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# ERRATUM FOR "UPPER AND LOWER BOUNDS FOR NORMAL DERIVATIVES OF DIRICHLET EIGENFUNCTIONS"

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The purpose of this note is to correct an error in the proof of Lemma 3.2 of our paper [2]. The error starts in the paragraph beginning "We use an integrating factor" on p295. The next displayed equation is incorrect, because we multiplied inequality (3.9) by the factor $2L'(r)L(r)^{-2}$, which does not have a fixed sign.

Fortunately, both the Lemma and the general strategy of the proof are correct. To correct the mistake, disregard the rest of the proof of Lemma 3.2 starting from "We use an integrating factor" and replace with the following:

"We now show that

$$L'(r) > 0 \implies (L'(r))^2 < 4C\lambda L(r) \text{ for all } r \in [0, \delta_3]. \tag{1}$$

For if not, then we have $L'(r_0) > 0$ and $(L'(r_0))^2 \geq 4C\lambda L(r_0)$ for some $r_0 \in [0, \delta/3]$. Plugging into (3.9) we obtain $L''(r_0) > 3C\lambda$, and this implies that

$$\frac{d}{dr}\Big((L'(r))^2 - 4C\lambda L(r)\Big) = 2L'(r)L''(r) - 4C\lambda L'(r) \geq 2C\lambda L'(r) > 0 \text{ for } r = r_0.$$

This means that the properties $L'(r)$ positive and increasing, and $(L'(r))^2 \geq 4C\lambda L(r)$ persist in some interval $[r_0, r_0 + \epsilon)$, $\epsilon > 0$. By a continuity argument we see that these properties hold for all $r \in [r_0, \delta]$. But then we would have

$$\frac{d}{dr}\sqrt{L(r)} \geq \sqrt{C\lambda}, \quad r \in [r_0, \delta].$$

This implies

$$L(r) \geq C\lambda(r - r_0)^2, \quad r \in [r_0, \delta],$$

which contradicts (3.10) for sufficiently large $\lambda$. We conclude (1).

Now (1) implies that we have

$$\frac{L'(r)}{2\sqrt{L(r)}} \leq \sqrt{C\lambda} \text{ for all } r \in [0, \delta/3].$$

Integrating this and using $L(0) = 0$, we find that

$$\sqrt{L(r)} \leq \sqrt{C\lambda} r \implies L(r) \leq C\lambda r^2, \quad r \in [0, \delta/3],$$

as desired."

We also take this opportunity to mention that some of the results of this paper are a consequence of previously known results in the control theory literature, for example in the paper [1]. This was not known to us at the time of writing the paper. We thank Nicolas Burq for subsequently pointing this out to us.

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## References

[1] C. Bardos, G. Lebeau, J. Rauch, *Sharp sufficient conditions for the observation, control, and stabilization of waves from the boundary*, SIAM J. Control Optim. **30** (1992), no. 5, 1024–1065.

[2] A. Hassell and T. Tao, *Upper and lower bounds for normal derivatives of Dirichlet eigenfunctions*, Math. Res. Lett. **9** (2002), 289–305.

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