_M(p, v_1 \wedge \cdots \wedge v_k)$$

and

$$k_{M'}(F(p), dF_p(v_1) \wedge \cdots \wedge dF_p(v_k)) \leq k_M(p, v_1 \wedge \cdots \wedge v_k)$$

for every  $p \in M$  and every  $v_1, \dots, v_k \in T_pM$ . In particular,  $c_M$  and  $k_M$  are invariant under the action of biholomorphic maps.

The proof of the Lemma 2.4 follows directly from the definitions of  $c_M$  and  $k_M$ .

The following Proposition gives the exact value of the Carathéodory  $k$ -volume in the unit ball  $B^N$ ; that will be essential for the proof of Theorem 1.1.

**Proposition 2.5.** *Denote by  $(e_1, \dots, e_N)$  the standard basis of  $\mathbb{C}^N$  and by  $B^N(R)$  the ball centered at 0 with radius  $R > 0$  in  $\mathbb{C}^N$ . Let  $k$  be a positive integer satisfying  $1 \leq k \leq N$ . Then for every  $p \in B^N(R)$  we have :*

$$c_{B^N}(p, e_1 \wedge \cdots \wedge e_k) = \frac{R}{(R^2 - |p|^2)^{\frac{k+1}{2}}}.$$

*Proof.* Let  $f$  be a holomorphic mapping from  $B^N$  to  $B^k$  such that  $f(0) = 0$ . If we denote by  $\tilde{f}$  the restriction of  $f$  to the  $k$ -dimensional space  $\text{span}(e_1, \dots, e_k)$ , then we have

$$df_0(e_j) = d\tilde{f}_0(e_j)$$

for every  $1 \leq j \leq k$ . Therefore

$$df_0(e_1) \wedge \cdots \wedge df_0(e_k) = d\tilde{f}_0(e_1) \wedge \cdots \wedge d\tilde{f}_0(e_k).$$

In particular, according to Cartan's Lemma, the condition

$$df_0(e_1) \wedge \cdots \wedge df_0(e_k) = \lambda e_1 \wedge \cdots \wedge e_k$$

implies that

$$|\lambda| \leq 1.$$

Moreover, since  $|\lambda| = 1$  is achieved by the natural projection  $\pi$  from  $\mathbb{C}^N$  to  $\mathbb{C}^k$ , we conclude that

$$c_{B^N}(0, e_1 \wedge \cdots \wedge e_k) = 1.$$

Since the unitary group  $U(N)$  is a subgroup of the holomorphic automorphism group  $\text{Aut}(B^N)$ , we see that

$$c_{B^N}(0, v_1 \wedge \cdots \wedge v_k) = 1$$

for every  $v_1, \dots, v_k$  that form an orthonormal family for the Euclidean Hermitian structure.

Finally, by considering the dilation map  $z \mapsto rz$  from  $B^N$  to the ball  $B^N(r)$  centered at 0 with radius  $r > 0$ , we also have

$$c_{B^N(r)}(0, v_1 \wedge \cdots \wedge v_k) = \frac{1}{r^k}$$

for any orthonormal family of vectors  $v_1, \dots, v_k$ .

Let now  $p \in B^N$ . Then by a unitary change, we may assume that  $p = (a, 0, \dots, 0)$  for some  $a \in \mathbb{C}$  with  $|a| < 1$ . If we denote by  $\phi$  the automorphism

of  $B^N$  defined by

$$\phi(z^1, \dots, z^N) = \left( \frac{z^1 - a}{1 - \bar{a}z^1}, \frac{\sqrt{1 - |a|^2}z^2}{1 - \bar{a}z^1}, \dots, \frac{\sqrt{1 - |a|^2}z^N}{1 - \bar{a}z^1} \right),$$

then  $\phi(p) = 0$ . Moreover, we have

$$d\phi_p(e_1) = \frac{1}{1 - |a|^2} e_1$$

and

$$d\phi_p(e_j) = \frac{1}{\sqrt{1 - |a|^2}} e_j$$

for every integer  $2 \leq j \leq N$ . Therefore

$$\begin{aligned} c_{B^N}(p, e_1 \wedge \dots \wedge e_k) &= c_{B^N}(0, d\phi_p(e_1) \wedge \dots \wedge d\phi_p(e_k)) \\ &= (1 - |a|^2)^{-\frac{k+1}{2}}. \end{aligned}$$

Finally for  $p \in B^N(R)$  we have

$$\begin{aligned} c_{B^N(R)}(p, e_1 \wedge \dots \wedge e_k) &= \frac{1}{R^k} c_{B^N} \left( \frac{p}{R}, e_1 \wedge \dots \wedge e_k \right) \\ &= \frac{1}{R^k} \left( 1 - \left| \frac{p}{R} \right|^2 \right)^{-\frac{k+1}{2}} \\ &= \frac{R}{(R^2 - |p|^2)^{\frac{k+1}{2}}}. \end{aligned}$$

□

Notice that similar computations are valid for  $k_{B^N}$ .

### 3. Proof of Theorem 1.1

A *holomorphic symplectic structure* is by definition a non-degenerate closed holomorphic 2-form  $\sigma$  defined on a complex manifold  $M$ . We call  $(M, \sigma)$  a *holomorphic symplectic manifold*. The simplest and most important example is the standard holomorphic symplectic form  $\sigma_0$  on  $\mathbb{C}^{2n}$  defined by

$$\sigma_0 = dz^1 \wedge dw^1 + \dots + dz^n \wedge dw^n,$$

where  $(z^1, w^1, \dots, z^n, w^n)$  denote the standard complex coordinates of  $\mathbb{C}^{2n}$ .

**Definition 3.1.** Let  $(M, \sigma)$  and  $(M', \sigma')$  be two holomorphic symplectic manifolds. A holomorphic map  $f$  from  $(M, \sigma)$  to  $(M', \sigma')$  is called a *holomorphic symplectic map* if it satisfies

$$(3.1) \quad f^* \sigma' = \sigma.$$

Obviously, it is assumed  $\dim M \leq \dim M'$ .

**Remark 3.2.** In Definition 3.1, we do not need to assume  $f$  is holomorphic at the beginning step. One can easily show that a  $C^1$ -smooth function  $f$  satisfying (3.1) is automatically holomorphic.

The proof of Theorem 1.1 proceeds as follows. First notice that the group of holomorphic symplectic automorphisms of  $B^2$  is the special unitary group  $SU(2)$ . Therefore, if  $F(0) = (p, q) \in B^2 \times M$ , then by a  $SU(2)$  change of coordinates, we may assume that  $p = (a, 0)$  for some  $a \in \mathbb{C}$  with  $|a| < 1$ . Indeed let  $v$  be a unit vector in  $\mathbb{C}^{2n}$  such that  $dF_0(v) = \lambda e_1$  for some  $\lambda \in \mathbb{C} \setminus \{0\}$ , then we may assume that  $v = e_1$ . In fact, the holomorphic symplectic automorphism group of  $B^{2n}(r)$  contains two special subgroups. The first one is  $SU(2) \times \cdots \times SU(2)$  such that each the  $j$ -th  $SU(2)$ -block acts on the plane  $\text{span}(e_{2j-1}, e_{2j})$ . The second one is  $U(n)$  acting on  $B^{2n}(r)$  by  $(z, w) \rightarrow (Az, \overline{A^t w})$  for  $A \in U(n)$ . Combining these linear automorphisms, we can transform every unit vector in  $\mathbb{C}^{2n}$  to  $e_1$ . Therefore, we may assume without loss of generality that

$$\pi \circ F(0) = p = (a, 0) \in B^2, \quad dF_0(e_1) = \lambda e_1.$$

Since  $F$  is a holomorphic symplectic embedding, then we have

$$dF_0(e_2) = (\lambda^{-1} e_2 + \mu e_1, V)$$

for some  $\mu \in \mathbb{C}$  and some vector  $V$  tangent to  $M$ . Therefore,  $d(\pi \circ F)_0(e_1) = \lambda e_1$ ,  $d(\pi \circ F)_0(e_2) = \lambda^{-1} e_2 + \mu e_1$  and hence

$$d(\pi \circ F)_0(e_1) \wedge d(\pi \circ F)_0(e_2) = e_1 \wedge e_2.$$

Therefore,

$$\begin{aligned} \frac{1}{r^2} &= c_{B^{2n}(r)}(0, e_1 \wedge e_2) \\ &\geq c_{B^2(R)}(\pi \circ F(0), d(\pi \circ F)_0(e_1) \wedge d(\pi \circ F)_0(e_2)) \\ &= c_{B^2(R)}(p, e_1 \wedge e_2) = \frac{R}{(R^2 - |p|^2)^{\frac{3}{2}}} \end{aligned}$$

by Lemma 2.4 and Proposition 2.5. This yields Statement (1.1). □

### 4. Holomorphic symplectic invariants and a generalization of the non-squeezing theorem

Let  $(M, \sigma)$  be a  $2n$ -dimensional holomorphic symplectic manifold. We denote by  $\tilde{G}(2, M)$  the complement of the set of isotropic complex planes in the Grassmannian bundle  $G(2, M)$  over  $M$ . For  $p \in M$  and  $H \in \tilde{G}_p(2, M)$ , we define

$$\tilde{c}_M(p, H) := \frac{c_M(p, v_1 \wedge v_2)}{|\sigma(v_1 \wedge v_2)|},$$

where  $v_1, v_2$  is a basis for  $H$ . Obviously, this definition does not depend on the choice of basis, and  $\tilde{c}_M$  is well-defined. Similarly, one can define  $\tilde{k}_M$  on  $\tilde{G}(2, M)$ . By definition, it is immediate that

$$\tilde{c}_M(p, H) \geq \tilde{c}_{M'}(f(p), f_*(H)),$$

for every holomorphic symplectic map  $f : (M, \sigma) \rightarrow (M', \sigma')$ . Hence if  $f$  is a biholomorphic symplectic mapping, then

$$\tilde{c}_M(p, H) = \tilde{c}_{M'}(f(p), f_*(H)).$$

In particular, if we define

$$C_M(p) = \inf\{\tilde{c}_M(p, H) : H \in \tilde{G}_p(2, M)\},$$

then there exists no holomorphic symplectic map from  $(M, \sigma)$  to another  $(M', \sigma')$  sending  $p \in M$  to  $p' \in M'$  if

$$C_M(p) < C_{M'}(p').$$

We will call  $C_M$  the *Carathéodory invariant* for  $(M, \sigma)$ . Obviously, one can also define the *Kobayashi invariant*  $K_M(p)$  in the same way.



Now we extend our definition of holomorphic symplectic mappings to the case where the dimension of the target manifold is less than that of the source manifold.

**Definition 4.1.** Let  $(M, \sigma)$  and  $(M', \sigma')$  be two holomorphic symplectic manifolds of dimension  $2n$  and  $2n'$ , respectively. Assume that  $n' \leq n$ . For  $p \in M$ , let  $V$  be a  $2n'$ -dimensional symplectic subspace of  $T_pM$ . We call a holomorphic mapping  $f : M \rightarrow M'$  a *holomorphic symplectic map with respect to  $V$  at  $p$* , if  $f^*\sigma'_{|V} = \sigma_{|V}$ . If  $\mathcal{V}$  is a distribution of  $2n'$ -dimensional symplectic subspaces on  $M$  and if  $f : M \rightarrow M'$  is a surjective holomorphic symplectic map with respect to  $\mathcal{V}_p$  for every  $p$ , we call  $f$  a *holomorphic symplectic submersion with respect to  $\mathcal{V}$* .

Let  $p \in M$  and let  $V$  be a symplectic  $2n'$ -dimensional subspace of  $T_pM$ . We define a *relative Carathéodory invariant*  $C_M(p; V)$  for  $V$  by

$$C_M(p; V) = \inf\{\tilde{c}_M(p, H) : H \in \tilde{G}_p(2, M), H \subset V\}.$$

With that notation, we obtain the following generalization of Theorem 1.1.

**Theorem 4.2.** *Let  $(M, \sigma)$  and  $(M', \sigma')$  be two holomorphic symplectic manifolds of dimension  $2n$  and  $2n'$ , respectively. Assume that  $n' \leq n$ . For given points  $p \in M$  and  $p' \in M'$ , suppose that*

$$C_M(p, V) < C_{M'}(p')$$

*for some symplectic  $2n'$ -dimensional subspace  $V$  in  $T_pM$ . Then there exists no holomorphic symplectic map  $f : (M, \sigma) \rightarrow (M', \sigma')$  with respect to  $V$  at  $p$  which maps  $p$  to  $p'$ .*

We consider now the Euclidean case  $(\mathbb{C}^{2n}, \sigma_0)$ . Let  $H$  be a 2-plane in  $\mathbb{C}^{2n}$ . Let  $v_1$  and  $v_2$  be an orthonormal basis for  $H$ . By a unitary symplectic linear change of coordinates, we assume that  $v_1 = e_1$ . Let  $v_2 = \sum_j a_j e_j$  where  $\sum |a_j|^2 = 1$ . Then

$$a_2 = \sigma_0(e_1, v_2).$$

The quantity  $|a_2|$  measures the defect for the plane  $H$  to be an isotropic 2-plane. We denote it by  $\lambda(H)$  and we call it the *symplectic cosine* of  $H$ . Obviously, it does not depend on the choice of an orthonormal basis for  $H$ , it satisfies the inequalities  $0 \leq \lambda(H) \leq 1$  and  $\lambda(H) = 0$  means that  $H$  is isotropic.

**Corollary 4.3.** *Let  $H$  be a non-isotropic 2-plane in  $\mathbb{C}^{2n}$  with symplectic cosine  $\lambda(H)$ . Suppose that there exists a holomorphic symplectic mapping  $f : B^{2n}(r) \rightarrow B^2(R)$  with respect to  $H$  at 0 and let  $p = f(0) \in B^2(R)$ . Then*

$$r^2 \leq \frac{(R^2 - |p|^2)^{3/2}}{R} \frac{1}{\lambda(H)}.$$

*Proof.* Let  $v_1$  and  $v_2$  be an orthonormal basis for  $H$ . Then  $\sigma_0(v_1, v_2) = \lambda(H)$ . Therefore, by Proposition 2.5,

$$C_{B^{2n}(r)}(0, H) = \frac{c_{B^{2n}(r)}(0, v_1 \wedge v_2)}{\sigma_0(v_1 \wedge v_2)} = \frac{1}{\lambda(H)r^2}.$$

On the other hand,

$$C_{B^2(R)}(p) = \frac{R}{(R^2 - |p|^2)^{3/2}}$$

by Proposition 2.5. Then Theorem 4.2 yields the conclusion.  $\square$

Notice that if  $F : (B^{2n+2}(r), \sigma_0) \rightarrow (B^2(R) \times M, \sigma_0 \oplus \sigma)$  is a holomorphic symplectic mapping for some  $2n$ -dimensional holomorphic symplectic manifold  $(M, \sigma)$ , then  $f := \pi \circ F : (B^{2n+2}(r), \sigma_0) \rightarrow (B^2(R), \sigma_0)$  is a holomorphic symplectic mapping with respect to a symplectic plane  $H \subset T_0 B^{2n+2}(r) = \mathbb{C}^{2n+2}$  with  $\lambda(H) = 1$ . Notice that as we have seen in the proof of Theorem 1.1, we may assume that  $H$  is  $\text{span}(e_1, e_2)$ . Hence Theorem 1.1 is a consequence of Corollary 4.3.

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