Obituary – Vaughan Jones

by Zhengwei Liu

Sir Vaughan Frederick Randal Jones, a famous New Zealand mathematician, passed away on September 6, 2020 in Nashville, Tennessee, USA. Many people know about his knot invariant, the Jones polynomial. This work has brought about profound changes in many fields of mathematics and physics, and it has also been used in the identification of DNA knot structures in biology.

Jones was born on December 31, 1952 in Gisborne, a small town in New Zealand. During his years at boarding school, St. Peter's School Cambridge, and high school, Auckland Grammar School, Jones became interested in mathematics and natural science. Jones obtained a Bachelor's degree from the University of Auckland in 1972 and a Master's degree the following year.

A Swiss government Scholarship supported his early academic career at the University of Geneva in Switzerland, where he was a Ph.D. student of André Haefliger. During his doctoral study, Jones met his future wife Martha, and Jones always called her by her nickname, Wendy. In 1979, they entered into marriage in Wendy's hometown of Westfield, New Jersey. Like many famous mathematicians, Jones completed a series of outstanding work nourished by love. The first contribution was his doctoral dissertation.

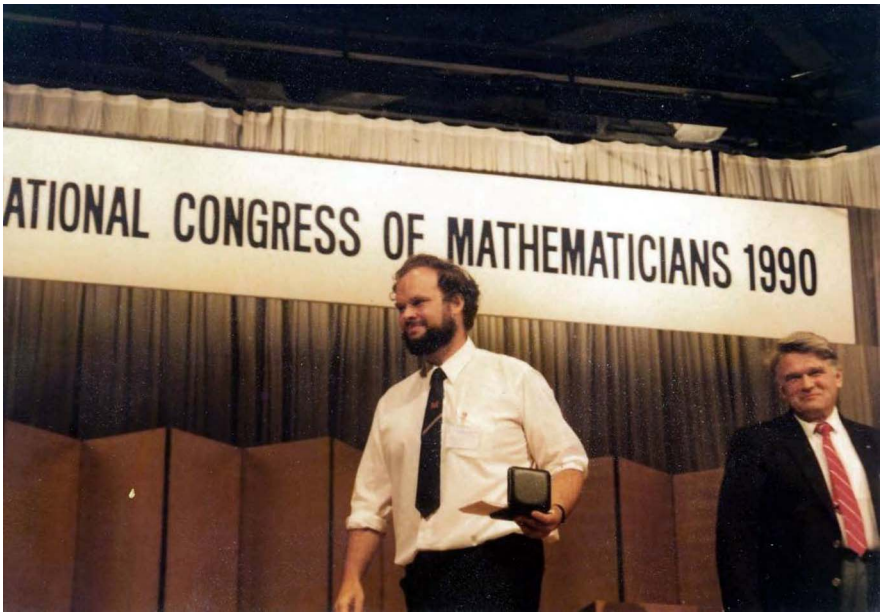
In 1979, Jones received his Ph.D. In his thesis, Jones gave the classification of the finite group actions on the hyperfinite II_1 factor. This topic was recommended by Alain Connes and inspired by Connes' previous classification of cyclic group actions. This work can also be interpreted as the classification of certain group-like subfactors, a foreshadowing of the subsequent connection between subfactor theory and three-dimensional topological quantum field theory.

After receiving his Ph.D., Jones went to the United States and worked as a Hedrick Assistant Professor from 1980 to 1981 at the University of California, Los Angeles, as an assistant professor from 1981 to 1984 and associate professor from 1984 to 1985 at the University of Pennsylvania, and as a professor from 1985 to 2013 at U.C. Berkeley. He served as the Stevenson Distinguished Professor from 2011 to 2020 at Vanderbilt University.

A childhood photo of Jones

Throughout Jones' academic career, I believe that Jones would choose the subfactor as a representative term since most of his research work was carried out around subfactor theory, which he began during his graduate studies.

The size of a subfactor can be characterized by its index, which can be seen as the generalization of the order of a group. In contrast to the usual index theory, the range of the index of subfactors can be continuous. This phenomenon can be traced back to Murray and von Neumann's early classification of factor modules. Jones published his classification of indices of subfactors in 1983. The magic is that the index can take any positive number from 4 to infinity, and a discrete sequence below 4. This quantization phe-



nomenon finally led to a profound connection with quantum physics. This work is the beginning of modern subfactor theory, and the index of a subfactor is also called a Jones index.

The most challenging part of the initial classification of the Jones indices is the construction of subfactors with indices less than 4. Jones found that the existence of such subfactors corresponds to the positive semi-definiteness of the trace on a *-algebra generated by a sequence of Jones projections. Temperley and Lieb had looked at the same sequence of projections in two-dimensional statistical mechanics, and the *-algebra is called the Temperley-Lieb algebra. This led to a deep connection between subfactor theory and integrable models in statistical mechanics. Moreover, while studying the representation theory of this algebra, Jones constructed novel braid group representations.

When Jones first met Joan Birman, a leading expert in knot theory, in 1984, they had a long discussion about Jones' discovery and Markov's theorem for knots. Jones realized that the trace on braid groups that he constructed through subfactor theory was consistent with the Markov condition in knot theory, and this led to a new knot invariant, namely the famous Jones polynomial. The definition and calculation of the Jones polynomial is very concise and can be used to distinguish a variety of knots. This led to answering a series of long-standing questions in knot theory, including many conjectures put forward by Tait in the 19th century, which has opened a new chapter for modern knot theory.

In 1990, Jones won the Fields Medal at the International Congress of Mathematicians in Kyoto in recognition of his series of major breakthroughs.

Jones' work in the 1980s not only established a close connection between operator algebras and knot theory, but also led to the connection between low-

dimensional topology, quantum groups, representation theory and many other mathematical fields, as well as statistical physics and quantum field theory. At that time, Michael Atiyah organized a seminar at the Institute for Advanced Study in Princeton, hoping to find the connection between low-dimensional topology and representation theory, and Jones' work was exactly what they were looking for. With the joint efforts of many mathematicians and physicists, many fields of mathematics and physics once again merged and flourished. Mathematics and physics complement each other, giving birth to a number of emerging, intersecting fields, such as quantum topology. One representative breakthrough is Witten's interpretation of the Jones polynomial as path integrals in topological quantum field theory through Chern-Simons theory. Witten was also awarded a Fields medal at the ICM in Kyoto in 1990.

In the 1990s, Jones and many experts systematically studied the representation theory of subfactors, and gave the ADE classification of hyperfinite subfactors with Jones' indices less than 4. This classification result is mutually corroborated with the ADE classification in conformal field theory. Spending 10 years, Jones completed the monograph preprint "Planar Algebras I" in 1999. (For some special reasons, this monograph has not been officially published.) Planar algebra is a way to characterize the representation theory of subfactors, which integrates ideas in operator algebras, knot theory, representation theory, category theory, topological quantum field theory, integrable systems, and many other areas in mathematics and physics. This monograph contains a large number of representative and profound examples. Jones prefers subtle examples to abstract theory.

In the last 20 years, Jones and his colleagues and students have given the classification of subfactor planar algebras with the Jones indices up to 5.25 using the theory of planar algebras. In the monograph "Planar Algebra I," Jones also proposed another classification scheme based on generators and relations. In these classification works, a series of interesting new examples have been discovered. Based on planar algebras, Jones also further studied the relationship between subfactor theory and free probability theory, Thompson groups and other theories. The development of the field of subfactors includes the representative work of many experts, for which I refer to my talk for further details. [1]

Jones' work is extremely original and has had a profound impact on many fields of mathematics and physics. He trained more than 30 doctoral students. His students and collaborators are all over the world. Jones won many important international awards. Jones made great contributions to the development of mathematics in New Zealand. He won the New Zealand Government Science Medal (Rutherford Prize) and he was elected Honorary Fellow of the Royal Society of New Zealand in1991. He was made a Distinguished Companion of the NZ Order of Merit (DCNZM) re-designated Knight Companion KNZM in 2009. Jones strongly supported the development of Chinese mathematics and he won the Cooperation Award of the International Congress of Chinese Mathematicians in 2019. Jones is a member of many national associations, including the Royal Society of the London, the Royal Society of New Zealand, the Australian Academy of Sciences, the American Academy of Sciences and Arts, and the U.S. National Academy of Sciences. Jones served as the Vice President of the American Mathematical Society from 2004 to 2006, and as the Vice President of the International Mathematical Union from 2014 to 2018.

My doctoral experience with Vaughan Jones

Recently, I unfortunately learned that my advisor, Vaughan Jones, had passed away. It saddened me so much and I could not calm down for a long time. There are too many memories between Jones and myself, I want to write all of them down, but I do not know where to start. Here, I want to talk about my doctoral experience with Vaughan Jones and the story of completing my doctoral dissertation, in order to cherish the memory of my advisor.

The scene of my first meeting with Vaughan is still fresh in my mind. In early 2011, I generalized the 2000 and 2003 classification results of Vaughan Jones and Dietmar Bisch. Recommended by Vaughan's student Emily Peters, Vaughan and I met at a conference in Dartmouth. Before that, I had been learning about Vaughan's work through his articles and I got a feeling for his mathematical style. I was very excited and nervous since I would be meeting Jones face to face soon. After the meeting, Vaughan took me to a local cafe. His collaborative nature allowed me to quickly transition into a discussion of mathematics. Vaughan told me that the classification results I generalized had previously been obtained by Dietmar and himself. They thought that this subsequent classification would only get Birman-Murakami-Wenzl (BMW) subfactor planar algebras, and no new examples would appear, so they did not continue, and the relevant results were not published either. Vaughan explained that this classification problem should be viewed from the perspective of generators and generative skein relations, so that his and Dietmar's classification results could be summarized as the classification of subfactor planar algebras of one generator with a commute/exchange relation. So Vaughan suggested that I con-

sider the corresponding classification of two generators and he hoped to find new examples in the classification. After discussing mathematics, Vaughan invited me to study with him for a Ph.D. This is what I had always dreamed of. (At that time, Vaughan was considering whether to retire from UC Berkeley to work at Vanderbilt University.) My first impression of Vaughan had been that he was more like an athlete than an academic. After one afternoon of discussing mathematics with him, however, I fully felt his demeanor as a mathematical master. From the background of mathematical problems and research motives to the latest progress, Jones could always combine exquisite examples and explain things to me in a simple way.

Two months later, I solved the classification problem of two generators. Vaughan was very happy and asked me to talk about the results at the annual meeting "Subfactors in Maui 2011" in Hawaii in July. In August, I came to Vanderbilt University with Vaughan and became his first student at Vanderbilt. My classification work also became the opening report of my graduate study.

The original proof of my classification results requires solving a large number of complicated algebraic equations through graphical calculations. Vaughan felt that there was more profound mathematics behind this and asked me to improve the proof. I cannot count how many versions I have written. During this period, I investigated the positivity on subfactors and proved several quantum inequalities which captured algebraic structures as the conditions of their equal sign. Through these analytical methods, I was able to solve the algebraic equations in a much simpler way, and to extend the classification result from two generators to any number of generators, with commute relations. Different from previous classifications in subfactor theory, this skein-theoretical classification has no restriction on the Jones index nor dimension. Vaughan was finally satisfied with my classification results and proofs, and he asked me to sort them out and to publish them. When I was writing my article, Vaughan patiently guided me, helping me with revisions word-by-word and teaching me how to use software to draw pictures. Obviously, my writing ability was far less than my mathematical ability and Vaughan spent a lot of energy in guiding my writing. Jones did not care much about which journal to submit the work to and asked me not to care about it either, he told me to focus on mathematics. Vaughan believes in his judgment of academic value.

These analytic methods have been further developed as Quantum Fourier Analysis. Using these analytical methods, I also found several other classification results, and completed two other papers with my collaborators. While investigating the skein-theoretic classification of subfactors, I discovered a potential family of planar algebras generated by a crossing with a Yang-Baxter relation (a generalization of Yang-Baxter equation), but it is not Birman-Murakami-Wenzl (BMW). The appearance of this family does not match the classification that Dietmar and Vaughan expected. After explaining this phenomenon to Vaughan, he was very excited and asked me to focus on this problem. (In the fall of 2013, I had solved an important open problem proposed by Dietmar and Uffe Haagerup in 1994. Vaughan had asked me to use it as my graduation thesis and graduate in May 2014. After discovering this new phenomenon, however, we both thought that I should not consider graduation for the time being. The main focus was on constructing a potentially new family of examples.)

Vaughan told me that it took him about 10 years to write the monograph "Planar Algebra I." One of the main motivations for proposing the theory of planar algebras is to construct new planar algebras and subfactors through generators and skein relations. He and Dietmar hoped to discover new examples from such classifications. Unfortunately, no new examples had been found in the classification results in this regard. It would be a major breakthrough, if one could construct this potential family of planar algebras. At that time, we knew little about the construction of a continuous family of subfactor planar algebras by generators and skein relations, and we needed to start from scratch. There were three essential problems to overcome: Evaluation, Consistency and Positivity.

One key suggestion from Vaughan was solving the Yang-Baxter equation. I admire Vaughan's amazing intuition; he could always point out the crux of the problem. This family of planar algebras has three different solutions to Yang-Baxter equations. By referring to the methods in knot theory, I overcame the first two problems. The third problem was very challenging. By combining methods in analysis, algebra, topology and other areas, I conquered this challenge several months later. This result was finally compiled into my graduation thesis. Vaughan praised this work very much: I had become the Ph.D. student of Vaughan that I had always wished to be and I was graduated in May 2015.

On the eve of the graduation ceremony in May 2015, my parents came to Vanderbilt and met Vaughan and his wife Wendy for the first time, and they had a very happy conversation.

I was fortunate to be a Ph.D. student of Vaughan, and I learned a lot in those four years. He substantially improved my understanding of mathematics

and always encouraged me to explore unknown areas. Vaughan's guidance to me was more directional than knowledge-based. More importantly, he would shield me from outside interference in various ways and allowed me to maintain a state of study without distraction. Vaughan and I have similar tastes in mathematics. Every time I talked about mathematics with him, there would be endless topics, and I often miss the time discussing problems in his office.

Vaughan, I wish you a good journey in heaven!

Your student,
Zhengwei Liu

Acknowledgement

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References

[1] Zhengwei Liu, "Subfactors-Dedicated to Vaughan Jones", Literature Lecture Series, November 23, 2020, Harvard CMSA. https://cmsa.fas.harvard.edu/literature-lecture-series/