ON LENS SPACES AND THEIR SYMPLECTIC FILLINGS

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ABSTRACT. The standard contact structure ξ_0 on the three–sphere S^3 is invariant under the action of $\mathbb{Z}/p\mathbb{Z}$ yielding the lens space $L(p,q)$, therefore every lens space carries a natural quotient contact structure $\overline{\xi}_0$. A theorem of Eliashberg and Mc-Duff classifies the symplectic fillings of $(L(p,1),\overline{\xi}_0)$ up to diffeomorphism. Here we announce a generalization of that result to every lens space. In particular, we give an explicit handlebody decomposition of every symplectic filling of $(L(p,q),\overline{\xi}_0)$ for every p and q. Our results imply:

(a) There exist infinitely many lens spaces $L(p,q)$ with $q \neq 1$ such that the contact 3–manifold $(L(p,q),\overline{\xi}_0)$ admits only one symplectic filling up to blowups and diffeomorphisms.

(b) For any natural number N, there exist infinitely many lens spaces $L(p,q)$ such that $(L(p,q),\overline{\xi}_0)$ admits more than N symplectic fillings up to blowups and diffeomorphisms.

1. Introduction

Four–dimensional symplectic fillings are objects of central interest in symplectic and contact topology. They can be used to prove tightness of contact structures on three–dimensional manifolds [5, 10], to distinguish tight contact structures [18], and they arise in symplectic cut–and–paste constructions [23]. Symplectic fillings also have an intrinsic interest. For example, they behave like closed symplectic four–manifolds from the point of view of the Seiberg–Witten invariants and this fact has a number of consequences [14, 15, 16]. A difficult and interesting problem in this area is the diffeomorphism classification of the symplectic fillings of a given contact three–manifold. Eliashberg and McDuff solved this problem in some cases [4, 19]. The purpose of this note is to announce a generalization of their results. A (coorientable) *contact three–manifold* is a pair (Y, ξ), where Y is a three–manifold and $\xi \subset TY$ is a two–dimensional distribution given as the kernel of a one–form $\alpha \in \Omega^1(Y)$ such that $\alpha \wedge d\alpha$ is a volume form.

A *symplectic filling* of a closed contact three–manifold (Y, ξ) is pair (X, ω) consisting of a smooth, compact, connected four–manifold X with $\partial X = Y$ and a symplectic form ω on X such that $\omega|_\xi \neq 0$ at every point of ∂X. Moreover, if X is oriented by $\omega \wedge \omega$ and $\xi = \{\alpha = 0\}$, the boundary orientation on Y must coincide with the orientation induced by $\alpha \wedge d\alpha$ (which only depends on ξ).

A basic fact to bear in mind is that any diffeomorphism classification of symplectic fillings will always be "up to blowups". This is because a blowup, i.e. a connected sum with $\overline{\mathbb{CP}}^2$, is a local operation which can be performed in the symplectic category. Therefore, if (X, ω) is a symplectic filling of (Y, ξ) then $\widehat{X} = X \# N \mathbb{CP}^2$, $N \geq 1$, carries a symplectic form $\widehat{\omega}$ such that $(\widehat{X}, \widehat{\omega})$ is still a symplectic filling of (Y, ξ).

The *standard contact structure* ξ_0 on the three–sphere is the 2–dimensional distribution of complex lines tangent to $S^3 \subset \mathbb{C}^2$. The unit four–ball $B^4 \subset \mathbb{C}^2$ endowed with the restriction of the standard Kähler form on $\mathbb{C}^2$ is a symplectic filling of (S^3, ξ_0). The following result, due to Eliashberg, yields the diffeomorphism classification of the symplectic fillings of (S^3, ξ_0).

Theorem 1.1 ([4]). *Let (X, ω) be a symplectic filling of (S^3, ξ_0). Then, X is diffeomorphic to a blowup of B^4.*

The standard contact structure ξ_0 is invariant under the natural action of $U(2)$ on S^3. Thus, ξ_0 is a fortiori invariant under the induced action of the subgroup

$$G_{p,q} = \left\{ \begin{pmatrix} \zeta & 0 \\ 0 & \zeta^q \end{pmatrix} \mid \zeta^p = 1 \right\} \subset U(2),$$

where $p, q \in \mathbb{Z}$. It follows that when $p > q \geq 1$ and p, q are coprime, ξ_0 induces a contact structure $\bar{\xi}_0$ on the lens space $L(p, q) = S^3 / G_{p,q}$.

Let D_p denote the disk bundle over the 2–sphere with Euler class p. It is not difficult to construct a symplectic form ω on D_{-p} such that (D_{-p}, ω) is a symplectic filling of $(L(p, 1), \bar{\xi}_0)$. Another symplectic filling of $(L(4, 1), \bar{\xi}_0)$ is given by $\mathcal{C} = \mathbb{CP}^2 \setminus \nu(C)$, where $\nu(C)$ is a pseudo–concave neighborhood of a smooth conic $C \subset \mathbb{CP}^2$, endowed with the restriction of the standard Kähler form. The four–manifold $\mathcal{C}$ is easily shown to have the handlebody decomposition given by the right–hand side of Figure 2 in section 3.

The following theorem is a by–product of Dusa McDuff's classification of closed symplectic four–manifolds containing symplectic spheres of non–negative self–intersection. It yields the diffeomorphism classification of the symplectic fillings of $(L(p, 1), \bar{\xi}_0)$.

Theorem 1.2 ([19]). *Let (X, ω) be a symplectic filling of $(L(p, 1), \bar{\xi}_0)$. Then, X is diffeomorphic to a blowup of:*

(a) D_{-p} *if $p \neq 4$,*
(b) D_{-4} *or $\mathcal{C}$ if $p = 4$.*

2. Symplectic fillings of $(L(p,q),\overline{\xi}_0)$

From now on, we shall assume that p and q are two coprime integers such that

(2.1)
$$p > q \geq 1, \qquad \frac{p}{p-q} = b_1 - \cfrac{1}{b_2 - \cfrac{1}{\ddots - \cfrac{1}{b_k}}},$$

where $b_i \geq 2$, $i = 1, \ldots, k$. Note that there is a unique continued fraction expansion of $\frac{p}{p-q}$ as in (2.1). We shall use the shorthand $[b_1, \ldots, b_k]$ for such continued fractions.

Let $\mathbf{n} = (n_1, \ldots, n_k)$ be a k–tuple of non–negative integers such that

(2.2)
$$[n_1, \ldots, n_k] = 0.$$

We can view the "thin" framed link in Figure 1 as a three–dimensional surgery presentation of a closed, oriented three–manifold $N(\mathbf{n})$. The assumption (2.2) ensures the existence of an orientation–preserving diffeomorphism

(2.3)
$$\varphi \colon N(\mathbf{n}) \to S^1 \times S^2.$$

Definition 2.1. Let $W_{p,q}(\mathbf{n})$ be the smooth four–manifold with boundary obtained by attaching 2–handles to $S^1 \times D^3$ along the framed link $\varphi(\mathbf{L}) \subset S^1 \times S^2$, where $\mathbf{L} \subset N(\mathbf{n})$ is the thick framed link in Figure 1.
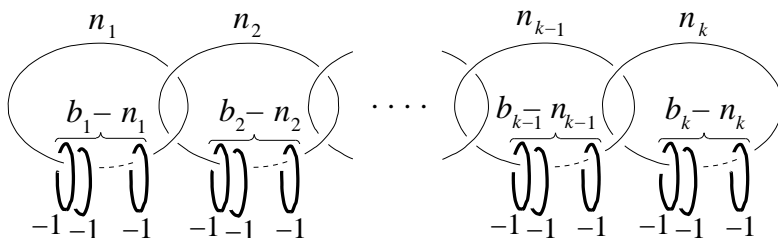

FIGURE 1. The manifold $W_{p,q}(\mathbf{n})$

Observe that the manifold $W_{p,q}(\mathbf{n})$ does not depend on the choice of the diffeomorphism φ, because every self–diffeomorphism of $S^1 \times S^2$ extends to $S^1 \times D^3$ [8]. Also, since $\mathbf{L}$ is canonically embedded in S^3, the framings of its components can be identified with integers.

Define $\mathbf{Z}_{p,q} \subset \mathbb{Z}^k$ as follows:

$$\mathbf{Z}_{p,q} = \{(n_1, \ldots, n_k) \in \mathbb{Z}^k \mid [n_1, \ldots, n_k] = 0, \ 0 \leq n_i \leq b_i, \ i = 1, \ldots, k\}.$$

We are now ready to state our main result.

Theorem 2.2 ([17]). *Let $p > q \geq 1$ be coprime integers. Then,*

(a) *For each $\mathbf{n} \in \mathbf{Z}_{p,q}$, the four–manifold $W_{p,q}(\mathbf{n})$ carries a symplectic form ω such that $(W_{p,q}(\mathbf{n}), \omega)$ is a symplectic filling of $(L(p,q), \overline{\xi}_0)$. Moreover, $W_{p,q}(\mathbf{n})$ does not contain smoothly embedded spheres with self–intersection -1.*

(b) *If (W, ω) is a symplectic filling of $(L(p,q), \overline{\xi}_0)$, then W is diffeomorphic to a blowup of $W_{p,q}(\mathbf{n})$ for some $\mathbf{n} \in \mathbf{Z}_{p,q}$.*

It is interesting to compare Theorem 2.2 with properties of the versal deformations of isolated complex bidimensional singularities. The lens space $L(p,q)$, endowed with the contact structure $\overline{\xi}_0$, can be viewed as the link of a cyclic quotient singularity of type (p,q). Stevens [22] used deep results of Kollár and Shepherd-Barron [13] to show that the irreducible components of the reduced base space S_{red} of the versal deformation of such a singularity are in one–to–one correspondence with the set $\mathbf{Z}_{p,q}$ (see also [2]).

Every irreducible component of S_{red} gives a smoothing of the singularity and therefore a symplectic filling of $(L(p,q), \overline{\xi}_0)$. It follows by Theorem 2.2 that every smoothing of a quotient (p,q)–singularity is diffeomorphic to a manifold of the form $W_{p,q}(\mathbf{n})$.

Indeed, it seems very likely that the smoothing coming from the irreducible component indexed by $\mathbf{n} \in \mathbf{Z}_{p,q}$ be diffeomorphic to $W_{p,q}(\mathbf{n})$. For example, it is easy to check that $\mathbf{Z}_{p,q}$ always contains the k–tuple $(1, 2, \ldots, 2, 1)$ as long as $k > 1$. The corresponding irreducible component of S_{red} is called the *Artin component*. The associated smoothing is diffeomorphic to the canonical resolution the singularity $R_{p,q}$, which is a well–known four–manifold obtained by plumbing according to a certain linear graph (see for example [1]). It is not difficult to see that $R_{p,q}$ is orientation–preserving diffeomorphic to $W_{p,q}(1, 2, \ldots, 2, 1)$.

If $\frac{p}{q} = [a_1, \ldots, a_h]$ with $a_i \geq 5$, $i = 1, \ldots, k$, it turns out that $\mathbf{Z}_{p,q} = \{(1, 2, \ldots, 2, 1)\}$. Thus, in this case S_{red} coincides with the Artin component, a fact which had been conjectured by Kollár [12].

Under the same assumption, Theorem 2.2 implies that every symplectic filling of $(L(p,q), \overline{\xi}_0)$ is diffeomorphic to a blowup of $W_{p,q}(1, 2, \ldots, 2, 1)$. In particular, we see that *there exist infinitely many lens spaces $L(p,q)$ with $q \neq 1$ such that $(L(p,q), \overline{\xi}_0)$ admits only one symplectic filling up to blowups and diffeomorphisms.*

On the other hand, suppose that $k \geq 4$, $b_2, \ldots, b_{k-2} \geq 3$ and $b_k \geq k - 2$. Then,

$$\mathbf{n}(r, s) = (1, \overbrace{2, \ldots, 2}^{r}, 3, \overbrace{2, \ldots, 2}^{s}, 1, s + 2) \in \mathbf{Z}_{p,q}$$

for every $0 \leq r, s \leq k - 4$ with $r + s + 4 = k$. Moreover, one can easily check that

$$(2.4) \qquad \operatorname{rk} H_2 \left(W_{p,q}(\mathbf{n}(r,s)); \mathbb{Z} \right) = \sum_{i=1}^{k} b_i - 2k - s.$$

Since the manifolds $W_{p,q}(\mathbf{n})$ do not contain exceptional spheres and therefore are not blowups of one another, Equation (2.4) implies that $(L(p,q), \overline{\xi}_0)$ admits at least $k - 3$ symplectic fillings up to blowups and diffeomorphisms. This shows that *for any natural number N, there exist infinitely many lens spaces $L(p,q)$ such that $(L(p,q), \overline{\xi}_0)$ admits more than N symplectic fillings up to blowups and diffeomorphisms.*

3. Examples

In this section we illustrate Theorem 2.2 by analyzing its implications in a few particular cases.

Given a k–tuple of positive integers

$$(n_1, \ldots, n_{s-1}, n_s, n_{s-1}, \ldots, n_k)$$

with $n_s = 1$, we say that the $k - 1$–tuple

$$(n_1, \ldots, n_{s-1} - 1, n_{s+1} - 1, \ldots, n_k)$$

is obtained by a *blowdown at n_s* (with the obvious meaning of the notation when $s = 1$ or $s = k$). The reverse process is a *blowup*.

We say that a k–tuple of positive integers $(n_1, \ldots, n_k)$ is *admissible* if the continued fraction $[n_1, \ldots, n_k]$ makes sense, i.e. if none of the denominators appearing in $[n_1, \ldots, n_k]$ vanishes.

The following Lemma 3.1 can be proved by an easy induction.

Lemma 3.1. *Let $(n_1, \ldots, n_k)$ be an admissible k–tuple of positive integers. Then, $[n_1, \ldots, n_k] = 0$ if and only if $(n_1, \ldots, n_k)$ is obtained from (0) by a sequence of blowups.*

First Example. Let us apply Theorem 2.2 to determine all the symplectic fillings of $(L(p,1), \overline{\xi}_0)$. We have

$$\frac{p}{p-1} = [\overbrace{2, \ldots, 2}^{p-1}].$$

Using Lemma 3.1 one can easily check that:

$$\mathbf{Z}_{p,1} = \begin{cases} \{(1, 2, \ldots, 2, 1)\}, & \text{if } p \neq 4, \\ \{(1, 2, 1), (2, 1, 2)\}, & \text{if } p = 4. \end{cases}$$

Therefore, Theorem 2.2 implies that the symplectic fillings of $(L(p,1),\bar\xi_0)$ up to blowups and diffeomorphisms are $W_{p,1}(1,2,\ldots,2,1)$ for $p \neq 4$ and either $W_{4,1}(1,2,1)$ or $W_{4,1}(2,1,2)$ for $p = 4$.

But $W_{p,1}(1,2,\ldots,2,1)$ is diffeomorphic to the canonical resolution $R_{p,1} = D_{-p}$, while Figure 2 shows that $W_{4,1}(2,1,2) = \mathcal{C}$. As expected, we have just re–obtained Theorem 1.2.
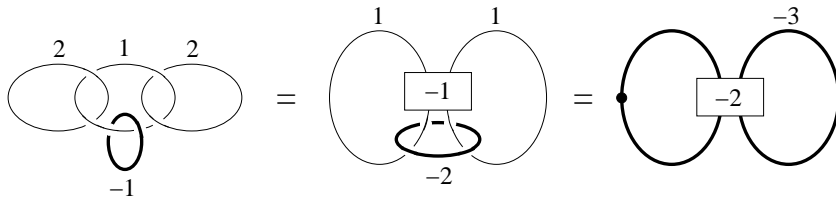

FIGURE 2. The manifold $W_{4,1}(2,1,2) = \mathcal{C}$

Second Example. Let us consider $L(p^2, p-1)$. Since

$$\frac{p^2}{p^2 - p + 1} = [p, \overbrace{2, \ldots, 2}^{p}],$$

applying Lemma 3.1 one sees that

$$\mathbf{Z}_{p^2,p-1} = \{(1,2,\ldots,2,1), (p,1,2,\ldots,2)\}.$$

Thus, by Theorem 2.2, the symplectic fillings of $(L(p^2, p-1), \bar\xi_0)$ are given, up to blowups and diffeomorphisms, by the canonical resolution $R_{p^2,p-1}$ and the manifold $W_{p^2,p-1}(p,1,2,\ldots,2)$, which is a rational homology ball as is apparent from Figure 3. In fact, Figure 3 shows that $W_{p^2,p-1}(p,1,2,\ldots,2)$ is precisely the rational homology ball used in the symplectic rational blowdown construction [6, 23], and therefore carries a symplectic form ω such that $(W_{p^2,p-1}(p,1,2,\ldots,2), \omega)$ is a symplectic filling of $(L(p^2, p-1), \bar\xi_0)$.
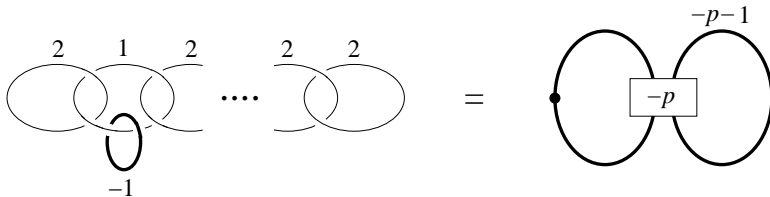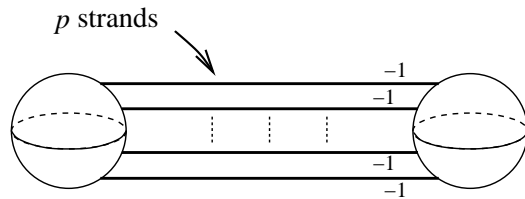

FIGURE 3. The manifold $W_{p^2,p-1}(p,1,2,\ldots,2)$

Third Example. This time we look at $L(p, p-1)$. We have $k = 1$ and $\mathbf{Z}_{p,p-1} = \{(0)\}$. Hence, by Theorem 2.2(b) every filling of $L(p,p-1)$ is diffeomorphic to a blowup of $W_{p,p-1}(0)$, which is given in Figure 4. Theorem 2.2(a)

FIGURE 4. The manifold $W_{p,p-1}(0)$

says that $W_{p,p-1}(0)$ carries a symplectic structure ω such that $(W_{p,p-1}(0), \omega)$ is a symplectic filling of $(L(p, p-1), \bar{\xi}_0)$ and moreover $W_{p,p-1}(0)$ does not contain smoothly embedded (-1)–spheres. In fact, $W_{p,p-1}(0)$ is easily seen to be diffeomorphic to the canonical resolution $R_{p,p-1}$, which gives a symplectic filling of $(L(p, p-1), \bar{\xi}_0)$ with even intersection form.

Alternatively, using [9] one can check that $W_{p,p-1}(0)$ carries a structure of Stein surface, so in particular it carries a symplectic structure ω such that $(W_{p,p-1}(0), \omega)$ is a symplectic filling of $(L(p, p-1), \xi)$ for some contact structure ξ. On the other hand, on the lens space $L(p, p-1)$ there exists only one tight contact structure up to isotopy [7, 11]. Hence, since fillable contact structures are tight we have $\xi = \bar{\xi}_0$ and $(W_{p,p-1}(0), \omega)$ is a symplectic filling of $(L(p, p-1), \bar{\xi}_0)$. As explained in the next section, the argument we have just used can be generalized to prove Theorem 2.2(a).

4. The proof of Theorem 2.2

In this section we outline the proof of Theorem 2.2. Complete arguments will appear elsewhere [17].

Part (a). The argument consists of two steps. First, we show that each of the manifolds $W_{p,q}(\mathbf{n})$ is a Stein surface (see e.g. [9] for the definition).

A knot K in a closed, contact three–manifold (Y, ξ) is called *Legendrian* if it is everywhere tangent to the distribution ξ. The contact structure induces a framing of K often called the *contact framing*.

The standard tight contact structure ζ_0 on $S^1 \times S^2$ is the kernel of the pull–back of the one–form $zd\theta + xdy - ydx$ under the inclusion $S^1 \times S^2 \subset S^1 \times \mathbb{R}^3$, where $d\theta$ is the standard angular form on S^1 and x, y, z are coordinates on $\mathbb{R}^3$.

The manifolds $W_{p,q}(\mathbf{n})$ are obtained by attaching 2–handles to the boundary of $S^1 \times D^3$. If the 2–handles are attached along knots K_i which are Legendrian with respect to ζ_0 and whose framings equal their contact framings minus one, then for every choice of orientations of the K_i's the resulting smooth four–manifold carries a structure of Stein surface. This construction is sometimes referred to as *Legendrian surgery*. It is originally due to Eliashberg [3], and

was sistematically investigated by Gompf [9]. The first step of the proof of Theorem 2.2(a) is achieved using the following Proposition 4.1, which can be proved via an induction argument.

Proposition 4.1 ([17]). *Let $\widetilde{\zeta}_0$ be the pull-back of ζ_0 under the diffeomorphism (2.3) and let $\mathbf{L}$ be the "thick" framed link of Figure 1. Then, $\mathbf{L}$ is isotopic to a link $\mathcal{L}$ Legendrian with respect to $\widetilde{\zeta}_0$, all of whose components have contact framing equal to zero.*

The existence of a Stein structure on $W_{p,q}(\mathbf{n})$ implies that $W_{p,q}(\mathbf{n})$ does not contain smoothly embedded (-1)–spheres. Moreover, each Stein structure induces an exact symplectic form ω on $W_{p,q}(\mathbf{n})$ and a contact structure ξ on $\partial W_{p,q}(\mathbf{n}) = L(p,q)$ such that $(W_{p,q}(\mathbf{n}), \omega)$ is a symplectic filling of $(L(p,q), \xi)$.

The second step of the argument consists of showing that $W_{p,q}(\mathbf{n})$ carries a Stein structure inducing a contact structure ξ isomorphic to $\overline{\xi}_0$.

Given a 2–plane field ξ on a three–manifold M, there is a Spin^c structure $\mathbf{s}_\xi$ on M which only depends on ξ up to homotopy. The following result is proved via an induction argument which relies on the homotopy invariant $\Gamma(\xi, -)$ introduced in [9].

Proposition 4.2 ([17]). *The Legendrian link $\mathcal{L}$ of Proposition 4.1 can be chosen and oriented so that the contact structure ξ induced by the corresponding Stein structure satisfies $\mathbf{s}_\xi = \mathbf{s}_{\overline{\xi}_0}$.*

By the classification of tight contact structures on lens spaces [7, 11], $\mathbf{s}_\xi$ determines ξ up to isotopy, hence $\xi = \overline{\xi}_0$. This concludes the proof of Theorem 2.2(a).

Part (b). Let (X, ω) be a symplectic four–manifold. A *symplectic string* in X is an immersed symplectic surface

$$\Gamma = \bigcup_{i=0}^{k} C_i \subset X$$

such that:

(a) C_i is a connected, embedded symplectic surface, $i = 0, \ldots, k$;
(b) C_i meets transversely C_{i+1} at one point, $i = 0, \ldots, k-1$;
(c) $C_i \cap C_j = \emptyset$ if $|i - j| > 1$.

We say that Γ as above is *of type* $(m_0, \ldots, m_k)$ if, furthermore,

(d) $C_i \cdot C_i = m_i$, $i = 0, \ldots, k$.

Let (W, ω) be a symplectic filling of $(L(p,q), \overline{\xi}_0)$. Using a cut–and–paste argument which combines results from [3] and [21], it is possible to prove:

Theorem 4.3 ([17])**.** *There exist a symplectic four–manifold X_W and a symplectic string*

$$\Gamma = \bigcup_{i=0}^{k} C_i \subset X_W$$

of type $(1, 1 - b_1, -b_2, \ldots, -b_k)$ such that W is diffeomorphic to $X_W \setminus \nu(\Gamma)$, where $\nu(\Gamma) \subset X_W$ is a regular neighborhood of Γ.

The results of [19] imply that if (M, ω) is a closed symplectic four–manifold containing an embedded symplectic 2–sphere C of self–intersection $+1$ such that $M \setminus C$ is minimal (i.e. not containing embedded symplectic (-1)–spheres), then (M, ω) is symplectomorphic to $\mathbb{CP}^2$ with the standard Kähler structure. Morever, the symplectomorphism can be chosen so that C is sent to a complex line. Thus, since non–minimal (possibly non–compact) symplectic four–manifolds can be reduced to minimal ones by blowing down exceptional symplectic spheres, applying [19] to the pair (X_W, C_0) we conclude that, for some $M \geq 0$, there is a symplectomorphism

$$\psi \colon X_W \to \mathbb{CP}^2 \# M \overline{\mathbb{CP}}^2$$

such that $\psi(C_0)$ is a complex line in $\mathbb{CP}^2$. Clearly, $\psi(\Gamma) \subset \mathbb{CP}^2 \# M \overline{\mathbb{CP}}^2$ is a symplectic string of type $(1, 1 - b_1, -b_2, \ldots, -b_k)$. Now we have:

Theorem 4.4 ([17])**.** *Let $\mathbb{CP}^2 \# M \overline{\mathbb{CP}}^2$ be endowed with a blowup of the standard Kähler structure on $\mathbb{CP}^2$. Let*

$$\Delta = \bigcup_{i=0}^{k} D_i \subset \mathbb{CP}^2 \# M \overline{\mathbb{CP}}^2$$

be a symplectic string of type $(1, 1 - b_1, -b_2, \ldots, -b_k)$ such that $D_0 \subset \mathbb{CP}^2$ is a complex line. Then,

(a) *Δ is the strict transform of two distinct complex linesin $\mathbb{CP}^2$;*
(b) *The complement of a regular neighborhood of Δ is diffeomorphic to $W_{p,q}(\mathbf{n})$ for some $\mathbf{n} \in \mathbf{Z}_{p,q}$.*

Note that symplectic strings have well–defined stricttransforms. Part (b) of Theorem 2.2 follows immediately fromTheorem 4.4 together with the previousdiscussion. The proof of Theorem 4.4(a) relies onpositivity of intersections of J–holomorphic curves [20],while Theorem 4.4(b) follows directly fromTheorem 4.4(a) via a Kirby calculus argument.

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