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EQUIVARIANT CHERN NUMBERS AND THE NUMBER OF FIXED POINTS FOR UNITARY TORUS MANIFOLDS

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Abstract. Let M^{2n} be a unitary torus $(2n)$-manifold, i.e., a $(2n)$-dimensional oriented stable complex connected closed T^n-manifold having a nonempty fixed point set. In this paper, we show that M bounds equivariantly if and only if the equivariant Chern numbers $\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = 0$ for all $i, j \in \mathbb{N}$, where $c_l^{T^n}$ denotes the lth equivariant Chern class of M. As a consequence, we also show that if M does not bound equivariantly then the number of fixed points is at least $\lceil \frac{n}{2} \rceil + 1$.

1. Introduction

Let T^n denote the torus of rank n. An *oriented stable complex closed T^n-manifold* is an oriented closed smooth manifold M with an effective T^n-action such that its tangent bundle admits a T^n-equivariant stable complex structure. It is well known from [2] that the equivariant cobordism class of an oriented stable complex closed T^n-manifold with isolated fixed points is completely determined by its equivariant Chern numbers. In this paper, we shall pay more attention on the oriented stable complex (connected) closed T^n-manifolds of dimension $2n$ with nonempty fixed point set, which are also called the *unitary torus manifolds* or *unitary toric manifolds* (see [3] and [6]). These geometrical objects are the topological analogues of compact non-singular toric varieties, and constitute a much wider class than that of quasi-toric manifolds introduced by Davis and Januszkiewicz in [1]. Also, the nonempty fixed point set of a unitary torus manifold must be isolated since the action is assumed to be effective. In this case, we shall show that the equivariant cobordism class of a unitary torus manifold is determined by only those equivariant Chern numbers produced by the first and the second equivariant Chern classes. Our result is stated as follows.

Theorem 1.1. *Let M be a unitary torus manifold. Then M bounds equivariantly if and only if the equivariant Chern numbers $\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = 0$ for all $i, j \in \mathbb{N}$, where $[M]$ is the fundamental class of M with respect to the given orientation.*

In [4], Kosniowski studied unitary S^1-manifolds and got some interesting results on the fixed points of the action, where "unitary" means that the tangent bundle of M admits an S^1-equivariant stable complex structure. In particular, when the fixed points are isolated, he proposed the following conjecture.

Conjecture 1.1 (Kosniowski)**.** Suppose that M^{2n} is a unitary S^1-manifold with isolated fixed points. If M does not bound equivariantly then the number of fixed points is greater than $f(n)$, where $f(n)$ is some linear function.

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Remark 1.1. As was noted by Kosniowski in [4], the most likely function is $f(n) = \frac{n}{2}$, so the number of fixed points of M^{2n} is at least $[\frac{n}{2}] + 1$.

With respect to this conjecture, recently some related works have been done. For example, Pelayo and Tolman in [8] studied compact symplectic manifolds with symplectic circle actions, and proved that if the weights induced from the isotropy representations on the fixed points of such an S^1-manifold satisfy some subtle condition, then the action has at least $n + 1$ isolated fixed points. In [5], Ping Li and Kefeng Liu showed that if M^{2mn} is an almost complex manifold and there exists a partition $\lambda = (\lambda_1, \dots, \lambda_r)$ of weight m such that the corresponding Chern number $\langle (c_{\lambda_1} \cdots c_{\lambda_r})^n, [M]\rangle$ is nonzero, then for any S^1-action on M, it must have at least $n + 1$ fixed points.

In the case of the unitary torus manifolds, comparing with Kosniowski's Conjecture 1.1, we can apply Theorem 1.1 to obtain the following result:

Theorem 1.2. *Suppose that M^{2n} is a $(2n)$-dimensional unitary torus manifold. If M does not bound equivariantly, then the number of fixed points is at least $\lceil \frac{n}{2} \rceil + 1$, where $\lceil \frac{n}{2} \rceil$ denotes the minimal integer no less than $\frac{n}{2}$.*

Remark 1.2. It should be interesting to discuss whether there exists an example of $(2n)$-dimensional unitary torus manifolds, which does not bound equivariantly but has exactly $\lceil \frac{n}{2} \rceil + 1$ isolated fixed points for every n.

2. Preliminaries

2.1. Equivariant Chern characteristic numbers. The *equivariant Chern characteristic numbers* $c_\omega^{T^n}(M)$ of an oriented stable complex closed T^n-manifold M are defined as

$$c_\omega^{T^n}(M) = \langle (c_1^{T^n})^{i_1} \dots (c_k^{T^n})^{i_k}, [M]\rangle \in H^*(BT^n; \mathbb{Z}),$$

where $\omega = (i_1, \dots, i_k)$ is a multi-index and $c_l^{T^n}$ is the lth equivariant Chern class of M. Unlike the ordinary Chern characteristic numbers, these equivariant Chern characteristic numbers can be nonzero polynomials in $H^*(BT^n; \mathbb{Z})$ even if the degree of the product $(c_1^{T^n})^{i_1} \cdots (c_k^{T^n})^{i_k}$ is greater than $\dim M/2$.

If the oriented stable complex closed T^n-manifold M has only isolated fixed points, then it is known from [2] that at each fixed point $p \in M^{T^n}$ the tangent space T_pM is equipped with the induced T^n-action, orientation and complex structure, and the T^n-equivariant cobordism class of M is determined by the complex T^n-representations T_pM at all $p \in M^{T^n}$ and their orientations. Then Guillemin et al. in [2] applied Atiyah–Bott–Berline–Vergne localization theorem to give the following theorem.

Theorem 2.1 (Guillemin–Ginzburg–Karshon)**.** *Let M be an oriented stable complex closed T^n-manifold with isolated fixed points. Then M bounds equivariantly if and only if all equivariant Chern characteristic numbers of M are equal to zero.*

2.2. Unitary torus manifolds and Atiyah–Bott–Berline–Vergne localization theorem. Let M^{2n} be a $(2n)$-dimensional unitary torus manifold. Following [6], we say that a closed, connected, real codimension-2 submanifold of M^{2n} is called *characteristic* if it is a connected component of a fixed point set by some circle subgroup of T^n and contains at least one fixed point of the whole T^n-action. Then M^{2n}

has finitely many such characteristic submanifolds. By $M_i, i \in [m] = \{1, \dots, m\}$ we denote all characteristic submanifolds of M^{2n}, and by ζ_i denote the corresponding normal bundle over M_i, and by T_i denote the circle subgroup fixing M_i pointwise. Then, for each $p \in M^{T^n}$, we can write the tangent T^n-representation at p as

$$T_pM = \bigoplus_{i \in I(p)} \zeta_i|_p,$$

where $I(p) = \{i | p \in M_i\} \subset [m]$ and $\zeta_i|_p$ is the restriction of ζ_i to p. So $|I(p)| = n$. Each M_i may define an element λ_i in the equivariant cohomology $H^2_{T^n}(M; \mathbb{Z})$. Actually, the inclusion $M_i \hookrightarrow M$ may induce an equivariant Gysin homomorphism: $H^*_{T^n}(M_i; \mathbb{Z}) \longrightarrow H^{*+2}_{T^n}(M; \mathbb{Z})$, so that $\lambda_i \in H^2_{T^n}(M; \mathbb{Z})$ can be chosen as the image of the identity in $H^0_{T^n}(M_i; \mathbb{Z})$. It was shown in [6, Theorem 3.1] that the total equivariant Chern characteristic class $c^{T^n}(TM)$ of the tangent bundle TM of M can be expressed as

$$c^{T^n}(TM) = \prod_{i \in [m]} (1 + \lambda_i)$$

in $\widehat{H}^*_{T^n}(M; \mathbb{Z}) = H^*_{T^n}(M; \mathbb{Z})/S$-torsion where S is the subset of $H^*(BT^n; \mathbb{Z})$ generated multiplicatively by nonzero elements of $H^2(BT^n; \mathbb{Z})$. It is well known that the restriction $\lambda_i|_p$ can be regarded as the top equivariant Chern class of $\zeta_i|_p$. Hence, the total equivariant Chern characteristic class of the vector bundle $T_pM \longrightarrow \{p\}$ is

$$c^{T^n}(T_pM) = c^{T^n}(TM)|_p = \prod_{i \in I(p)} (1 + \lambda_i|_p).$$

In particular, Masuda in [6] also showed the following result, which will be very useful in our discussion later.

Lemma 2.1 ([6, Lemma 1.3(1)]). *$\{\lambda_i|_p \big| i \in I(p)\}$ forms a basis of $H^2_{T^n}(\{p\}; \mathbb{Z}) \cong H^2(BT^n; \mathbb{Z})$.*

On the other hand, the normal bundle to p in M^{2n} is T_pM with the orientation inherited from M^{2n}. Thus, the equivariant Euler class $e^{T^n}(T_pM)$ of this bundle is $\pm c_n^{T^n}(T_pM) = \pm \prod_{i \in I(p)} \lambda_i|_p$, where the sign is positive if the orientation of T_pM agrees with the complex orientation and negative otherwise.

Each $c^{T^n}(T_pM) = \prod_{i \in I(p)}(1+\lambda_i|_p) = 1+\sigma_1(p)+\cdots+\sigma_n(p)$ determines a collection $\sigma(p) = (\sigma_1(p), \dots, \sigma_n(p))$, where $\sigma_j(p)$ denotes the jth elementary symmetric function over n variables $\lambda_i|_p, i \in I(p)$. Clearly, $\sigma(p)$ determines the representation T_pM, but not the orientation of T_pM inherited from M.

Now let $\{\sigma(p) \big| p \in M^{T^n}\} = \{\sigma^{(1)}, \dots, \sigma^{(t)}\}$ and for each $1 \le \ell \le t$, set,

$$\begin{aligned} m_\ell = \sharp\{p \in M^{T^n} \big| \sigma(p) = \sigma^{(\ell)}, e^{T^n}(T_pM) = \sigma_n^{(\ell)}\} \\ - \sharp\{p \in M^{T^n} \big| \sigma(p) = \sigma^{(\ell)}, e^{T^n}(T_pM) = -\sigma_n^{(\ell)}\}. \end{aligned}$$

Then we can state the Atiyah–Bott–Berline–Vergne localization theorem in our case as follows:

Theorem 2.2 (A–B–B–V localization theorem)**.** *Let M^{2n} be a $(2n)$-dimensional unitary torus manifold. Then*

$$c_{\omega}^{T^n}(M) = \sum_{p \in M^{T^n}} \frac{\sigma_1(p)^{i_1} \cdots \sigma_n(p)^{i_n}}{\pm\sigma_n(p)} = \sum_{\ell=1}^{t} m_\ell (\sigma_1^{(\ell)})^{i_1} \cdots (\sigma_n^{(\ell)})^{i_n - 1},$$

where $\omega = (i_1, \ldots, i_n)$ is a multi-index.

3. Proofs of main results

First we prove two lemmas that will be used in the proof of Theorem 1.1. Let M^{2n} be a $(2n)$-dimensional unitary torus manifold and let $p, q \in M^{T^n}$ be two fixed points.

Lemma 3.1. *If $\sigma_1(p) = \sigma_1(q)$ and $\sigma_n(p) = \pm\sigma_n(q)$ then $\sigma(p) = \sigma(q)$.*

Proof. If $\sigma_n(p) = \pm\sigma_n(q)$, then $\prod_{i \in I(p)} \lambda_i|_p = \pm \prod_{i \in I(q)} \lambda_i|_q$. So, by Lemma 2.1 we have that $\{\lambda_i|_p \big| i \in I(p)\} = \{\varepsilon_i \lambda_i|_q \big| i \in I(q)\}$ where $\varepsilon_i = \pm 1$. Furthermore, if $\sigma_1(p) = \sigma_1(q)$, then

$$\sigma_1(p) = \sum_{i \in I(p)} \lambda_i|_p = \sum_{i \in I(q)} \varepsilon_i \lambda_i|_q = \sum_{i \in I(q)} \lambda_i|_q = \sigma_1(q)$$

so $\sum_{i \in I(q)} (1 - \varepsilon_i)\lambda_i|_q = 0$. This implies that $\varepsilon_i = 1$ for all $i \in I(q)$ since $\lambda_i|_q, i \in I(q)$ are linearly independent, and the lemma then follows. □

Lemma 3.2. *$\sigma(p) = \sigma(q)$ if and only if $\sigma_1(p) = \sigma_1(q)$ and $\sigma_2(p) = \sigma_2(q)$.*

Proof. It suffices to show that $\sigma(p) = \sigma(q)$ if $\sigma_1(p) = \sigma_1(q)$ and $\sigma_2(p) = \sigma_2(q)$. Consider $s_2(p) = \sum_{i \in I(p)} (\lambda_i|_p)^2$ and $s_2(q) = \sum_{i \in I(q)} (\lambda_i|_q)^2$. If $\sigma_1(p) = \sigma_1(q)$ and $\sigma_2(p) = \sigma_2(q)$, then $s_2(p) = s_2(q)$ since $s_2 = \sigma_1^2 - 2\sigma_2$ by [7]. Since both $\{\lambda_i|_p \big| i \in I(p)\}$ and $\{\lambda_i|_q \big| i \in I(q)\}$ are two bases of $H^2(BT^n; \mathbb{Z})$ by Lemma 2.1, there is an $n \times n$ nondegenerate $\mathbb{Z}$-matrix A such that

$$(\lambda_i|_p \big| i \in I(p)) = (\lambda_i|_q \big| i \in I(q))A.$$

Moreover, we have that

$$s_2(p) - s_2(q) = (\lambda_i|_q \big| i \in I(q))(AA^\top - E_n)(\lambda_i|_q \big| i \in I(q))^\top = 0,$$

so we conclude that $AA^\top = E_n$, where E_n is the identity matrix. This implies that each row of A contains only one ± 1 and the other elements in this row are all 0. Hence

$$\sigma_n(p) = \pm \prod_{i \in I(q)} \lambda_i|_q = \pm\sigma_n(q)$$

and then the proof is completed by Lemma 3.1. □

Let $\{\sigma_1(p) \big| p \in M^{T^n}\} = \{\tau_1, \ldots, \tau_s\}$ and $\{\sigma_2(p) \big| p \in M^{T^n}\} = \{\eta_1, \ldots, \eta_u\}$. Then $s, u \leq t$. Set

$$\mathcal{A}_k = \{p \in M^{T^n} \big| \sigma_1(p) = \tau_k\}$$

for $1 \leq k \leq s$ and

$$\mathcal{B}_l = \{p \in M^{T^n} \big| \sigma_2(p) = \eta_l\}$$

for $1 \leq l \leq u$. Then $|M^{T^n}| = \sum_{k=1}^{s} |\mathcal{A}_k| = \sum_{l=1}^{u} |\mathcal{B}_l|$.

Proof of Theorem 1.1. By Theorem 2.1 it suffices to prove that if the equivariant Chern numbers $\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = 0$ for all $i, j \in \mathbb{N}$, then M^{2n} bounds equivariantly. Now suppose $\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = 0$ for all $i, j \in \mathbb{N}$. By Theorem 2.2, we can write these equivariant Chern numbers in the following way:

$$\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = \sum_{\ell=1}^{t} \frac{m_\ell (\sigma_1^{(\ell)})^i (\sigma_2^{(\ell)})^j}{\sigma_n^{(\ell)}} = \sum_{k=1}^{s} \tau_k^i \sum_{l\in\mathcal{L}_k} \eta_l^j \sum_{\ell\in\mathcal{C}_{k,l}} \frac{m_\ell}{\sigma_n^{(\ell)}} \tag{3.1}$$

where $\mathcal{L}_k = \{l \big| \mathcal{A}_k \cap \mathcal{B}_l \neq \emptyset, 1 \le l \le u\}$, and $\mathcal{C}_{k,l} = \{\ell \big| \sigma_1^{(\ell)} = \tau_k, \sigma_2^{(\ell)} = \eta_l, 1 \le \ell \le t\}$. Obviously, $|\mathcal{L}_k| \le |\mathcal{A}_k|$ for every k. Let i vary in the range $0, 1, \dots, s-1$. Then (τ_k^i) is an $s \times s$ van der Monde matrix, so for each k,

$$\sum_{l\in\mathcal{L}_k} \eta_l^j \sum_{\ell\in\mathcal{C}_{k,l}} \frac{m_\ell}{\sigma_n^{(\ell)}} = 0.$$

Next, let j vary in the range $0, 1, \dots, |\mathcal{L}_k| - 1$. Then (η_l^j) is a $|\mathcal{L}_k| \times |\mathcal{L}_k|$ van der Monde matrix, hence for each k and each $l \in \mathcal{L}_k$,

$$\sum_{\ell\in\mathcal{C}_{k,l}} \frac{m_\ell}{\sigma_n^{(\ell)}} = 0.$$

Furthermore, by Lemma 3.2 we have that $\mathcal{C}_{k,l}$ contains only an element, so $m_\ell = 0$ for all ℓ. Thus, by Theorem 2.2, all equivariant Chern characteristic numbers of M are equal to zero, as desired. $\square$

Now we focus on the proof of Theorem 1.2. First we give a general result.

Proposition 3.1. *Let M^{2n} be a $(2n)$-dimensional unitary torus manifold. If $s + 2\max_{1\le k\le s}\{|\mathcal{A}_k|\} - 3 < n$ or $2u + \max_{1\le l\le u}\{|\mathcal{B}_l|\} - 3 < n$, then M^{2n} bounds equivariantly.*

Proof. In a similar way to the proof of Theorem 1.1, we can write the equivariant Chern numbers $\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle$ in the following two ways:

$$\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = \sum_{k=1}^{s} \tau_k^i \sum_{l\in\mathcal{L}_k} \eta_l^j \sum_{\ell\in\mathcal{C}_{k,l}} \frac{m_\ell}{\sigma_n^{(\ell)}}, \tag{3.2}$$

where $\mathcal{L}_k = \{l \big| \mathcal{A}_k \cap \mathcal{B}_l \neq \emptyset, 1 \le l \le u\}$ with $|\mathcal{L}_k| \le |\mathcal{A}_k|$ for every k and $\mathcal{C}_{k,l} = \{\ell \big| \sigma_1^{(\ell)} = \tau_k, \sigma_2^{(\ell)} = \eta_l, 1 \le \ell \le t\}$ as before, and

$$\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = \sum_{l=1}^{u} \eta_l^j \sum_{k\in\mathcal{K}_l} \tau_k^i \sum_{\ell\in\mathcal{C}_{k,l}} \frac{m_\ell}{\sigma_n^{(\ell)}} \tag{3.3}$$

where $\mathcal{K}_l = \{k \big| \mathcal{A}_k \cap \mathcal{B}_l \neq \emptyset, 1 \le k \le s\}$, satisfying that and $|\mathcal{K}_l| \le |\mathcal{B}_l|$ for every l. We note that if $i + 2j < n$, then $\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = 0$.

If $s+2\max_{1\le k\le s}\{|\mathcal{A}_k|\}-3 < n$, then we can let i vary in the range $0, 1, \dots, s-1$ and for every k, let j vary in the range $0, 1, \dots, |\mathcal{L}_k| - 1 \le \max_{1\le k\le s}\{|\mathcal{A}_k|\} - 1$ in Equation (3.2). Similarly, if $2u+\max_{1\le l\le u}\{|\mathcal{B}_l|\}-3 < n$, then we can let j vary in the range $0, 1, \dots, u-1$ and for every l, let i vary in the range $0, 1, \dots, |\mathcal{K}_l| - 1 \le \max_{1\le l\le u}\{|\mathcal{B}_l|\} - 1$ in Equation (3.3). Using the proof method of Theorem 1.1 as above, we can obtain van der

Monde matrices, which imply that $m_\ell = 0$ for all ℓ, and hence $\langle (c_1^{T^n})^i (c_2^{T^n})^j, [M]\rangle = 0$ for all $i, j \in \mathbb{N}$. Therefore, M bounds equivariantly by Theorem 1.1. $\square$

Lemma 3.3. *Let $a_1, \dots, a_r$ be positive integers. If $a_1 + \cdots + a_r = \wp$, then $r + 2\max\{a_i | 1 \le i \le r\} \le 2\wp + 1$.*

Proof. Obviously, $\max\{a_i|1 \le i \le r\} \le \wp - r + 1$, and the equation holds if and only if there is only some one $a_i = \wp - r + 1$ and all others are equal to 1. Then we have the required inequality $r + 2\max\{a_i|1 \le i \le r\} \le 2\wp + 1$, where the last equation holds if and only if $r = 1$. $\square$

Proof of Theorem 1.2. If $|M^{T^n}| = |\mathcal{A}_1| + \cdots + |\mathcal{A}_s| < \frac{n}{2} + 1$, then by Lemma 3.3, we have $s + 2 \max_{1\le k\le s}\{|\mathcal{A}_k|\} \le 2|M^{T^n}| + 1 < n + 3$, so M bounds equivariantly by Proposition 3.1. $\square$

Remark 3.1. Let us look at the case in which M does not bound equivariantly and $|M^{T^n}| = \lceil \frac{n}{2} \rceil + 1$. When n is even, we have $s + 2\max_{1\le k\le s}\{|\mathcal{A}_k|\} = 2|M^{T^n}| + 1$ by Proposition 3.1 and Lemma 3.3. This implies that $s = 1$ by the proof of Lemma 3.3, which means that all σ_1 are the same. When n is odd, we see that $n + 3 \le s + 2\max_{1\le k\le s}\{|\mathcal{A}_k|\} \le 2|M^{T^n}| + 1 = n + 4$. An easy argument shows that $n + 3 = s + 2\max_{1\le k\le s}\{|\mathcal{A}_k|\}$ is impossible, so we must have $s + 2\max_{1\le k\le s}\{|\mathcal{A}_k|\} = 2|M^{T^n}| + 1$. Thus, in this case s must be 1 and then all σ_1 are the same, too. Moreover, in a similar way to the proof of Theorem 1.1, we can show easily that $|M^{T^n}| = u$ so all σ_2 are distinct. These observations seemingly imply the existence of a nonbounding unitary torus manifold M^{2n} with $|M^{T^n}| = \lceil \frac{n}{2} \rceil + 1$. Indeed, we can see an example in the case $n = 1$, as shown in [4, Theorem 5].

Finally we conclude this paper with the following conjecture:

Conjecture 3.1. $\lceil \frac{n}{2} \rceil + 1$ is the best possible lower bound of the number of fixed points for $(2n)$-dimensional nonbounding unitary torus manifolds.

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