

## Preface

While the widespread prominence of combinatorial methods in applications dates back to at least the 1960s telecommunications research at Bell Labs, in recent years the scope of applications for which combinatorial methods have relevance has increased dramatically. Combinatorial methods are now common across nearly all areas of science. Accordingly, applied combinatorics has evolved to engage methods from probability, topology, linear algebra, mathematical logic, computer science, and more.

Building off a series of AMS Special Sessions at the 2021 and 2022 Joint Mathematics Meetings organized by the guest editors, this special issue of the Journal of Combinatorics highlights a diversity of applied combinatorics research ranging from quantum computing to RNA branching. Below, we briefly introduce each article and highlight the application areas they concern:

- In “Generalizations of leaky forcing”, Alameda, Kritschgau, and Young consider a variant of zero-forcing, a topic well-known for its applications in phasor measurement unit (PMU) placement in power systems, quantum control, and inverse eigenvalue problems. They focus on so-called *edge leaks*, disruptions in the zero-forcing process that naturally model those one might expect in real systems.
- Deniskin and Benzi introduce and investigate a series of questions on subgraph centrality in “New results and open problems on subgraph centrality”. They investigate a popular measure of graph vertex importance in network science, as well as highlight rich connections to spectral theory and analytic matrix functions.
- Regarding the problem of quantifying the expressiveness of parameterized quantum circuits, Bennink’s work in “Counting Abelian Squares Efficiently for a Problem in Quantum Computing” elucidates how, for a particular subclass of circuits, this problem reduces to that of counting abelian squares over an exponentially large alphabet. Leveraging this connection, Bennink derives and then applies a new formula to quantify the expressiveness of this subclass of circuits.
- In “Pursuit-evasion games on Latin square graphs”, Ahirwar et al. study a class of combinatorial models for detecting or neutralizing adversarial activity on a graph; such games have rich connections to

areas in computer science and AI, including robotics and network security. They analyze the cop number, metric dimension, and localization number of graphs arising from Latin squares.

- With “On a barrier height problem for RNA branching” Heitsch, Huynh, and Johnston use ordered trees as a model for RNA folding. In this framework, they investigate the thermodynamic cost of transitioning between RNA branching configurations.
- Garapaty, Lokshtanov, Maji and Pothen obtain tight bounds on the chromatic number of squares and higher powers of Erdős-Rényi random graphs in “The Chromatic Number of Squares of Random Graphs”. As they explain, such work is relevant in generating benchmark instances to evaluate parallel algorithms that color the input graph in the context of estimating a sparse Jacobian for optimization.
- “Hessian Chain Bracketing” by Naumann and Burela considers the combinatorial Hessian Accumulation problem aiming to minimize the number of floating-point operations required for the computation of a Hessian. They show their dynamic programming formulation for the solution of this problem yields improvements by factors of ten and higher over the current state of the art.

Finally, the issues concludes with an eclectic collection of open problems in applied combinatorics, with hopes of inviting further research. Compiled from contributions from multiple authors, each section is devoted to an open problem, consisting of a self-contained description as well as a discussion of relevant applications and any existing work. We thank the reader for their attention and the authors for their contributions in realizing this special issue.

Sinan G. Aksoy  
Stephen J. Young