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A COMPACT SYMMETRIC SYMPLECTIC NON-KAEHLER MANIFOLD: REVISIT

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Abstract. Lerman constructed a twelve dimensional Hamiltonian circle action with an isolated fixed point on a non-Kaehler manifold. In this report, the author constructs such an example which is eight dimensional.

1. Introduction

In [Ler], Lerman constructed a twelve dimensional Hamiltonian circle action with an isolated fixed point on a non-Kaehler manifold. In this report, the author constructs an eight dimensional semifree Hamiltonian circle action with an isolated fixed point on a simply connected non-Kaehler manifold. The example raises the following question: Can we construct such an example which is six dimensional?

2. Construction

Gompf constructed a six dimensional simply connected symplectic non-Kaehler manifold M^6 such that $M^6 \times S^2$ is also non-Kaehler [Gom, Theorem 7.1]. He shows that there exists a nontrivial element q in $H^2(M^6 \times S^2)$ such that $q \wedge w^2 = 0$ for all w in $H^2(M^6 \times S^2)$. Hence by the Hard Lefschetz Theorem, the manifold $M^6 \times S^2$ (also $M^6 \times S^2 \# \overline{\mathbb{C}P^4}$) is non-Kaehler.

We give the trivial circle action on M^6 and the usual rotation on S^2. Hence the manifold $M^6 \times S^2$ has a Hamiltonian circle action with two copies of M^6 as the fixed set. If we blow up $M^6 \times S^2$ at a fixed point with the action of weight $(1, 0, 0, 0)$, then a simple computation shows that the blown up manifold has an isolated fixed point with the action of weight $(1, -1, -1, -1)$. Also, it is non-Kaehler since it is diffeomorphic to $M^6 \times S^2 \# \overline{\mathbb{C}P^4}$.

References

[Gom] R. E. Gompf, *A new construction of symplectic manifolds*, Ann. of Math. **142** (1995), 527–595.

[Ler] E. Lerman, *A compact symmetric symplectic non-Kaehler manifold*, Math. Res. Lett. **3** (1996), 587–590.

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