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PROPER DISCS IN STEIN MANIFOLDS AVOIDING COMPLETE PLURIPOLAR SETS

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1. Introduction and the results

Denote by $\triangle$ the open unit disc in $\mathbb{C}$. Recall that a subset Y in a complex manifold X is called *complete pluripolar* if there exists a plurisubharmonic function ρ on X such that $Y = \{z;\, \rho(z) = -\infty\}$.

In this paper we prove the following result.

Theorem 1.1. *Let X be a Stein manifold of dimension at least 2. Given a closed complete pluripolar set $Y \subset X$, a point $p \in X \setminus Y$ and a vector v tangent to X at p, there exists a proper holomorphic map $f\colon \triangle \to X$ such that $f(0) = p$, $f'(0) = \lambda v$ for some $\lambda > 0$ and $f(\triangle) \cap Y = \emptyset$.*

Clearly, every closed complex analytic subset A of a connected Stein manifold X, $A \neq X$, is locally complete pluripolar, that is, for any point $a \in A$ there is an open neighborhood U of a such that $A \cap U$ is complete pluripolar in U. By [Col] every closed locally complete pluripolar set in a Stein manifold is complete pluripolar, thus every closed complex analytic subset is closed complete pluripolar. Therefore our theorem answers the question posed in [FG2] on the existence of proper holomorphic discs in the complements of hypersurfaces.

J. Globevnik [Glo] proved in 2000 that for any point p in a Stein manifold X of dimension at least 2 there exists a proper holomorphic map from the unit disc to X with the point p in its image.

The most general result on avoiding certain sets by proper holomorphic discs was given by H. Alexander [Ale] in 1975: he proved that for a closed polar set $E \subset \mathbb{C}$ there exists a proper holomorphic map $F = (F_1, F_2)\colon \triangle \to \mathbb{C}^2$ such that $F_1(\triangle) \cap E = \emptyset$. On the other hand, a proper holomorphic disc in $\mathbb{C}^2$ cannot avoid a non-polar set of parallel complex lines (see [Jul, Tsu, Ale, FG2]). F. Forstnerič and J. Globevnik [FG2] in 2001 constructed a proper holomorphic disc in $\mathbb{C}^2$ omitting both coordinate axes and proper holomorphic discs avoiding large real cones in $\mathbb{C}^2$. However, it was unknown if the image of a proper holomorphic map from the disc can miss three or more complex lines. Our theorem provides a positive answer to this question since a finite union of complex lines in $\mathbb{C}^2$ is closed complete pluripolar. Note that closed convex sets in $\mathbb{C}^2$ which can be avoided by the image of proper holomorphic maps from the disc were characterized in [Dri].

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We shall prove the following approximation theorem, which easily implies Theorem 1.1. In fact, Theorem 1.1 will follow directly from Lemma 2.3.

Theorem 1.2. *Let X be a Stein manifold of dimension at least 2 and let $Y \subset X$ be a closed complete pluripolar subset. Let d be a complete metric on X which induces the manifold topology. Assume that $f\colon \triangle \to X$ is a holomorphic map such that there is an open subset $V \subset\subset \triangle$ with the property $f(\zeta) \notin Y$ for $\zeta \in \triangle \setminus V$. Given $\epsilon > 0$ there is a proper holomorphic map $g\colon \triangle \to X$ satisfying*
(i) *$g(\zeta) \notin Y$ for $\zeta \in \triangle \setminus V$,*
(ii) *$d(g(\zeta), f(\zeta)) < \epsilon$ for $\zeta \in V$,*
(iii) *$g(0) = f(0)$ and $g'(0) = \lambda f'(0)$ for some $\lambda > 0$.*

We will prove Theorem 1.2 in section 2.

Corollary 1.3. *Let X be a Stein manifold of dimension at least 2 and let $Y \subset X$ be a closed complete pluripolar subset. Assume that S is a discrete subset of X such that $S \cap Y = \emptyset$. Then there are proper holomorphic maps $f_n\colon \triangle \to X$ such that $f_n(\triangle)$ are pairwise disjoint, $f_n(\triangle)$ avoids Y $(n \in \mathbb{N})$ and $\cup_n f_n(0) = S$.*

Proof. We first note that a finite union of complete pluripolar sets is complete pluripolar, since a finite sum of plurisubharmonic functions is plurisubharmonic. We will also need the fact that a discrete set S in a Stein manifold is complete pluripolar. Namely, by [Col] it is enough to prove that S is locally complete pluripolar, which follows from the fact that S is a complex analytic subset of X.

Let $S = \{s_n; n \in \mathbb{N}\}$. We shall construct the maps f_n inductively. By Theorem 1.1 there is a proper holomorphic map $f_1\colon \triangle \to X$ such that $f_1(\triangle) \cap (Y \cup S \setminus \{s_1\}) = \emptyset$ and $f_1(0) = s_1$. Assume that for some $n \in \mathbb{N}$ we have already constructed proper holomorphic maps $f_j\colon \triangle \to X$, $1 \leq j \leq n$, such that $f_j(\triangle)$ are pairwise disjoint, $f_j(\triangle)$ avoids Y and $f_j(\triangle) \cap S = \{s_j\}$ $(1 \leq j \leq n)$. By Remmert's proper mapping theorem [Re1, Re2], [Ch2, p. 65] the image of a proper holomorphic map is a closed analytic subset of X and therefore closed complete pluripolar. Thus $Y \cup f_1(\triangle) \cup \cdots \cup f_n(\triangle) \cup S \setminus \{s_{n+1}\}$ is closed complete pluripolar. Then by Theorem 1.1 there is a proper holomorphic map $f_{n+1}\colon \triangle \to X$ such that

$$f_{n+1}(\triangle) \cap (Y \cup f_1(\triangle) \cup \cdots \cup f_n(\triangle) \cup S \setminus \{s_{n+1}\}) = \emptyset$$

and $f_{n+1}(0) = s_{n+1}$. The inductive construction is finished and the proof is complete. □

Let $\mathcal{R}$ be a bordered Riemann surface. By the theorem of Ahlfors [Ahl], there are inner functions on $\mathcal{R}$. Recall that a nonconstant continuous function $f\colon \mathcal{R} \to \overline{\triangle}$, which is holomorphic on $\mathcal{R} \setminus b\mathcal{R}$, is called an *inner function* (or an *Ahlfors function*) on $\mathcal{R}$ if $|f| = 1$ on $b\mathcal{R}$. Therefore Theorem 1.1 implies the following:

Corollary 1.4. *Let X be a Stein manifold of dimension at least 2 and let $Y \subset X$ be a closed complete pluripolar subset. Given a bordered Riemann surface $\mathcal{R}$ there is a proper holomorphic map $f\colon \mathcal{R} \setminus b\mathcal{R} \to X$ such that $f(\mathcal{R} \setminus b\mathcal{R}) \cap Y = \emptyset$.*

2. Proof of Theorem 1.2

As it was observed in [FG2] the methods developed in [FG1, Glo] actually prove the following:

Theorem 2.1. *Let X be a Stein manifold of dimension at least 2 and $\rho\colon X \to \mathbb{R}$ a smooth exhaustion function which is strongly plurisubharmonic on $\{\rho > M\}$ for some $M \in \mathbb{R}$. Let $f\colon \overline{\triangle} \to X$ be a continuous map which is holomorphic on $\triangle$ such that $\rho(f(\zeta)) > M$ for each $\zeta \in b\triangle$. Let d be a complete metric on X which induces the manifold topology. For any numbers $0 < r < 1$, $\epsilon > 0$, $N > M$ and for any finite set $A \subset \triangle$ there exists a continuous map $g\colon \overline{\triangle} \to X$, holomorphic on $\triangle$, satisfying*
(i) $\rho(g(\zeta)) > N$ *for* $\zeta \in b\triangle$,
(ii) $\rho(g(\zeta)) > \rho(f(\zeta)) - \epsilon$ *for* $\zeta \in \overline{\triangle}$,
(iii) $d(f(\zeta), g(\zeta)) < \epsilon$ *for* $|\zeta| \leq r$, *and*
(iv) $g(\zeta) = f(\zeta)$ *and* $g'(\zeta) = f'(\zeta)$ *for* $\zeta \in A$.

In the proof of Theorem 1.2 we also need the following lemma which is a slight generalization of [Ch1, Lemma 1]. Since its proof is essentially the same, we omit it.

Lemma 2.2. *Let X be a Stein manifold and $Y \subset X$ a complete pluripolar set. Let $L_1 \subset L_2 \subset X$ be holomorphically convex compact sets. Then the set $(L_1 \cup Y) \cap L_2$ is holomorphically convex.*

The main tool in the proof of Theorem 1.2 is the following

Lemma 2.3. *Let X be a Stein manifold of dimension at least 2 and let $Y \subset X$ be a closed complete pluripolar subset. Let d be a complete metric on X which induces the manifold topology. Assume that $f\colon \triangle \to X$ is a holomorphic map such that there is an open subset $V \subset\subset \triangle$ with the property $f(\zeta) \notin Y$ for $\zeta \in \triangle \setminus V$. Given $\epsilon > 0$ there are a domain Ω, $\{0\} \cup V \subset\subset \Omega \subset\subset \triangle$, conformally equivalent to the unit disc and a proper holomorphic map $g\colon \Omega \to X$ with the following properties*
(i) $g(\zeta) \notin Y$ *for* $\zeta \in \Omega \setminus V$,
(ii) $d(g(\zeta), f(\zeta)) < \epsilon$ *for* $\zeta \in V$,
(iii) $g(0) = f(0)$ *and* $g'(0) = f'(0)$.

Proof. One can choose a simply connected domain Ω_1 such that $\{0\} \cup V \subset\subset \Omega_1 \subset\subset \triangle$. By [Hör, Theorem 5.1.6] there is a smooth strongly plurisubharmonic exhaustion function ρ for Stein manifold X. Sard's theorem implies that one can choose a strictly increasing sequence $\{M_n\}$ of regular values of ρ converging to ∞ with M_1 so big that $\rho(f(\zeta)) < M_1$ for $\zeta \in \overline{\Omega}_1$. By continuity there is a simply connected domain $\triangle_1$, $\Omega_1 \subset\subset \triangle_1 \subset\subset \triangle$, such that $\rho(f(\zeta)) < M_1$ for $\zeta \in \overline{\triangle}_1$. Let $U_0 = \emptyset$ and for $n \in \mathbb{N}$ denote by U_n the sublevel set $\{z \in X;\, \rho(z) < M_n\}$. Since M_n is a regular value of ρ it holds that $\overline{U}_n = \{z \in X;\, \rho(z) \leq M_n\}$. This implies that $\overline{U}_n$ is a holomorphically convex compact set, because on a Stein manifold plurisubharmonic hull equals holomorphic hull.

We shall construct inductively a decreasing sequence of domains $\{\triangle_n\}$ conformally equivalent to $\triangle$, an increasing sequence of domains $\{\Omega_n\}$ conformally equivalent to $\triangle$, $\Omega_n \subset\subset \triangle_n$ ($n \in \mathbb{N}$), a sequence of continuous maps $g_n \colon \overline{\triangle}_n \to X$, holomorphic on $\triangle_n$, and a decreasing sequence of positive numbers $\{\epsilon_n\}$, satisfying for each $n \in \mathbb{N}$ the following

(I) $g_n(\zeta) \in U_n \setminus (\overline{U}_{n-1} \cup Y)$ $(\zeta \in \overline{\triangle_n \setminus \Omega_n})$,
(II) $g_{n+1}(\zeta) \notin \overline{U}_{n-1} \cup Y$ $(\zeta \in \overline{\triangle_{n+1} \setminus \Omega_n})$,
(III) $d(g_n(\zeta), g_{n+1}(\zeta)) < \frac{\epsilon_n}{2^n}$ $(\zeta \in \Omega_n)$,
(IV) $g_{n+1}(0) = g_n(0)$ and $g'_{n+1}(0) = g'_n(0)$,
(V) if $z \in X$ such that $d(z, g_n(\overline{\triangle_n \setminus V})) < \epsilon_n$ then $z \in U_n \setminus Y$,
(VI) if $z \in X$ such that $d(z, g_{n+1}(\overline{\triangle_{n+1} \setminus \Omega_n})) < \epsilon_{n+1}$ then $z \notin \overline{U}_{n-1}$.

Let $g_1 = f$ and let $\triangle_1$ and Ω_1 as above. Then (I) holds. Choose ϵ_1, $0 < \epsilon_1 < \epsilon$, so small that (V) holds for $n = 1$. Suppose that $j \in \mathbb{N}$ and that we have constructed g_n, $\triangle_n$, Ω_n and ϵ_n, $1 \le n \le j$, such that (I) and (V) hold for $1 \le n \le j$ and (II), (III), (IV) and (VI) hold for $1 \le n \le j-1$. It follows by Lemma 2.2 that the set $(\overline{U}_{j-1} \cup Y) \cap \overline{U}_{j+1}$ is holomorphically convex. Therefore there is a smooth strongly plurisubharmonic exhaustion function ρ_{j+1} on X such that $\rho_{j+1}(z) < 0$ $(z \in (\overline{U}_{j-1} \cup Y) \cap \overline{U}_{j+1})$ and $\rho_{j+1}(g_j(\zeta)) > 0$ $(\zeta \in \overline{\triangle_j \setminus \Omega_j})$ [Hör, Theorem 5.1.6]. There is N so big that $U_j \subset \{z;\ \rho_{j+1}(z) < N\}$. We use Theorem 2.1 to get a continuous map $g_{j+1} \colon \overline{\triangle}_j \to X$, holomorphic on $\triangle_j$, with the following properties

(a) $\rho_{j+1}(g_{j+1}(\zeta)) > N$ for $\zeta \in b\triangle_j$,
(b) $\rho_{j+1}(g_{j+1}(\zeta)) > 0$ for $\zeta \in \overline{\triangle_j \setminus \Omega_j}$,
(c) $d(g_{j+1}(\zeta), g_j(\zeta)) < \frac{\epsilon_j}{2^j}$ for $\zeta \in \Omega_j$, and
(d) $g_{j+1}(0) = g_j(0)$ and $g'_{j+1}(0) = g'_j(0)$.

By (a) and by the choice of N we get that $\rho(g_{j+1}(\zeta)) > M_j$ $(\zeta \in b\triangle_j)$. Thus there is M, $M_j < M < M_{j+1}$, such that the holomorphic disc $g_{j+1}(\triangle_j)$ and the level set $\{z;\ \rho(z) = M\}$ intersect transversally. It follows by (c) and (V) that $g_{j+1}(\Omega_j \setminus V) \subset U_j$. This and the fact that $\rho \circ g_{j+1}$ is subharmonic imply that there is a simply connected component of $\{\zeta \in \triangle_j;\ \rho(g_{j+1}(\zeta)) < M\}$ which contains Ω_j. We denote such component by $\triangle_{j+1}$. Choose a simply connected domain Ω_{j+1}, $\Omega_j \subset\subset \Omega_{j+1} \subset\subset \triangle_{j+1}$, such that $\rho(g_{j+1}(\zeta)) > M_j$ $(\zeta \in \overline{\triangle_{j+1} \setminus \Omega_{j+1}})$. It is easy to see that $\triangle_{j+1}$, Ω_{j+1}, g_{j+1} satisfy (I) for $n = j+1$ and (II), (III) and (IV) for $n = j$. Since we have $g_{j+1}(\overline{\triangle}_{j+1}) \subset U_{j+1}$ and $g_{j+1}(\overline{\triangle_{j+1} \setminus \Omega_j}) \cap Y = \emptyset$ and since by (V) for $n = j$ and by (c) we get that $g_{j+1}(\overline{\Omega_j \setminus V}) \cap Y = \emptyset$ it follows that $g_{j+1}(\overline{\triangle_{j+1} \setminus V}) \subset U_{j+1} \setminus Y$. This together with (II) for $n = j$ implies that there is ϵ_{j+1}, $0 < \epsilon_{j+1} < \epsilon_j$, so small that (V) holds for $n = j+1$ and that (VI) holds for $n = j$. The construction is finished.

Denote by Ω the union $\cup_{j=1}^{\infty} \Omega_j$. As Ω is a union of an increasing sequence of simply connected open sets it is simply connected and therefore conformally equivalent to the unit disc. It follows by (III) that for $\zeta \in \Omega$ the sequence $g_n(\zeta)$ is Cauchy with respect to the complete metric d therefore it converges to $g(\zeta)$.

Since the convergence is uniform on compact sets it follows that the map g is holomorphic on Ω.

Next we show that the map g and the domain Ω have all the required properties. Fix $j \in \mathbb{N} \cup \{0\}$. It follows by (III) that

$$\begin{aligned} d(g(\zeta), g_{j+1}(\zeta)) &\leq d(g_{j+1}(\zeta), g_{j+2}(\zeta)) + d(g_{j+2}(\zeta), g_{j+3}(\zeta)) + \cdots < \\ &< \frac{\epsilon_{j+1}}{2^{j+1}} + \frac{\epsilon_{j+2}}{2^{j+2}} + \cdots < \epsilon_{j+1} \ (\zeta \in \Omega_{j+1}). \end{aligned} \tag{1}$$

Thus for $\zeta \in \Omega_{j+1} \setminus \Omega_j$ it holds by (VI) that $g(\zeta) \notin U_{j-1}$. This implies that $g\colon \Omega \to X$ is a proper map. To prove that $g(\Omega \setminus V)$ avoids Y, choose $\zeta \in \Omega \setminus V$. There is $j \in \mathbb{N}$ so large that $\zeta \in \Omega_{j+1}$. It follows by (1) and by (V) that $g(\zeta) \notin Y$. By (1) for $j = 0$ we obtain that $d(g(\zeta), f(\zeta)) < \epsilon$ for $(\zeta \in V)$. We get by (IV) that $g(0) = f(0)$ and $g'(0) = f'(0)$. This completes the proof. □

Proof of Theorem 1.2. There are r and R, $0 < r < R < 1$, such that $V \subset\subset r\triangle$. One can choose ϵ_0, $0 < \epsilon_0 < \epsilon$, so small that

(2) for $z \in X$ such that $d(z, f(r\triangle \setminus V)) < \epsilon_0$ it holds that $z \notin Y$.

There is $\delta > 0$ so small that

$$V \subset\subset (r-\delta)\triangle \subset (r+\delta)\triangle \subset\subset R\triangle, \tag{3}$$

(4) $\zeta \in r\triangle,\ \zeta' \in \triangle$ such that $|\zeta - \zeta'| < \delta$ then $d(f(\zeta), f(\zeta')) < \frac{\epsilon_0}{2}$.

Let $\Omega_0 = \emptyset$ and choose an increasing sequence $\{R_n\}$ of positive numbers converging to 1 with $R_1 > R$. We shall construct inductively an increasing sequence of simply connected domains $\{\Omega_n\}$ such that $R_n\triangle \cup \Omega_{n-1} \subset\subset \Omega_n \subset\subset \triangle$, a decreasing sequence of positive numbers $\{\epsilon_n\}$, $\epsilon_1 < \frac{\epsilon_0}{2}$, and a sequence of proper holomorphic maps $g_n\colon \Omega_n \to X$ such that

(a) $g_n(\zeta) \notin Y$ for $\zeta \in \Omega_n \setminus V$,
(b) $d(g_n(\zeta), f(\zeta)) < \epsilon_n$ for $\zeta \in R_n\triangle \cup \Omega_{n-1}$,
(c) $g_n(0) = f(0)$ and $g_n'(0) = f'(0)$.

Assume that we have already constructed Ω_n and ϵ_n $(0 \leq n \leq j)$ and g_n $(1 \leq n \leq j)$ for some $j \in \mathbb{N} \cup \{0\}$. One can choose ϵ_{j+1}, $0 < \epsilon_{j+1} < \frac{\epsilon_j}{2}$, with the following property

(5) for $z \in X$ such that $d(z, f((R_{j+1}\triangle \cup \Omega_j) \setminus V)) < \epsilon_{j+1}$ it holds that $z \notin Y$.

Using Lemma 2.3 for $V = R_{j+1}\triangle \cup \Omega_j$ and $\epsilon = \epsilon_{j+1}$ we obtain a simply connected domain Ω_{j+1}, $R_{j+1}\triangle \cup \Omega_j \subset\subset \Omega_{j+1} \subset\subset \triangle$, and a proper holomorphic map $g_{j+1}\colon \Omega_{j+1} \to X$ which satisfy (b) and (c) for $n = j+1$ and it holds that $g_{j+1}(\zeta) \notin Y$ for $\zeta \in \Omega_{j+1} \setminus (R_{j+1}\triangle \cup \Omega_j)$. By (5) and (b) we get $g_{j+1}(\zeta) \notin Y$ for $\zeta \in (R_{j+1}\triangle \cup \Omega_j) \setminus V$, thus (a) holds for $n = j+1$. This completes the construction.

Note that $\cup_n \Omega_n = \triangle$. Caratheodory kernel theorem [Car, Pom] implies that the sequence of conformal maps $h_n\colon \triangle \to \Omega_n$, such that $h_n(0) = 0$, $h_n'(0) > 0$, converges uniformly on compact sets to identity. Choose n so big that

$$|h_n(\zeta) - \zeta| < \delta \quad (\zeta \in r\triangle). \tag{6}$$

Let $g = g_n \circ h_n$. By the above $g\colon \triangle \to X$ is a proper holomorphic map and (c) implies (iii). Take $\zeta \in r\triangle$. By (6) and (3) we get that $h_n(\zeta) \in R\triangle$ which by (b) implies that $d(g_n(h_n(\zeta)), f(h_n(\zeta))) < \frac{\epsilon_0}{2}$. It follows by (6) and (4) that $d(f(h_n(\zeta)), f(\zeta)) < \frac{\epsilon_0}{2}$. Therefore $d(g(\zeta), f(\zeta)) < \epsilon_0$ $(\zeta \in r\triangle)$. This proves (ii) and for $\zeta \in r\triangle \setminus V$ this together with (2) implies that $g(\zeta) \notin Y$. Choose $\zeta \in \triangle \setminus r\triangle$. By (6) it follows from Rouché's theorem that $(r-\delta)\triangle \subset h_n(r\triangle)$ and thus we get by (3) that $h_n(\zeta) \notin V$. By (a) it follows that $g(\zeta) \notin Y$. This proves (i). The proof is complete. $\square$

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