Remembering John Coates, the Great Friend of Chinese Mathematics

Shing-Tung Yau

Figure 1. John Henry Coates (January 26, 1945 – May 9, 2022).

On May 9, 2022, Fellow of the Royal Society and internationally renowned number theorist John Henry Coates passed away. Professor Coates worked hard to advance the development of mathematics and mathematical training in China. He was the first recipient of the International Collaboration Prize of the International Consortium of Chinese Mathematicians (ICCM), in recognition of the spirit of selfless service which he exemplified in his efforts training Chinese mathematicians. The following is a tribute to Professor Coates which appeared in "Shu Li Ren Wen".

Translated and adapted by Nathan Thomas Carruth.

Today [May 9, 2022] I received a letter from my old friend John Coates's son David, informing me that his father had passed away. It is a great sadness to see the passing of my friend of fifty years.

Five years ago, he and I, together with other Tsinghua faculty, made a trip to Dunhuang. When we came out of the Han tomb, he didn't feel well, and his face was pale, but he kept pushing himself on. I think of that event over and over.

Coates always treated people well. He wasn't particularly well-off, but was happy to help others, and never felt he had done enough. He put decades of work into helping China, South Korea, and other countries establish modern number theory, training countless number theorists. He could truly be called a peerless international scholar!

Coates's experience was similar to mine. He lost his mother when he was young, and when he was thirteen his father became very ill. He entered Australian National University on a scholarship, was noticed by B. Neumann, and learned number theory from K. Mahler. At the encouragement of these two he went to Paris, where he was noticed by J. W. S. Cassels and went to Cambridge to study, finally completing his doctorate under the direction of Alan Baker. Baker was a very accomplished number theorist and (later) 1970 Fields Medal laureate. Coates studied Baker's theory in a p-adic setting, taking Baker's theory to a new level. They published several important papers together; these papers reflected Mahler's influence on him, and also showed that he had already gone beyond Baker's area of work and into algebraic K-theory.

In 1969, Coates went to Harvard as a Benjamin Pierce assistant professor. There he met John Tate and Barry Mazur. He did important work with Mazur and also made contributions towards the resolution of the Weil conjectures.

In 1972, Stanford University hired him as an associate professor. In 1973, I also went to Stanford to visit. At that beautiful campus, Coates and I became coworkers. Those were the best moments of my life. I also got to know another Australian scholar, Leon Simon, and our student Richard Schoen; they subsequently became a generation of great mathematicians. Together we opened the branch of mathematics which is known today as geometric analysis.

At that point I wasn't married yet, and one of the reasons I went to Stanford was to pursue my girlfriend. I lived in a small room in a mansion on University Avenue, and would drive to campus early in the morning to work. After work my colleagues would go home, but I would stay at the school with the graduate students, find a place to eat, and keep working, often not returning home to rest until 11 o'clock at night. Sometimes I would go out to the suburbs, or go visit some colleague's house; it was a happy life. Coates and his wife Julie often invited me to go to their house for dinner, but to tell the truth, his level of cultural refinement was altogether too high, beyond most others', and I preferred the lively conversations and unrestrained atmosphere at Leon Simon's house. Nevertheless, through Coates I came to understand English etiquette. Afterwards, in interactions with great mathematicians such as E. Bombieri, A. Borel, and J.-P. Serre, I always heard it said that Coates was not only an outstanding mathematician; he was also a very cultured gentleman with refined tastes.

At that time Stanford didn't have any researchers focusing on algebra; none of our colleagues were working in algebra or number theory. (Once, when I was taking the elevator with P. Cohen, R. Philips, and Kai Lai Chung, Chung even boldly said that the field of algebra had been dead and buried since Hilbert, and there was no need to spend any more time on it!) At Coates's behest, in the fall of 1974 Stanford hired Iwasawa's student Larry Washington as an assistant professor. This good fellow's scholarship wasn't bad, but he was quite a character: he would spend every evening with the graduate students talking about all kinds of random things, and he even came to my class looking for girls. This year Coates taught a class on Iwasawa theory and p-adic L-functions which we all attended. He taught deep subjects simply, and his auditors learned a great deal. He ran number theory solo at Stanford, but still trained a great many well-known number theorists. Thinking back on it today really makes one admire him.

In the fall of 1975, Coates left Stanford to become a fellow at Emmanuel College of the University of Cambridge. This college was where Harvard's founder John Harvard had studied; the layout of Harvard Yard is based on it. Here, he met his most important student, Andrew Wiles. Through their collaboration they wrote many extremely important papers, mostly related to elliptic curves. Particularly important were those which contributed to the Birch and Swinnerton-Dyer conjecture, which were published between 1977 and 1978. Afterwards, Wiles proved

Fermat's Last Theorem and the Shimura-Taniyama-Weil conjecture. The methods he used were particularly influenced by his work in the Iwasawa theory of symmetric squares of elliptic curves.

In 1977 Coates returned to his native Australia, and began working to establish a research institute at Australian National University. Unfortunately, things proved too complicated, and in 1978 he went to Paris to work as a professor. He stayed there until he returned to Cambridge in 1986 as a fellow at Emmanuel College. In 1991 he became the head of the Department of Pure Mathematics and Mathematical Statistics at the University of Cambridge.[1] We didn't see each other much in those ten years, just writing each other occasionally.

In the spring of 1993, M. Atiyah invited me and other researchers in mirror symmetry to spend a few months at the Newton Institute at the University of Cambridge. The Newton Institute was established by Atiyah while he was master of Trinity College, Cambridge, and this was an invitation not to be turned down. I suggested that he should also invite some of my friends who had made great contributions to mathematical physics, including Andy Strominger. My first visit to Cambridge had been in August of 1978, at Hawking's invitation. When I returned this time, Hawking was still there, but a lot had happened since then: his was a household name by this point, and he lived in a much nicer place than he had previously. My stay this time was longer than the previous time. I didn't stay in the Blue Boar hotel in front of the university, but in the guest room at Emmanuel College where Coates was. John Harvard previously studied at this college, and the college has a fellow position named after him, specifically for visiting scholars from Harvard, so naturally I became the Harvard Fellow. The title of 'fellow' sounds very nice, but the room was in the basement: it was cold and the ventilation was poor. Coates probably didn't know this. He had a beautiful study and seminar room on the second floor of the college. We would often meet in the seminar room, and he would explain English literature to me. He loved art, and had collected a number of pieces of Eastern art, including Chinese porcelain, Japanese paintings, Indian sculptures, and so on. For twenty years, every time he saw me, he would find the opportunity to spend time alone with me, and give me one of his treasured pieces of Chinese porcelain. Now, with his passing, these gifts remain, and looking at them makes me think of him, and grieve.

No matter whether it was at Stanford or at Cambridge, he and Julie often invited me over to their home to eat. His wife was a true gentlewoman in word and in deed; they were a special couple. When they had me over for dinner, they would often also invite several other visiting scholars from China. Julie was a very good cook, and her food was far better than what one would have gotten in a restaurant. All of us had a good time.

I pointed out to Coates many times that number theory research in China was too narrow: it needed to be broadened and include algebraic number theory

[1] *Translator's note:* the Faculty of Mathematics at Cambridge consists of this department together with the Department of Applied Mathematics and Theoretical Physics.

and computational geometry. He readily agreed to help me and develop modern number theory in China. He hosted many Chinese scholars, some of whom were from Peking University, and all of whom were inspired by him. After they returned to China, they guided their students to new directions of research in number theory. Shortly afterwards, he came himself, and lectured on number theory at the Morningside Center of Mathematics, inspiring many young people. I was very moved. He was very hospitable, and especially liked entertaining friends from Asia. He was an admirably generous man.

When he had just been chosen as vice-president of the International Mathematical Union [in 1991], he told me how he had, a few years previously, asked Atiyah to help the Chinese Mathematical Society join the IMU. I asked him to support China's hosting the International Congress of Mathematicians (ICM) in Beijing, and he agreed without reservation. Ten years later, in 2002, after effort by many people, the ICM was indeed held in Beijing. He was a very chivalrous man who was always willing to fight for what was right.

I remember during my visit at Emmanuel College, in May, when the weather began to get warm, the two of us went for a walk around the college grounds, and seeing a pool of clear water, I suggested that we should go swimming together. He considered for a moment but then agreed. After we changed clothes, I went into the water first, but to my surprise the water in Cambridge in May was still icy cold, as though it were winter. I could not but immediately climb out again, and, not without some chagrin, watch him swim around freely in the pool. He told me an interesting story: one winter he was driving along the seashore with

the famous Russian mathematician L. Faddeev, when Faddeev suddenly got up, shed his clothes, and jumped into the sea. This made me realize that I was not as strong as other people, and needed more exercise.

In 1996, Lo Yang and myself started the Morningside Center of Mathematics, which was aimed at mathematicians throughout the country, or even the whole world. In this effort we got help from Yongxiang Lu, the president of the Chinese Academy of Sciences, and from the central government, as well as support from Coates, for which I was most grateful. This allowed us to successfully establish the Morningside Center of Mathematics as a small but bold research institute, which led to new discoveries in many fields of mathematics and benefited top schools throughout the country. One thing in particular of which we can feel proud is the following: because we had the support of Coates for such a long time, and also had the support of Shou-Wu Zhang and scholars from the Institute of Mathematics and Tsinghua University, the Morningside Center of Mathematics trained a large number of talented number theorists. Several young scholars at Peking University were trained at this center. Ye Tian later returned to China, and his growth and efforts were also supported by Coates. We are forever grateful for all of this.

In 1998, the year the Morningside Center of Mathematics was formally opened, the International Congress of Chinese Mathematicians was held in the Great Hall of the People in Beijing, with the full support of Coates and other foreign guests.[2] The success of this conference was greatly influenced by these foreigners who loved China.

Today's Chinese number theorists, though they may not be the most seasoned experts, are also not neophytes who know only a modicum of analytic number theory. Coates's contributions to Chinese number theory were tremendous. At the same time, he also cared about the development of Korean number theory; he cared about number theory everywhere without distinction.

Five years ago, I organized an inspection tour at Tsinghua, with members including my old friend Coates, Wilfried Schmid, Jun Li, and Xiangyang Shen and his wife, together with a large group of teachers and students from Tsinghua. We travelled all over, from Yangguan, through Dunhuang, back to Lanzhou, and on to Tianshui. We were able to take in Qilian and Gebi, the Ruoshui and the Yellow River: all the scenery of western China. It was such a joy to take this marvellous journey through these ancient areas and talk about many topics, old and new. Coates was familiar with the cultures of both India and western China, and talking with him drew us even closer. It was a pity that his physical condition was not what it used to be.

In the winter of 2019, Coates and Schmid accepted our invitation to participate in Tsinghua University's Master Lectures series held at Sanya. He gave three comprehensive lectures on the history of Iwasawa theory. Every time I listened to him talk, I felt that I had gained a lot. It didn't occur to me that that was the last time I would see Coates.

[2] *Translator's note:* A short report on this inaugural conference was later published in these *Notices*, see [1].

Figure 2. From left: Xiangyang Shen, Coates, Yau, Wilfried Schmid.

May Professor Coates rest in peace; Chinese mathematicians will always remember your friendship.

Every three years, at the International Congress of Chinese Mathematicians, we will award the John Coates International Collaboration Prize to remember this great mathematician, our eternal friend! This international collaboration prize was first given in the 2004 meeting in Hong Kong, but didn't have a name. Coates was the first person to receive this prize; now we name it after him, which everyone feels is very appropriate. We give his remarks of August 2004 at the Zhejiang University Center of Mathematical Sciences:[3]

"The international summer workshop on p-adic computational geometry is about to conclude. I regret that I will soon depart from Hangzhou, leaving Zhejiang University to return to the University of Cambridge. I very much like the rich academic atmosphere on Zhejiang University's campus. Even during the heat of summer, a large number of students were still diligently studying at Qiushi College. They made a very positive impression on me. Zhejiang University's campus is peaceful and attractive. I am certain that the ground of Chinese mathematics is as fertile as the rich fields of Zhejiang Province, and we have sowed many seeds

[3] *Translator's note:* I have been unable to locate the original for the following quote and can only render a translation of what is presumably itself a Chinese translation of the original English.

on this rich soil with high expectations."

The back of the prize is engraved with a few sentences I learned from the *Shi Jing*:[4]

> Stones of other hills,
> Polish gems of jade –
> As by the knife and file,
> The engraving pick, the grind.
> Let drums roll and bells ring,
> Let the Huai waters swell;
> Let us rejoice in you, noble man,
> Let the music of your virtue resound.

He appreciated these lines very much:[5]

> Dear Yau,
> I have attached photographs of the two sides of the Medal. I am very moved that I was awarded this Medal by the ICCM. I assume that it was you who selected (and translated?) the poem on the back of the Medal?
> Very best wishes,
> John

Letters exchanged with Coates
S.-T. Yau to Coates, May 5 2022:

> Dear John,
> I got the sad news that your illness is getting very serious. I feel really sorry for this. We have been close friends since I met you first time about fifty years ago in Stanford. I was young and so were you. But I always looked at you as my big brother. Indeed, you have always treated me like your little brother, with love for my career, for my family and for my ambition to help Chinese to build a new program in number theory. You have never disappointed me when I need your helps. Your kindness and your deep contribution to number theory has served as my role model in the past fifty years.
> You were the only number theorist in Stanford. I knew little about the subject. I was struggling to learn the Iwasawa theory, the Mazur-Coates theory and all that interesting subjects from the class that you taught. You wrote beautifully and explained patiently to all beginners like me. You hired Larry Washington as assistant professor and we had a lot of fun together. But beyond mathematics, I am always fond of and respect your elegant taste in life. Those lessons I learn from you and Julia went far beyond those I learnt from you in class on the very high level mathematics you taught me.
> It was a big loss to Stanford when you left But I think your settlement in Paris and later in Cambridge were very good decisions. You have trained

[4] *Translator's note:* the translation here is that given on the prize itself.

[5] *Translator's note:* the three letters exchanged between Coates and Yau are presented here verbatim as they appeared in the original article.

so many spectacular students who became great leaders in number theory. You are so kind to all people who are interested in number theory that I was able to convince you to come to the Morningside institute to train and greatly broaden the scope of number theory in China.
We were really happy to have Shou Wu to come back constantly to help. Now several new generations of good number theorist grew up, China and the world of number theory owed a big debt to both of you.
As a small token, we decided to name our international cooperation award in ICCM after you. From now on, the award will be called John Coates international award. I hope you are pleased, as an award to your love and contribution to China.
I really miss you, John! For all those days and nights together, I am fond of talking with you on all nice topics. We went out together many times to buy antiques and you always gave me those precious gifts. I am truly grateful to your friendship. The trip to Dun Huang with you is so memorizable that I like to write much more beyond those poems that I composed in that trip.
John, god bless you and your Chinese friends will always remember yous. If there is any need from Julia and you sons, we shall always be there.
Best regards
Your friend YAU

Dear Yau,
Thank you for your kind email which my brother forwarded to me and which I read to my father earlier.
He dictated the following response for me to send as a reply.
Best,
Philip Coates
Dear Yau,
I was very touched by your very kind letter. It was indeed a happy chain of circumstances that brought us together as young mathematicians at Stanford long ago. Always you have given your wonderful support to the development of number theory in China, and this made my time there both practical and fulfilling. Thank you very much again. I hope to write myself later if I am stronger.
John

References

[1] *Reports of ICCM from 1998 to 2010*, ICCM Notices **1** (2013), no. 1, 104–106. MR3155201

Shing-Tung Yau
styau@tsinghua.edu.cn
YMSC, Tsinghua University, Beijing (China)