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ERRATUM - A MODULARITY LIFTING THEOREM FOR WEIGHT TWO HILBERT MODULAR FORMS

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1. Introduction

Unfortunately, the proof of Theorem 3.2 of [Gee06] (and thus the main Theorem stated in the introduction to the paper) is incomplete; in particular, the proof of Lemma 3.3 is incorrect. Specifically, one cannot automatically assume that the type of $\rho_{f'}$ is $\tilde{\omega}_1 \oplus \tilde{\omega}_2$; to do so is to make a rather strong assumption about the Serre weights of $\overline{\rho}_f$. In addition one cannot conclude that the type determines the descent data on $\mathcal{G}_1$ and $\mathcal{G}_2$, at least in the case where $\overline{\rho}_f|_{G_{F_v}}$ is split; either $\mathcal{G}_1$ can correspond to $\tilde{\omega}_1$ and $\mathcal{G}_2$ to $\tilde{\omega}_2$, or vice versa.

As a consequence, we are only able to obtain a slightly weaker modularity lifting theorem; the most general result we can obtain is:

Theorem 1.1. *Let $p > 2$, let F be a totally real field, and let E be a finite extension of $\mathbb{Q}_p$ with ring of integers $\mathcal{O}$. Let $\rho : G_F \to \mathrm{GL}_2(\mathcal{O})$ be a continuous representation unramified outside of a finite set of primes, with determinant a finite order character times the p-adic cyclotomic character. Suppose that*

(1) *ρ is potentially Barsotti-Tate at each $v|p$.*
(2) *There exists a Hilbert modular form f of parallel weight 2 over F such that $\overline{\rho}_f \sim \overline{\rho}$, and for each $v|p$, if ρ is potentially ordinary at v then so is ρ_f.*
(3) *$\overline{\rho}|_{G_{F(\zeta_p)}}$ is absolutely irreducible, and if $p = 5$ and the projective image of $\overline{\rho}$ is isomorphic to $\mathrm{PGL}_2(\mathbb{F}_5)$ then $[F(\zeta_5) : F] = 4$.*

Then ρ is modular.

Proof. The proof is extremely similar to that of Theorem 3.1 of [Gee06]. Hypothesis (3) has been weakened because of a corresponding weakening of (3.2.3)(3) in the final version of [Kis07]. As for (2), we need only check that after making a base change, we may assume that at each place dividing p, ρ is potentially ordinary if and only if ρ_f is. This is easily achieved by employing Lemma 3.1.5 of [Kis07] at each place where ρ is not potentially ordinary. □

Additionally, we would like to thank Fred Diamond and Florian Herzig for independently bringing to our attention a minor error in the proof of Proposition 2.3. The points D^j constructed in the proof are not necessarily points on $\mathcal{GR}_{V_{\mathbb{F}},0}$. However,

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their only use is in showing that the points D and D' lie on the same component, and this in fact follows immediately from an application of Lemma 2.4 with $N = (N_i)$,

$$N_i = \begin{pmatrix} 0 & -w_i u^{-b_i} \\ 0 & 0 \end{pmatrix}.$$

References

[Gee06] Toby Gee, *A modularity lifting theorem for weight two Hilbert modular forms*, Math. Res. Lett. **13** (2006), no. 5-6, 805–811.

[Kis07] Mark Kisin, *Moduli of finite flat group schemes, and modularity*, to appear in Annals of Mathematics (2007).

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