The 5th New World Mathematics Awards

Introduction of the New World Mathematics Awards

Co-initiated in 2007 by Dr. Henry Cheng, Chairman and Executive Director of New World Development Co. Ltd. and Professor Shing-Tung Yau, New World Mathematics Awards (NWMA) aim to select and encourage outstanding Chinese mathematics students worldwide in their pursuit of mathematical truth and contribution to the development of mathematics in China and the rest of the world. NWMA is a triennial event coinciding with the ICCM. The appraisal and election of NWMA winners takes place on an annual basis, and the Award Ceremony, on a triennial basis. So far, it has been held successfully for four times, and discovered a group of outstanding young talents in Fundamental Mathematics, Applied Mathematics, Probability and Statistics, Computational Mathematics, Operational Research and Control, etc. Some winners have now become young leaders in mathematics.

More than 1100 students from a total of 190-plus colleges and universities in Mainland China, Hong Kong and Taiwan as well as the rest of the world have participated in the NWMA event since its kickoff. Of the approximate 200 winners of NWMA, 80% are pursuing advanced studies or teaching at prestigious universities such as Tsinghua University, Peking University, Stanford University, and UC, Berkeley; some of them have now become the leaders in mathematics. NWMA winners are given priority when joining major research institutes or admission to colleges and universities.

Qualification

Candidates: PhD, masters and bachelors of the Department of Mathematics who have successfully defended their theses. Candidates' submission: their graduation thesis. The topics of thesis can be Fundamental Mathematics, Applied Mathematics, Probability and Statistics, Computational Mathematics, Operational Research and Control, Financial Mathematics, and Biomathematics, etc.; and the thesis cannot be accepted unless it is recommended from a senior mathematician. Recommendation mechanism: A senior mathematician can nominate a qualified student to participate in the selection process, and recommend excellent graduates to take part in the event. Application Process Please visit the official website of NWMA at http://ymsc.tsinghua.edu.cn/NWMA/ for registration and application. In the case of a non-Award year: Application starts from April, and ends in mid-September each year. In the case of an Award year: Application starts from March, and ends in mid-April of the year. Selection Process To ensure fair, equitable and high-standard requirements of the thesis review process, all submissions are subject to two rounds of review:

1. Firstly, a professional review committee conducts a preliminary review. For each thesis, 2–3 renowned mathematicians are assigned to conduct a correspondence review. This round of review ends either in mid-November each year (in the case of a non-Award year), or in late April of an Award year.
2. According to the scores from the correspondence review, these are selected for the second round of review. A comprehensive review committee conducts a final review, and selects the final list of winners. This round of review ends either in December each year (in the case of a non-Award year), or in May of an Award year.

The Awards NWMA falls into three categories: PhD, Master and Bachelor awards, each including gold, silver and excellence awards. Each year, all categories can have 3 gold awards and 5 silver awards; and every three years, there will be a total of 9 gold awards and 15 silver awards for the PhD, Master and Bachelor categories.

Visit http://ymsc.tsinghua.edu.cn/nwma/.

Organization

International Committee

Shing-Tung Yau

Professor Shing-Tung Yau is the William Casper Graustein Professor of Mathematics at Harvard University. He is the inaugural Director of the Yau Mathematical Sciences Center of Tsinghua University. Professor Yau has made fundamental contribution to differential geometry, differential equations and mathematical physics. He is a member of the United States National Academy of Sciences, a member of Russian Academy of Sciences, a foreign member of the Chinese Academy of Sciences, and a member of Academia Sinica.

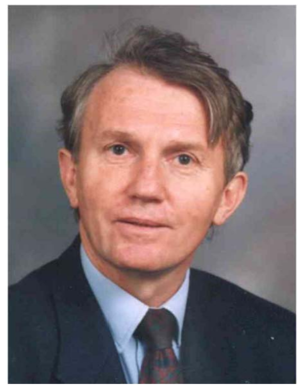
Prizes and Awards:

1981, Oswald Veblen Prize
1982, Fields Medal
1984, MacArthur Fellow
1994, Crafoord Prize
1997, United States National Medal of Science
2003, China International Scientific and Technological Cooperation Award
2010, Wolf Prize in Mathematics
2018, Marcel Grossmann Award

Ben Andrews

Professor Ben Andrews is a senior fellow of Mathematical Sciences Institute, Australian National University, and Professor of the Mathematical Sciences Center of Tsinghua University. Professor Andrews works on differential geometry and related partial differential equation and is well known for his work in geometric evolutions. He is a leading international geometric analyst, who specializes in Riemannian geometry, submanifold geometry, heat flows, image processing, interface model and reaction-diffusion system.

John H.Coates



Professor John H. Coates is the Sadleirian Professor of Pure Mathematics at the University of Cambridge. His research work concerns number theory, arithmetical algebraic geometry and Iwasawa theory. He became Head of the Department of Pure Mathematics and Mathematical Statistics at Cambridge in 1991, served as president of the London Mathematical Society during 1988–1990 and as vice-president of the International Mathematical Union from 1991 to 1995, as a member of Council of the Royal Society during 1992–1994. Professor Coates was elected a fellow of Emmanuel College in Cambridge twice, a fellow of the Royal Society of London in 1985 and awarded Senior Whitehead Prize by the London Mathematical Society in 1997. Professor Coates is the first receipt of the ICCM International Cooperation Award (2004).

Björn Engquist

Professor Björn Engquist is Director of the Parallel and Scientific Computing Institute. Engquist currently holds the Computational and Applied Mathematics Chair at the Institute for Computational Engineering and Sciences at the University of Texas at Austin. He has been a leading contributor in the areas of multiscale modeling and scientific computing, and a productive educator of applied mathematicians. His research field is computational and applied mathematics and numerical methods for differential equations with applications to multi-scale modeling, electromagnetism, and fluid mechanics. Engquist has authored more than 100 scientific publications and advised 31 PhD students. He is a recipient of numerous distinctions and awards: a member of the Royal Swedish Academy of Sciences and the Royal Swedish Academy of Engineering Sciences, and an invited speaker at the International Congress of Mathematicians (1982 and 1998), European Congress of Mathematics (1992), and European Congress of Fluid Mechanics (1991). He was selected to the Norwegian Academy of Science and Letters in 2011.

Dorian Goldfeld

Dorian Goldfeld is a professor at Columbia University. He is a member of the editorial board of Acta Arithmetica and of The Ramanujan Journal. He is a co-founder and board member of SecureRF, a corporation that has developed the world's first linear-based security solutions. Professor Goldfeld's research interests include various topics in number theory. In his thesis, he proved a version of Artin's conjecture on primitive roots on the average without the use of the Riemann Hypothesis. In 1987 he received the Frank Nelson Cole Prize in Number Theory, for his solution of Gauss' class number problem for imaginary quadratic fields. He has also held the Sloan Fellowship (1977–1979) and in 1985 he received the Vaughan prize.

Eduard Looijenga

Professor Eduard Looijenga is a Professor of Yau Mathematical Sciences Center of Tsinghua University. His research started in singularity theory, but migrated via Torelli problems to locally symmetric varieties, then to mapping class groups and moduli spaces of curves. His recent work is concerned with automorphic forms with poles that are associated with moduli problems and with the algebraic geometry of Wess-Zumino-Witten systems. One of his major works is a solution of the Zucker conjecture concerning identification of the L^2-cohomology of an arithmetic Hermitian locally symmetric space and the intersection cohomology of the Baily-Borel compactification of the space. Professor Looijenga was an invited speaker at the ICM in 1978 and at the ECM in 1992. He is a member of the Royal Netherlands Academy of Arts and Sciences, and an editor of The Michigan Mathematical Journal and Journal of The European Mathematical Society.

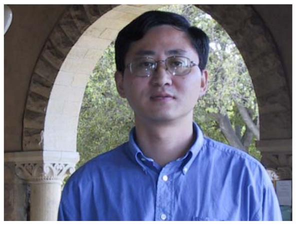
Stanley Osher

Stanley Osher is currently Director of Special Projects at the Institute for Pure and Applied Mathematics (IPAM) at the University of California Los Angeles, and Director of Applied Mathematics. Stanley Osher has made fundamental contributions to applied mathematics, computational science, and scientific computing, and has cofounded three companies based on his research. He has applied level set methods for partial differential equations to the field of image processing, to video image enhancement, and movie animation. He has been featured in international media such as Science News, Die Zeit, and Los Angeles Times. Stanley Osher is a recipient of the 2007 USACM Computational and Applied Sciences Award, he was awarded Docteur Honoris Causa in 2006, and elected to the National Academy of Sciences in 2005. Stanley Osher has received the SIAM Kleinman Prize in 2005, the SIAM ICIAM Pioneer Prize in 2003, the NASA Public Service Group Achievement Award, and the Japan Society of Mechanical Engineers Computational Mechanics Award.

Thomas Yizhao Hou

Thomas Yizhao Hou is Charles Lee Powell Professor of Applied and Computational Mathematics in the Department of Computing and Mathematical Sciences at the California Institute of Technology. Hou is known for research on multiscale analysis. He is an author of the monograph Multiscale finite element methods. He has worked extensively on numerical analysis and applied analysis of the Navier-Stokes equations. His recent work focuses on adaptive data analysis. He received an Alfred P. Sloan Research Fellowship in 1990. He was awarded the Feng Kang Prize in Scientific Computing in 1997. He received the James H. Wilkinson Prize in Numerical Analysis and Scientific Computing from the Society for Industrial and Applied Mathematics (SIAM) in 2001, Morningside Gold Medal in Applied Mathematics in 2004, and Computational and Applied Sciences Award in 2005. Hou has also been inducted into several scholarly societies. He was elected Fellow of the Society for Industrial and Applied Mathematics in 2009, Fellow of the American Academy of Arts and Sciences (AAAS) in 2011, and Fellow of the American Mathematical Society (AMS) in 2012.

Jun Li



Jun Li is a professor at the Department of Mathematics, Stanford University. Since receiving his Ph. D from Harvard University in 1989, he was on the faculty of UCLA from 1992–1996, before joining Stanford University. His research interest is in algebraic geometry; he has made significant contribution to the research on moduli of vector bundles, stable morphisms and Gromov-Witten invariants. He was the recipient of Sloan fellowship, Terman fellowship; he was awarded the morningside medal in 2001.

Akito Futaki

Akito Futaki is a Professor of Graduate School of Mathematical Sciences at University of Tokyo. His main research interests are differential geometry, complex algebraic geometry. He is interested in the existence problem of extremal Kähler metrics such as Kähler-Einstein metrics, and related problems in geometry. He received the Geometry Prize of Mathematical Society of Japan in 1990, Autumn Prize of Mathematical Society of Japan in 2011. He was elected professor emeritus of Tokyo Institute of Technology in 2012.

Lo Yang

Professor Lo Yang was the Director of Institute of Mathematics (1987–1995) and the President of Academy of Mathematics and System Science (1998–2002), CAS. Now he is the professor and Chairman of Scientific Committee of AMSS. He was elected as the academician of the Chinese Academy of Sciences in 1980. Besides, he was the President (1992–1995) of Chinese Mathematical Society. Professor Yang was mainly engaged in the research on complex analysis. He has made a through study of deficient values and deficient functions. He, cooperated with Guang-hou Zhang, established for the first time a close relation between the numbers of deficient values and Borel directions of entire and meromorphic functions. Among his research on normal families, he built the relationship between normal families and fix-points, as well as that between normal families and differential polynomials. He also made the systematic research on the angular distribution: finding a new kind of singular direction and establi-shing a necessary and sufficient condition for the distribution of singular directions. Distribution Theory was published by the Springer-Verlag.

Horng-Tzer Yau

Horng-Tzer Yau is a professor of Harvard University. Professor Yau is a leader in the fields of mathematical physics, analysis and probability. He is a powerful analyst who has introduced important tools and concepts to study probability, stochastic processes, nonequilibrium statistical physics and quantum dynamics. His insight and skilled teaching are invaluable to students. He has also been a member of the Institute for Advanced Study in Princeton, in 1987–1988, 1991–92, and 2003, and a member of the American Academy of Arts and Sciences. He is a member of the editorial boards of Communications in Mathematical Physics, Journal of Statistical Mathematics, Asian Journal of Mathematics and Communications on Pure and Applied Mathematics. He received Henri Poincaré Prize, MacArthur Fellowship and Morningside Gold Medal of Mathematics in 2000 and 2001. He is elected a fellow of the US National Academy of Sciences in 2013.

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Recipients of the 5th New World Mathematics Awards (2017–2019)

Doctor Thesis Awards, Gold Prize

Gao Yu, Harbin Institute of Technology

Thesis Title: Well-Posedness for Two Types of Partial Differential Equations by Partical Methods

Abstract: The past several decades have seen significant development in the design and numerical analysis of particle methods for approximating solutions of PDEs. Even though the most "natural"application of the particle methods is linear transport equations, over the years, the range of these methods has been extended for approximating solutions of convection-diffusion and dispersive equations and general nonlinear problems. In this thesis, we mainly introduce some particle methods to study two different types of partial differential equations. One is a nonlinear dispersive equation and the other one is a Vlasov type equation.

Firstly, we study the modified Camassa-Holm (mCH) equation with cubic nonlinearity in one dimension. Similarly to the famous KdV equation, the mCH equation is also used to describe shallow water waves. It is a nonlinear dispersive equation and a completely integrable system, which has a bi-Hamiltonian structure and a Lax-pair. As it is known, the strong solutions to this equation exit locally and they are unique. However, strong solutions may blow up in finite time for some initial data. One of our main purpose is to study how to extend the solutions after the blow up time, which means to study the global existence of weak solutions. We start with the special solutions, N-peakon solutions, to the mCH equation. Due to the collision between peakons, the characteristic equations for the trajectories of peakons have a non-Lipschitz vector field. Hence, the nature question is how to extend the trajectories globally. Inspired by the sticky particle model in astrophysics, we give a sticky particle model for the mCH equation. This method gives the global sticky N-peakon solutions. For general initial data in Radon measure space, we use the mean field limit of the sticky particle model to obtain global weak solutions. Moreover, we prove that the weak solutions are stable in some solution classes. We also provide some examples to show the non-uniqueness of N-peakon weak solutions. Secondly, we present a dispersive regularization for the uniqueness of the weak solutions to the mCH equation. This method is similar to the vortex blob method for incompressible Euler equation. As mentioned above, the peakons may collide in finite time, which makes the vector field of characteristic equations non-Lipschitz. To obtain unique global N-peakon solutions, we present a regularized system

by a double mollification for the characteristic equations. From this regularized system of ODEs, we obtain approximated N-peakon solutions with no collision between peakons. Then, a global N-peakon solution for the mCH equation is obtained, whose trajectories are globally Lipschitz functions and do not cross each other. Then, by a mean field limit process, we obtain global weak solutions for general initial data in Radon measure space.

At last, we use a particle method to study a Vlasov type equation with local alignment which is the kinetic partial differential equation of Motsch-Tadmor (MT) model when the number of particles goes to infinity. MT model is a N particle model with self-organized behavior, which is used to describe the flocking phenomenon of animals. The MT model provides a natural particle method. This particle method is different from the above two particle methods, which satisfies a second order ODE system. For N particle system, we study the unconditional flocking behavior for a weighted MT model and a model with a "tail". When N goes to infinity, global existence and stability (hence uniqueness) of measure valued solutions to the kinetic equation of this model are obtained. We also prove that measure valued solutions converge to a flock.

Advisor: Xue Xiaoping

Li Xinran, Harvard University

Thesis Title: New Asymptotic Results on Randomization Inference in Experiments

Abstract: This manuscript consists of three self-contained chapters about randomization inference in experiments (Neyman, 1923; Fisher, 1935; Rubin, 1978). The first chapter studies the asymptotic property of rerandomization (Morgan and Rubin, 2012) in treatment-control experiments, and the second chapter extends the discussion to 2K factorial experiments. The third chapter studies peer effects when the stable unit treatment value assumption (Rubin, 1980) fails. Chapter 1. Although complete randomization ensures covariate balance on average, the chance for observing significant differences between treatment and control covariate distributions increases with many covariates. Rerandomization discards randomizations that do not satisfy a predetermined covariate balance criterion, generally resulting in better covariate balance and more precise estimates of causal effects. Previous theory has derived finite sample theory for rerandomization under the assumptions of equal treatment group sizes, Gaussian covariate and outcome distributions, or additive causal effects, but not for the general sampling distribution of the difference-in-means estimator for the average causal effect. We develop asymptotic theory for rerandomization without these assumptions, which reveals a non-Gaussian asymptotic distribution for this estimator, specifically a linear combination of a Gaussian random variable and a truncated Gaussian random variable. This distribution follows because rerandomization affects only the projection of potential outcomes onto the covariate space but does not affect the corresponding orthogonal residuals.

We also demonstrate that, compared to complete randomization, rerandomization reduces the asymptotic sampling variances and quantile ranges of the difference-inmeans estimator. Moreover, our work allows the construction of accurate large-sample confidence intervals for the average causal effect, thereby revealing further advantages of rerandomization over complete randomization.

Chapter 2. With many pretreatment covariates and treatment factors, classical factorial experiments often fail to balance covariates across multiple factorial effects simultaneously. Therefore, it is intuitive to restrict the randomization of the treatment factors to satisfy certain covariate balance criteria, possibly conforming to the tiers of factorial effects and covariates based on their relative importances. This is rerandomization in factorial experiments. We study the asymptotic properties of this experimental design under the randomization inference framework without imposing any distributional or modeling assumptions of the covariates and outcomes. We derive the joint asymptotic sampling distribution of the usual estimators of the factorial effects, and show that it is symmetric, unimodal, and more "concentrated" at the true factorial effects under rerandomization than under classical factorial experiments.

This advantage of rerandomization is quantified by the mathematical notions of "central convex unimodality" and "peakedness" of the joint asymptotic sampling distribution, which also serve as theoretical bases for constructing conservative largesample confidence sets for the factorial effects. Chapter 3. Many previous causal inference studies require no interference among units, that is, the potential outcomes of a unit do not depend on the treatments of other units. This no-interference assumption, however, becomes unreasonable when units are partitioned into groups and they interact with other units within groups.

In a motivating application from Peking University, students are admitted through either the college entrance exam (also known as Gaokao) or recommendation (often based on Olympiads in various subjects). Right after entering college, students are randomly assigned to different dorms, each of which hosts four students. Because students within the same dorm live together and interact with each other extensively, it is very likely that peer effects exist and the no-interference assumption is violated. More importantly, understanding peer effects among students gives useful guidance for future room-

mate assignment to improve the overall performance of students. Methodologically, we define peer effects in terms of potential outcomes, and propose a randomization-based inference framework to study peer effects in general settings with arbitrary numbers of peers and arbitrary numbers of peer types. Our inferential procedure does not require any parametric modeling assumptions on the outcome distributions. Additionally, our analysis of the data set from Peking University gives useful practical guidance for policy makers.

Advisor: Donald Rubin, Liu Jun

Shao ShuHeng, Harvard University

Thesis Title: Supersymmetric Particles in Four Dimensions

Abstract: In this dissertation we study supersymmetric particles in four spacetime dimensions and their relations to other physical observables. For a large class of four-dimensional $N = 2$ systems, the supersymmetric particles are described by the ground states of certain quiver quantum mechanics in the low energy limit. We derive a localization formula for the index of quiver quantum mechanics with four supercharges. Our answer takes the form of a residue integral on the complexified Cartan subalgebra of the gauge group. The wall-crossing phenomenon appears as discontinuities in the value of the residue integral as the integration contour is varied. We then move on to study the ground states in the Kronecker model of quiver quantum mechanics. This is the simplest quiver with two gauge groups and bifundamental matter fields, and appears universally in four-dimensional $N = 2$ systems. The ground state degeneracy may be written as a multi-dimensional contour integral, and the enumeration of poles can be simply phrased as counting bipartite trees. We solve this combinatorics problem, thereby obtaining exact formulas for the degeneracies of an infinite class of models.

For large ranks, the ground state degeneracy is exponential with the slope being a modular function that we are able to compute at integral values of its argument. We also observe that the exponential of the slope is an algebraic number and determine its associated algebraic equation explicitly in several examples. The speed of growth of the degeneracies, together with various physical features of the bound states, suggests a dual string interpretation. In the last part of the dissertation, we conjecture a precise relationship between a limit of the superconformal index of four-dimensional $N = 2$ field theories, which counts local operators, and the spectrum of BPS particles on the Coulomb branch. We verify this conjecture for the case of free field theories, $N = 2$ QED, and $SU(2)$ gauge theories coupled to matter. Assuming the validity of our proposal, we compute the superconformal index of all Argyres-Douglas theories. Our answers match expectations from the connection of Schur operators with two-dimensional chiral algebras.

Advisor: Xin Yi

Yu Chenglong, Harvard University

Thesis Title: Picard-Fuchs Systems Arising From Toric and Flag Varieties

Abstract: This thesis studies the Picard-Fuchs systems for families arising as vector bundles zero loci in toric or partial ag varieties, including Riemann-Hilbert type theorems and arithmetic properties of the differential systems. The theory of tautological systems is proposed by Lian, Song and Yau in [51, 52]. These Picard-Fuchs type di erential systems arise from the variation of Hodge structures of complete intersections in variety X with large symmetries, generalizing Gel'fand-Kapranov-Zelevinski systems for toric varieties [25]. The form of tautological systems makes it natural to introduces the powerful tool of D-modules into the study of hypersurface family in X. Following this direction, Riemann-Hilbert type theorems are obtained by Bloch, Huang, Lian, Srinivas, Yau and Zhu in [6, 34, 35], including solution rank formula, completeness and geometric interpretation of solution sheaves.

In the first part of this thesis, we generalize these results in two aspects, one is the construction of tautological systems for vector bundles, the other is Riemann-Hilbert type theorems in this case.

In the second part of the thesis, we examine an explicit description of Jacobian rings for homogenous vector bundles. This can be viewed as a description of the graded quotients of tautological systems with respect to the natural ltered D-module structure. We consider a set of cohomological vanishing conditions that imply such a description, and we verify these conditions for some new cases. We also observe that the method can be directly extended to log homogeneous varieties. We apply the Jacobian ring to study the null varieties of period integrals and their derivatives, generalizing a result in [16] for projective spaces. As an additional application, we prove the Hodge conjecture for very generic hypersurfaces in certain generalized ag varieties.

The last part is devoted to the arithmetic properties of fundamental periods near large complex structure limit. Motivated by the work of Candelas, de la Ossa and Rodriguez-Villegas [14], we study the relations between Hasse-Witt matrices and period integrals of Calabi-Yau hypersurfaces in both toric varieties and partial ag varieties. We prove a conjecture

by Vlasenko [67] on higher Hasse-Witt matrices for toric hypersurfaces following Katz's method of local expansion [38, 39].

The higher Hasse-Witt matrices also have close relation with period integrals. The proof gives a way to pass from Katz's congruence relations in terms of expansion coefficients [39] to Dwork's congruence relations [21] about periods.

Advisor: Shing-Tung Yau

Zhou Rong, Harvard University

Thesis Title: Converse KAM Theory for Positive Definite Hamiltonian Systems

Abstract: We study the special fiber of the integral models for Shimura varieties of Hodge type with parahoric level structure constructed by Kisin and Pappas in *Integral models of shimura varieties with parahoric level structure, preprint*. We show that when the group is residually split, the points in the mod p isogeny classes have the form predicted by the Langlands Rapoport conjecture in *Shimuravarietäten und Gerben, J. Reine Angew. Math.* We also verify most of the He-Rapoport axioms for these integral models without the residually split assumption. This allows us to prove that all Newton strata are non-empty for these models.

Advisor: Mark Kisin

Zhou Yang, Stanford University

Thesis Title: Higher-genus wall-crossing in Landau-Ginzburg theory

Abstract: 1. Gromov-Witten invariants and quasi-maps

Let X be a smooth projective variety over $\mathbb{C}$. For decades people have been interested in counting holomorphic curves in X, motivated by string theory in physics. Instead of counting embedded curves, it is more natural to count maps from curves C to X. However, the moduli space of maps with a smooth domain curve is usually non-compact, and singular of the wrong dimension. In order to rigorously count curves, one needs to define a compact moduli space with nice properties so that we can "virtually" count maps via intersection theory. In Gromov-Witten theory, one studies the moduli $\overline{M}_{g,n}(X,\beta)$ of stable maps which parametrizes

$$(C,x_1,\dots,x_n,f)$$

where

1. $(C,x_1,\dots,x_n)$ is a genus-g nodal curve with marked points $x_1,\dots,x_n$ in the smooth locus $C^{sm}\subset C$;
2. $f: C\longrightarrow X$ is a morphism of class $f_\star([C])=\beta$,

such that the automorphism group of $(C,x_1,\dots,x_n,f)$ is finite. Note that although we view $\overline{M}_{g,n}(X,\beta)$ as a compactification of the moduli of maps from smooth curves to X, it might have extra irreducible components where the domain curves are all singular. The advantage is that $\overline{M}_{g,n}(X,\beta)$ is a proper Deligne-Mumford stack with a naturally defined virtual fundamental class $\overline{M}_{g,n}(X,\beta)^{vir}$, which coincides with the actual fundamental class when $\overline{M}_{g,n}(X,\beta)$ happens to have the expected dimension [6, 32]. The Gromov-Witten invariants are defined by integrating tautological classes against the virtual fundamental class.

When $X\subset\mathbb{P}^{s-1}$ is a smooth hypersurface in projective space defined by a degree-r polynomial W, the moduli space of maps has other natural compactifications, one of which is the moduli of quasi-maps. We view a map from C to $\mathbb{P}^{s?1}$ as a line bundle L over C together with sections $\varphi_1,\dots,\varphi_s\in H^0(C,L)$ without common zeros. When a family of maps degenerates, in Gromov-Witten theory we allow the curve to "bubble" some rational components; in quasi-maps theory, we allow $\varphi_1,\dots,\varphi_s$ to have some common zeros.

For each $\epsilon\in\mathbb{Q}\cap(0,\infty)$, the moduli $Q^\epsilon_{g,n}(X,d)$ of ϵ-stable degree-d quasi-maps parametrizes

$$(C,x_1,\dots,x_k,L,\varphi_1,\dots,\varphi_s)$$

where

1. $(C,x_1,\dots,x_k)$ is a genus-g nodal curve with marked points $x_1,\dots,x_k$ contained in the smooth locus $C^{sm}\subset C$;
2. L is a line bundle on C of degree d;
3. $\overrightarrow{\varphi}=(\varphi_1,\dots,\varphi_s)\in H^0(C,L^{\oplus s})$ such that $W(\overrightarrow{\varphi})=0$,

such that

1. the zero locus of $\overrightarrow{\varphi}$ is disjoint from the markings and nodes;
2. for each $x\in C^{sm}$, $l(x):=\min_i\{\mathrm{ord}_x\varphi_i\}\leq\frac{1}{\epsilon}$.
3. the $\mathbb{Q}$-line bundle $L^{\otimes\epsilon}\otimes\omega_{C,\log}$ is ample.

Theorem 3.1 ([17]). *The moduli $Q^\epsilon_{g,n}(X,d)$ is a proper Deligne-Mumford stack with a (relative) perfect relative obstruction theory.*

Thus $Q^\epsilon_{g,n}(X,d)$ has a virtual fundamental class $[Q^\epsilon_{g,n}(X,d)]^{vir}$ and quasi-maps invariants are defined by integrating tautological classes against $[Q^\epsilon_{g,n}(X,d)]^{vir}$. The space of stability conditions $\mathbb{Q}\cap(0,\infty)$ is divided into chambers by the walls

$$\frac{1}{d},\frac{1}{d-1},\dots,1.$$

The theory remains constant when ϵ varies within each chamber. When $\epsilon>1$, we get the moduli space of stable maps (when $g=0,n=0$, we need $\epsilon>2$ for the moduli to be non-empty). When $\epsilon<1/d$, we get the moduli of stable quotients [33, 42]. We denote those two stability conditions by $\epsilon = +\infty$ and $\epsilon = 0+$, respectively. Conjecturally all those theories contain the same information. The quasimap wall-crossing studies the change of $[Q^{\epsilon}_{g,n}(X,d)]^{vir}$ when a wall is crossed.

The theory of quasimaps have been defined for a broader class of targets [18]. A uniform wall-crossing formula is conjectured to hold for all targets and in all general.In genus 0, it has been proved for varieties with a torus action whose fixed points are isolated, and for smooth zero locus of a section of an equivariant convex vector bundle over those varieties [13]; in higher genus, it has been proved for semi-positive toric varieties [16] and for complete intersections in products of projective spaces [15]. The moduli space of quasimaps $Q^{0+}_{g,n}(X,d)$ is in general simpler than $\overline{M}_{g,n}(X,d)$, especially for a small genus. In genus 0, the study of quasimaps recovers the classical toric mirror symmetry. In genus 1, $\epsilon = 0+$ quasi-maps satisfies the hyper-plane property, i.e. for example, if $X \subset \mathbb{P}^{s-1}$ is a smooth complete intersection, then the virtual cycle of $Q^{0+}_{g,n}(X,d)$ is equal to the virtual cycle of $Q^{0+}_{g,n}(\mathbb{P}^{s-1},d)$ capped with the top Chern class of some vector bundle. Using the torus action on $\mathbb{P}^{s-1}$, Kim and Lho [29] gave a new proof of the genus-1 mirror theorem of Zinger and Popa [37,45] for Calabi-Yau complete intersections. Recently, quasi-map wall-crossing is used by Guo-Janda-Ruan in their project towards computing all-genera Gromov-Witten invariants of the quintic threefold.

2. The Landau-Ginzburg model and weighted Fan-Jarvis-Ruan-Witten theory

Let W be a Fermat type degree-r quasi-homogeneous polynomial of weights $(w_1,\dots,w_s) \in \mathbb{Z}^s$:

$$W(X_1,\dots,X_s) = X_1^{r/w_1} + \cdots + X_S^{r/w_s}.$$

We assume that each $r/w_\alpha \geq 2$ is an integer and $\gcd(r,w_1,\dots,w_s) = 1$. The polynomial W defines a smooth hypersurface X_W in the weighted projective space $\mathbb{P}(w_1,\dots,w_s)$. The Fan-Jarvis-Ruan-Witten (FJRW) theory is a cohomological field theory defined via intersection theory on the moduli of r-spin curves $\overline{M}^{1/r}_{g,n}$ which parametrizes

$$(C,x_1,\dots,x_n,L,p)$$

where C is a twisted curve of genus g with markings $x_1,\dots,x_n$ and balanced nodes, L is a representable line bundle on C and $p \in H^0(L^{\otimes r} \otimes \omega_{C,\log})$ is a non-vanishing section, such that the underlying coarse curve $(Cx_q,\dots,x_n)$ is Deligne-Mumford stable. See Definition 1 for the precise definition. The FJRW theory is the Landau-Ginzburg A-model for W. When W defines a Calabi-Yau hypersurface, the Landau-Ginzburg/Calabi-Yau correspondence relates the Gromov-Witten theory of X_W to the FJRW theory of $(W,\langle J\rangle)$, where J is the symmetry of W given by diagonal multiplication by the primitive r-th root of unity. They are the two phases of the *gauged linear sigma model* mathematically defined in [22].

An analogue of quasimap theory on the Landau-Ginzburg side is called weighted FJRW theory, in which one allows the single section $p \in H^0(L^{\otimes r} \otimes \omega_{C,\log})$ to have some zeros, together with some condition on positivity of the line bundles.

Our definition of the weighted FJRW theory will be different from the gauged linear sigma model construction in [22]. Since we consider only one section p on the Landau–Ginzburg side, on a family of curves the zero locus of p is a relative Cartier divisor. Hence we can view the zeros of p as additional weighted unordered markings in the sense of Hassett [26]. Ordering those additional markings results in an equivalent theory. Based on this observation, we will give an alternative definition (Definition 1), which is essential to our proof.

Indeed, there is a family of gauged linear sigma models interpolating between these two theories [22, 43], parametrized by a nonzero rational number ϵ. As ϵ varies, the change of the theories gives a wall-and-chamber structure on the parameter space. In this picture, for $\epsilon > 0$ it is equivalent to the theory of ϵ-stable quasimaps [7,10,30]; for $\epsilon < 0$ we get the weighted FJRW invariants, where we allow the section p to have zeros of length at most $1/|\epsilon|$ (Definition 1). Thus when $\epsilon < -1$, we get the original FJRW theory defined via the moduli of r-spin curves.

Unfortunately, in some literature the weighted FJRW theory are also index by positive ϵ [39]. Thus we will simply call the $\epsilon < -1$ theory the ∞-FJRW theory and call the $\epsilon = 0-$ theory the 0-FJRW theory. For simplicity we will only consider those two theories, but our method works for general $\epsilon < 0$.

3. Goal of the thesis

In Chapter 1 we will prove an all-genera wall-crossing formula relating the ∞-FJRW theory and the 0-FJRW theory, for extended narrow insertions.This is the Landau–Ginzburg analogue to the quasimap wall-crossing formula by Ciocan-Fontanine–Kim [15].

We expect that the Landau-Ginzburg side and the Calabi-Yau side are more directly related near $\epsilon = 0$. Using wall-crossing Ross-Ruan proved the Landau-Ginzburg/Calabi-Yau correspondence in genus 0 [38]. In genus 1, Guo–Ross proved the wall-crossing formula for the Fermat quintic and used it to compute the FJRW invariants of the quintic 3-fold explicitly [24] and verified the genus-1 Landau-Ginzburg/Calabi-Yau correspondence [25]. Our result generalizes their wall-crossing formulas to all genera. We hope this will be useful for establishing the all-genera Landau-Ginzburg/Calabi-Yau correspondence.

The higher-genus wall-crossing formula is also proved by Clader-Janda-Ruan [20] using different methods. Their method applies to the more general hybrid-model case, where the corresponding target on the Calabi-Yau side is a complete intersection in weighted projective spaces. However, they have to assume that there is at least one marked point.

In Chapter 2, we will apply our method in [44] to prove analogues of the dilaton and divisor equations in the context of [20]. Using those results, we are able to remove the assumption on the presence of markings and get the full wall-crossing formula.

Advisor: Jun Li

Doctor Thesis Awards, Silver Prize

Deng Ya, Université de Strasbourg

Thesis Title: Generalized Okounkov Bodies, on the Direct Image and Hyperbolicity Problems

Abstract: In Part 1 of this thesis, we construct "Okounkov bodies" for an arbitrary pseudo-effective $(1,1)$-class on a Kähler manifold. We prove the differentiability formula of volumes of big classes for Kähler manifolds on which modified nef cones and nef cones coincide. As a consequence we prove Demailly's transcendental Morse inequality for these particular Kähler manifolds; this includes Kähler surfaces. Then we construct the generalized Okounkov body for any big $(1,1)$-class, and give a complete characterization of generalized Okounkov bodies on surfaces. We show that this relates the standard Euclidean volume of the body to the volume of the corresponding big class as defined by Boucksom; this solves a problem raised by Lazarsfeld and Mustată in the case of surfaces. We also study the behavior of the generalized Okounkov bodies on the boundary of the big cone.

Part 2 deals with Kobayashi hyperbolicity-related problems. Chapter 2's goal is to study the degeneracy of leaves of the one-dimensional foliations on higher dimensional manifolds, along the lines of [McQ98, Bru99, McQ08, PS14]. The first part of Chapter 2 generalizes McQuillan's Diophantine approximations for one-dimensional foliations with absolutely isolated singularities, on higher dimensional manifolds. As an application, we give a new proof of Brunella's hyperbolicity theorem, that is, all the leaves of a generic foliation of degree $d \geq 2$ in $\mathbb{P}^n$ is hyperbolic. In the second part of Chapter 2 we introduce the so-called weakly reduced singularities for one-dimensional foliations on higher dimensional manifolds. The "weakly reduced singularities" assumption is less demanding than the one required for "reduced singularities", but play the same role in studying the Green-Griffiths-Lang conjecture. Finally we discuss a strategy to prove the Green-Griffiths-Lang conjecture for complex surfaces.

In Chapter 3, assuming that the canonical sheaf K_V is big in the sense of Demailly, we prove the Kobayashi volume-hyperbolicity for any (possibly singular) directed variety (X,V).

In Chapter 4, our first goal is to deal with effective questions related to the Kobayashi and Debarre conjectures, relying on the work of Damian Brotbek [Bro16] and his joint work with Lionel Darondeau [BD15]. We then combine these techniques to study the conjecture on the ampleness of the Demailly-Semple bundles raised by Diverio and Trapani [DT10], and also obtain some effective estimates related to this problem. Our result integrates both the Kobayashi and Debarre conjectures, with some (non-optimal) effective estimates.

The purpose of Chapter 5 is twofold: on the one hand we study a Fujita-type conjecture by Popa and Schnell, and give an effective (linear) bound on the generic global generation of the direct image of the twisted pluricanonical bundle. We also point out the relation between the Seshadri constant and the optimal bound. On the other hand, we give an affirmative answer to a question by Demailly-Peternell-Schneider in a more general setting. As applications, we generalize the theorems by Fujino and Gongyo on images of weak Fano manifolds to the Kawamata log terminal cases, and refine a result by Broustet and Pacienza on the rational connectedness of the image.

In Chapter 6, we give a concrete and constructive proof of the equivalence between the category of semistable Higgs bundles with vanishing Chern classes and the category of all representations of the fundamental groups [Cor88, Sim88] on smooth Kähler manifolds. This chapter is written for the complex geometers who are not familiar with the language of differential graded category used by Simpson to prove the above equivalence on smooth projective manifolds, and for those who would like to see an elementary proof of Corlette-Simpson correspondence for semistable Higgs bundles.

Advisor: Jean-Pierre Demailly

Gao Yuan, Fudan University

Thesis Title: Some Nonlinear Evolution Equations in Material Science with Dissipative Structures

Abstract: This thesis considers some nonlinear evolution equations with dissipative structures which are derived from some material models. These models are important to advances in manufacturing in the aspects like electronics manufacture, nanotechnology and building materials. By applying some theoretical methods in partial differential equations, we study existence, uniqueness and long-time behavior of solu-

tions to those equations, and hopefully, it could provide some theoretical instructions for the manufacture of those materials.

The thesis is divided into two parts. The first part (Chapter 1–6) concerns the epitaxial growth on vicinal surface which derives some degenerate parabolic equations while the second part (Chapter 7–9) concerns the noise control in build materials which derives a system of wave equation coupled with some acoustic boundary conditions. In Chapter 2, starting from the mesoscopic Burton-Cabrera-Frank (BCF) type model, we consider the rigorous derivation of continuum models of step motion separately in diffusion-limited (DL) regime and attachment-detachment-limited (ADL) regime. We prove that as the lattice parameter goes to zero, for a finite time interval, the discrete model converges to the strong solution of the limiting continuum equation with first order convergence rate.

In Chapter 3, we study the continuum model derived from step flow in ADL regime,

$$u_t = -u^2(u^3)_{hhhh}$$

where u, considered as a function of step height h, is the step slope of the surface. With periodic boundary conditions and positive initial data, we formulate a notion of weak solution to u-equation and then using regularized method and two Lyapunov energy functions, we prove the existence of a global weak solution, which is positive almost everywhere. We also study the long time behavior of weak solution and prove that it converges to a constant solution as time goes to infinity. The space-time Hölder continuity of the weak solution is also discussed as a byproduct.

In Chapter 4, we rewrite w-equation using another variable w, i.e.,

$$w_t = [(w_{hh} + c_0)^{-3}]_{hh}, \quad w(0) = w^0 \tag{3.1}$$

to understand the singularity of $1/u$. We establish a general framework for abstract evolution equation with maximal monotone operator in non-reflexive Banach space which can be used in a wide class of degenerate parabolic equations. Following this framework, we formulate a global strong solution to (3.1) by defining a proper convex functional which allows a Radon measure occurring. Then we prove the existence of such a global strong solution and obtain the almost everywhere positivity of $w_{hh} + c_0$.

In Chapter 5, we study a continuum model $u_t = -u^2(u^3 + \alpha u)_{hhhh}$ to incorporate a logarithmic factor. We prove the strict positivity, existence and uniqueness of global strong solution to this equation using two Lyapunov energy functions. The long time behavior of u converging to a constant that only depends on the initial data is also investigated.

In Chapter 6, we give some numerical pictures which show the previous theoretical results briefly and visually.

In Chapter 8, we study a system described by wave equation coupled with some nonlinear vibration equations on the interface. Under the mass-indispensable interface assumption, we need a local layer with damping effect to stabilize the whole system. By the method of Lyapunov functional and truncation function, we obtain the uniform decay of energy using iteration and give a uniform decay rate which is a solution to a differential equation.

In Chapter 9, the same coupled system with acoustic interface is considered. Under the mass-neglectable interface assumption, which allow us to stabilize the whole system with only one boundary damping effect. By the method of observability inequality, we obtain the uniform decay of energy indirectly and give a uniform decay rate using the standard optimal-weight convexity method.

Advisor: Liu Jianguo

Mathew Kwan, ETH Zürich

Thesis Title: Probabilistic Reasoning in Combinatorics

Abstract: In the 1940s, Erdös popularised the *probabilistic method* in combinatorics, in which one proves the existence of objects satisfying certain properties by showing that a *random* object satisfies such properties with positive probability. Since then, probabilistic tools and perspectives have come to play an inseparable role in combinatorics. This can be attributed in part to the development of the probabilistic method, but also to some other themes in modern combinatorics: increasing interest in the study of random discrete objects, and towards the question of what it might usefully mean for an object to be "random-like". In this thesis we explore a number of different aspects of the role that probability plays in combinatorics.

First, we consider the problem of estimating the maximum size of cuts in hypergraphs. An r-cut of a k-uniform hypergraph H is a partition of the vertex set of H into r parts and the size of the cut is the number of edges which have a vertex in each part. A classical result of Edwards says that every m-edge graph has a 2-cut of size $m/2 + \Omega(\sqrt{m})$, and this is best-possible. From a probabilistic viewpoint, this is precisely the statement that there exist cuts which exceed the expected size of a random cut by some multiple of the standard deviation. We study analogues of this and related results in hypergraphs. In particular we make the surprising discovery that while the standard deviation dictates the worst-case size of the maximum cut for 2-cuts in graphs and for 2-cuts in 3-uniform hypergraphs, in all other cases the maximum r-cut ex-

ceeds the expected size of a random rcut by significantly more than the standard deviation.

Second, we prove an old conjecture of Erdös, Faudree and Sós, and a more recent conjecture of Narayanan, Sahasrabudhe and Tomon, both regarding the structure of Ramsey graphs. An n-vertex graph is called *C-Ramsey* if it has no clique or independent set of size $C \log n$. All known constructions of Ramsey graphs involve randomness in an essential way, and there is an ongoing line of research towards showing that in fact all Ramsey graphs must obey certain "richness" properties characteristic of random graphs. We prove that in any C-Ramsey graph there are $\Omega(n^{5/2})$ induced subgraphs, no pair of which have the same numbers of vertices and edges, and we also prove that in any such graph there are $\Omega(n^2)$ induced subgraphs with different numbers of edges.

Third, we study the resilience of the Littlewood-Offord problem. Consider the sum $X(\xi) = \sum_{i=1}^{n} a_i \xi_i$, where $a = (a_i)_{i=1}^{n}$ is a sequence of nonzero reals and $\xi = (\xi_i)_{i=1}^{n}$ is a sequence of i.i.d. Rademacher random variables (that is, $Pr(\xi = 1) = Pr(\xi = -1) = 1/2$). This sum is nothing more than the outcome of an inhomogeneous unbiased random walk. The classical Littlewood–Offord problem asks for the best possible upper bound on the concentration probabilities $Pr(X = x)$, thereby quantifying the *anticoncentration* of random variables of this type. We ask, in a sense, how robustly this anti-concentration holds: how many of the xi is an adversary typically allowed to change without being able to force concentration on a particular value? We solve this problem asymptotically.

Finally, we turn our attention to the subject of random designs, a subject about which very little is currently known.We show that for any n divisible by 3, almost all order-n Steiner triple systems have a perfect matching (also known as a *parallel class or resolution class*). In fact, we prove a general upper bound on the number of perfect matchings in a Steiner triple system and show that almost all Steiner triple systems essentially attain this maximum. We accomplish this via a general theorem comparing a uniformly random Steiner triple system to the outcome of the triangle removal process. We hope this theorem will be more generally applicable and might open the door to further research on random designs.

Advisor: Benny Sudakov

He Siqi, California Institute of Technology

Thesis Title: The Kapustin-Witten Equations with Singular Boundary Conditions

Abstract: Witten proposed a fasinating program interpreting the Jones polynomial of knots on a 3-manifold by counting solutions to the Kapustin-Witten equations with singular boundary conditions.

In Chapter 1, we establish a gluing construction for the Nahm pole solutions to the Kapustin-Witten equations over manifolds with boundaries and cylindrical ends. Given two Nahm pole solutions with some convergence assumptions on the cylindrical ends, we prove that there exists an obstruction class for gluing the two solutions together along the cylindrical end. In addition, we establish a local Kuranishi model for this gluing picture. As an application, we show that over any compact four-manifold with S^3 or T^3 boundary, there exists a Nahm pole solution to the obstruction perturbed Kapustin-Witten equations. This is also the case for a four-manifold with hyperbolic boundary under some topological assumptions.

In Chapter 2, we find a system of non-linear ODEs that gives rotationally invariant solutions to the Kapustin-Witten equations in 4-dimensional Euclidean space. We explicitly solve these ODEs in some special cases and find decaying rational solutions, which provide solutions to the Kapustin-Witten equations. The imaginary parts of the solutions are singular. By rescaling, we find some limit behavior for these singular solutions. In addition, for any integer k, we can construct a $5|k|$ dimensional family of C^1 solutions to the Kapustin-Witten equations on Euclidean space, again with singular imaginary parts. Moreover, we get solutions to the Kapustin-Witten equation with Nahm pole boundary condition over $S^3 \times (0, +\infty)$.

In Chapter 3, we develop a Kobayashi-Hitchin type correspondence for the extended Bogomolny equations on $\Sigma\times$ with Nahm pole singularity at $\Sigma \times \{0\}$ and the Hitchin component of the stable $SL(2;\mathbb{R})$ Higgs bundle; this verifies a conjecture of Gaiotto and Witten. We also develop a partial Kobayashi-Hitchin correspondence for solutions with a knot singularity in this program, corresponding to the non-Hitchin components in the moduli space of stable $SL(2;\mathbb{R})$ Higgs bundles. We also prove the existence and uniqueness of solutions with knot singularities on $C \times \mathbb{R}^+$. This is joint a work with Rafe Mazzeo.

In Chapter 4, for a 3-manifold Y, we study the expansions of the Nahm pole solutions to the Kapustin-Witten equations over $Y \times (0, +\infty)$. Let y be the coordinate of $(0, +\infty)$ and assume the solution convergence to a flat connection at $y \to +\infty$, we prove the sub-leading terms of the Nahm pole solution is C^1 to the boundary at $y \to 0$ if and only if Y is an Einstein 3-manifold. For Y non-Einstein, the sub-leading terms of the Nahm pole solutions behave as $y \log y$ to the boundary. This is a joint work with Victor Mikhaylov.

Advisor: Ciprian Manolescu

Huang Hui, Tsinghua University

Thesis Title: Mean-field Limit for the Keller-Segel System and the Theory of Propagation of Chaos

Abstract: This thesis consists of two parts, in which we study the propagation of chaos for the N-particle chemotaxis system subject to Brownian diffusion in the $\mathbb{R}^d (d \geq 2)$ space. The first part is to present a probabilistic proof of the distance between the exact microscopic and the approximate mean-filed dynamics, which leads to a derivation of the Keller-Segel equation from the microscopic N-particle system. Specifically, With a blob size $\varepsilon = h^{\kappa}(1/2 < \kappa < 1)$, we prove a rate $h|\ln h|$ of convergence in $l_h^p(p > \frac{d}{1-\kappa})$ norm up to a probability $1 - h^C|\ln h|$, where h is the initial grid size. In this case, the initial positions of the particles are taken on the lattice points. As for the second part, by assuming the initial data are identically independent distributed, we analyze the discrete-in-time method of approximating solutions of the Keller-Segel equation by the interacting particles subject to Brownian diffusion. This work contains some technical results on the degree of the approximation of solutions to the mean-field equation by random evolving systems. More precisely, with a blob size $\varepsilon = N^{-\frac{1}{d(d+1)}\log(N)}$, we prove the convergence rate between the solution to the Keller-Segel equation and the empirical measure of the random particle method under L^2 norm in probability, where N is the number of the particles.

Advisor: Jian Huaiyu

Man-Chun LEE, The Chinese University of Hong Kong

Thesis Title: Kähler Ricci Flow and Chern Ricci Flow on Noncompact Hermitian Manifolds

Abstract: In this thesis, we will investigate the short-time existence of the Chern-Ricci flow and Kähler Ricci flow on complete noncompact manifolds together with its local curvature estimate and global behaviour.

In the first part, we will generalize the characterization of maximal existence time of the Chern-Ricci flow shown by Tosatti and Weinkove in [54] to complete noncompact Hermitian manifolds with possibly unbounded curvature. To construct a flow, we construct a approximating sequence of Hermitian manifolds by conformally blowing up the metric in a neighbourhood of the boundary of a compact exhaustion. After derivation of local a priori estimates, a Chern-Ricci flow can be constructed with an estimate on lifespan.

In the second part, we will derive local curvature estimates of Kähler-Ricci flow. When the bisectional curvature $BK_{g(t)}$ is bounded from below along the ow and the original metric is noncollapsing, a global curvature bound can be obtained. We also establish the preservation of nonnegative BK under curvature bound $at^{-\theta}$ for some $\theta < 2$. We will also discuss under what circumstances this curvature bound may hold. Finally, we will apply the existence of the Chern-Ricci flow to construct a Kähler Ricci flow starting from a non-collapsing complete noncompact Kähler manifold with nonnegative bisectional curvature. An application on Yau's uniformization conjecture will be discussed.

Advisor: Luen-Fai TAM

Luo Ma, Duke University

Thesis Title: Algebraic De Rham Theory for Completions of Fundamental Groups of Moduli Spaces of Elliptic Curves

Abstract: To study periods of fundamental groups of algebraic varieties, one requires an explicit algebraic de Rham theory for completions of fundamental groups. This thesis develops such a theory in two cases.

In the first case, we develop an algebraic de Rham theory for unipotent fundamental groups of once punctured elliptic curves over a field of characteristic zero using the universal elliptic KZB connection of Calaque-Enriquez-Etingof and Levin-Racinet. We use it to give an explicit version of Tannaka duality for unipotent connections over an elliptic curve with a regular singular point at the identity. In the second case, we develop an algebraic de Rham theory for relative completion of the fundamental group of the moduli space of elliptic curves with one marked point. This allows the construction of iterated integrals involving modular forms of the *second kind*, whereas previously Brown and Manin only studied iterated integrals of *holomorphic* modular forms.

Advisor: Richard Hain

Ma Siyuan, Albert Einstein Institute and University of Potsdam

Thesis Title: Analysis of Teukolsky equations on slowly rotating Kerr spacetimes

Abstract: In this thesis, we treat the extreme Newman-Penrose components of both the Maxwell field $(s = 1)$ and the linearized gravitational perturbations (or "linearized gravity" for short) $(s = 2)$ in the exterior of a slowly rotating Kerr black hole. Upon different rescalings, we can obtain spin s components which satisfy the separable Teukolsky master equation (TME). For each of these spin s components defined in Kinnersley tetrad, the resulting equations by performing some first-order differential operator on it once and twice (twice only for $s = 2$), together with the TME, are in the form of an "inhomogeneous spin-weighted wave equation" (ISWWE) with different potentials and constitute a linear spin-weighted wave system. We then prove energy and integrated local energy decay (Morawetz) estimates for this type of ISWWE, and utilize them to achieve both a uniform bound of a positive definite energy and a Morawetz estimate for the regular extreme Newman-Penrose components defined in the regular Hawking-Hartle tetrad. We also present some brief discussions on mode stability for TME for the case of real frequencies. This says that in a fixed subextremal Kerr spacetime, there is no nontrivial separated mode solutions to TME which are purely ingoing at horizon and purely outgoing at in infinity. This yields a representation formula for solutions to inhomogeneous Teukolsky equations, and will play a crucial role in generalizing the above energy and Morawetz estimates results to the full subextremal Kerr case.

Advisor: Lars Andersson

Qi Di, New York University

Thesis Title: Strategies for Reduced-Order Models in Uncertainty Quantification of Complex Turbulent Dynamical Systems

Abstract: Turbulent dynamical systems are ubiquitous in science and engineering. Uncertainty quantification (UQ) in turbulent dynamical systems is a grand challenge where the goal is to obtain statistical estimates for key physical quantities. In the development of a proper UQ scheme for systems characterized by both a high-dimensional phase space and a large number of instabilities, significant model errors compared with the true natural signal are always unavoidable due to both the imperfect understanding of the underlying physical processes and the limited computational resources available. One central issue in contemporary research is the development of a systematic methodology for reduced order models that can recover the crucial features both with model fidelity in statistical equilibrium and with model sensitivity in response to perturbations.

In the first part, we discuss a general mathematical framework to construct statistically accurate reduced-order models that have skill in capturing the statistical variability in the principal directions of a class of complex systems with quadratic nonlinearity. A systematic hierarchy of simple statistical closure schemes, which are built through a global statistical energy conservation principle combined with statistical equilibrium fidelity, are designed and tested for UQ. Second, the capacity of imperfect low-order stochastic approximations to model extreme events in a passive scalar field advected by turbulent flows is investigated. The effects in complicated flow systems are considered including strong nonlinear and non-Gaussian interactions, and much simpler and cheaper imperfect models with model error are constructed to capture the crucial statistical features in the stationary tracer field.

Several mathematical ideas are used to improve the prediction skill of the imperfect reduced-order models. Most importantly, empirical information theory and statistical linear response theory are applied for calibrating model errors in the training phase; and total statistical energy dynamics are introduced to improve the model sensitivity in the prediction phase. The validity of reduced-order models for predicting statistical responses and intermittency is demonstrated on a series of instructive models with increasing complexity, including the stochastic triad model, the Lorenz '96 model, and models for barotropic and baroclinic turbulence. The skillful low-order modeling methods developed here should also be useful for other applications such as efficient algorithms for data assimilation.

Advisor: Andrew J. Majda

Wan Chen, University of Minnesota

Thesis Title: A Local Trace Formula and the Multiplicity One Theorem for the Ginzburg-Rallis Model

Abstract: Following the method developed by Waldspurger and Beuzart-Plessis in their proof of the local Gan-Gross-Prasad conjecture, we are able to prove a local trace formula for the Ginzburg-Rallis model. By applying this trace formula, we proved a multiplicity formula for the Ginzburg-Rallis model for tempered representations. Then by applying this multiplicity formula, we proved the multiplicity one theorem for all tempered L-packets. In some cases, we also proved the epsilon dichotomy conjecture which gives a relation between the multiplicity and the exterior cube epsilon factor. Finally, in the archimedean case, we proved some partial results for the general generic representations by applying the open orbit method.

Advisor: Jiang Dihua

Yu Hui, University of Texas at Austin

Thesis Title: Several Regularity Results for Nonlocal Elliptic Equations

Abstract: Nonlocal elliptic equations have long been used by physicists and engineers to model diffusion processes involving jumps. Apart from several works from a probabilistic view, there had not been much development concerning their mathematical properties until the fundamental works of Caffarelli and Silvestre.

Here we establish several results concerning the regularity of viscosity solutions to nonlocal elliptic

equations. In particular, we show the existence of smooth solutions to two class of nonlocal fully nonlinear elliptic equations, an integrability estimate for the fractional order Hessian of solutions to nonlocal equations, as well as a theory of flat solutions.

Advisor: Luis Caffarelli

Zhang Ruixiang, Princeton University

Thesis Title: Separable Preconditioner for Time-Space Fractional Diffusion Equations

Abstract: In this thesis, we study the perturbed Brascamp-Lieb inequalities and its applications in translation-dilation systems. We prove the endpoint perturbed Brascamp-Lieb inequalities using polynomial partition techniques. We also look at the Parsell-Vinogradov system and verify the Brascamp-Lieb condition holds in its decoupling approach. As a corollary of this and the work of Guo, the main conjecture about the system is true in dimension 2 and can be proved by the decoupling approach.

Advisor: Peter Sarnak

Zhao Yiming, New York University

Thesis Title: Geometric Measures, Affine Invariants, and Their Characterizations

Abstract: The classical Brunn-Minkowski theory focused heavily on the study of geometric invariants and measures associated with convex bodies (compact convex subsets of $\mathbb{R}^n$, with non-empty interiors). Quermassintegrals—which include volume, surface area, and mean width—are the fundamental geometric invariants in the classical Brunn-Minkowski theory. Surface area measure and Aleksandrov's integral curvature are two important geometric measures that can be viewed as differentials of quermassintegrals. The classical Minkowski problem and the Aleksandrov problem characterizing surface area measure and integral curvature are two well-known problems.

In [66] and [68], Lutwak introduced L_p surface area measure and brought life to the L_p Brunn-Minkowski theory which is now becoming the center of modern convex geometry. The L_p surface area measure in the L_p theory is the corresponding notion of surface area measure in the classical Brunn-Minkowski theory. Its characterization problem is called the L_p Minkowki problem, which includes critical singular open cases such as the logarithmic Minkowski problem and the centro-affine Minkowski problem.

Another theory that receives great attention in modern convex geometry is the dual Brunn-Minkowski theory, which is parallel to the classical Brunn-Minkowski theory. The dual theory was introduced by Lutwak [61] in the 1970s and was crucial in the solution of the famous Busemann-Petty problem [25, 29, 64, 111]. Despite its fast development in the last three decades, the fundamental geometric measures in the dual theory remained elusive until the ground-breaking work [45] by Huang-Lutwak-Yang-Zhang. Dual curvature measures are the long-sought-for "dual" of Federer's curvature measures in the dual Brunn-Minkowski theory. Surprisingly, the family of dual curvature measures connects well-known measures such as the cone volume measure (a crucial case of the L_p surface area measure) and Aleksandrov's integral curvature. Dual curvature measures can be viewed as the differentials of dual quermassintegrals, the fundamental geometric invariants in the dual Brunn-Minkowski theory.

The problem characterizing dual curvature measures is called the dual Minkowski problem, which contains the Aleksandrov problem and the unsolved logarithmic Minkowski problem as special cases. A large portion of the current work will be dedicated to solving the dual Minkowski problem in various cases. In Chapter 2, basics regarding the theory of convex bodies will be covered. We will also agree on the notations that will be used in each of the following chapters.

In Chapter 3, an overview of the aforementioned geometric measures and their characterization problems will be presented.

In Chapter 4, a subspace mass inequality will be presented and will be shown to be sufficient for the existence of solutions to the dual Minkowski problem when the index $q \in (0, n)$ is an integer and the given data is even. It is worthwhile to note that this problem is much more challenging than the classical Minkowski problem or any other previous Minkowski-type problems in the sense that a full solution cannot be obtained through a solution when the given data is smooth and a simple approximation argument.

In Chapter 5, a complete solution, including the existence and the uniqueness part, to the dual Minkowski problem when the index q is negative will be presented.

In additional to the geometric measures and invariants discussed above, in Chapter 6, the notion of affine surface area will be studied. The notion of affine surface area traces back to affine differential geometry and has various applications in information theory, polytopal approximation, and image processing. It was originally defined only for convex bodies whose boundaries are sufficiently smooth. Extending affine surface area to one that works for general convex bodies (without smoothness assumptions) while keeping the basic properties of the classical notion was of huge interest in the late 80s and 90s (in the

previous century). This led to three seemingly different definitions. Through highly nontrivial efforts, the three definitions were eventually shown to be equivalent. In the current work, the relation between affine surface area and Federer's curvature measures will be investigated. As a result, a new formulation of affine surface area that depends only on curvature measures will be demonstrated. Properties of affine surface area using this new formulation will also be presented. Instead of treating affine surface area first and then extending the results to L_p affine surface area (the L_p analog of affine surface area), we will treat L_p affine surface area directly.

Advisor: Erwin Lutwak

Zhu Yihang, Harvard Unversity

Thesis Title: The Stabilization of the Frobenius-Hecke Traces on the Intersection Cohomology of Orthogonal Shimura Varieties

Abstract: The orthogonal Shimura varieties are associated to special orthogonal groups over $\mathbb{Q}$ of signature at infinity. In this work we prove a version of Morel's formula for the Frobenius-Hecke traces on the intersection cohomology of their Baily-Borel compactifications. We then stabilize the formula, in terms of Kottwitz's simplified expression for the geometric side of the Arthur-Selberg trace formula for test functions that are stable cuspidal at infinity.

Advisor: Mark Kisin

Master Thesis Awards, Silver Prize

Pui Tung CHOI, The Chinese University of Hong Kong

Thesis Title: Surface Conformal/Quasi-conformal Parameterization with Applications

Abstract: Surface conformal and quasi-conformal parameterizations are important in computer graphics and medical imaging. In this thesis, we develop efficient and practical algorithms for mesh and point cloud parameterizations with various applications. Firstly, we propose a linear algorithm for the spherical conformal parameterizations and an efficient algorithm called FLASH forlandmark-aligned spherical optimized conformal mappings of genus-0 closed triangular meshes. The algorithms are applied for the registration of human cortical surfaces, and the shape analysis of the carotid arteries and the hippocampal surfaces in medical imaging. Secondly, we propose two fast disk conformal parameterization algorithms for simply-connected open triangular meshes and apply the parameterizations for texture mapping. Thirdly, we develop a linear algorithm for computing spherical quasi-conformal parameterization and Teichmüller parameterization of genus-0 closed meshes. Fourthly, we develop an iterative scheme for the spherical conformal parameterizations of genus-0 point clouds with applications in meshing and multilevel representation. Fifthly, we propose a novel algorithm called TEMPO for the landmark aligned Teichmüller parameterization of disk-type point clouds with theoretical guarantee. The algorithm is applied for developing a dissimilarity metric of point clouds. Experimental results are presented for demonstrating the effectiveness of our proposed algorithms.

Advisor: Ronald Lok Ming LUI

Ding Chun, Nankai University

Thesis Title: On Dye's Theorem for C^*-algebras

Abstract: In this thesis, we first introduce the related theories on C^*-algebras and W^*-algebras, including representations of C^*-algebras, enveloping W^*-algebras, Borel functional calculus and projection lattice of atomic W^*-algebras. After that, we prove that a projection orthoisomorphism, between the sets of projections (not necessarily all, but containing all minimal projections) in atomic W^*-algebras with no 2-dimensional direct summand, extends to a unique Jordan $*$-isomorphism between the atomic W^*-algebras. At last, we give the Dye's theorem for C^*-algebras:

Suppose A and B are C^*-algebras and the atomic part of A^{**} has no direct summand $*$-isomorphic to $B(\mathbb{C}^2)$, then there is a Jordan $*$-isomorphism J from A onto B such that the double adjoint J^{**} extends a given projection orthoisomorphism between the sets of q-closed projections of A and B. If A and B are separable, the sets of q-closed projections can be replaced with the sets of closed projections.

Advisor: Wu Zhiqiang

Lin Xuelei, University of Macau

Thesis Title: Separable Preconditioner for Time-Space Fractional Diffusion Equations

Abstract: In this thesis, we consider a separable preconditioner for linear systems arising from time-space Caputo-Riesz fractional diffusion equations. We give an introduction in Chapter 1. In Chapter 2, we propose a separable preconditioner for linear systems arising from full discretization of time-space Caputo-Riesz fractional diffusion equations. Theoretically, we show that if the diffusion coefficient function is temporal-independent or spatial-independent, singular values of the preconditioned matrix are bounded above and below by positive constants

which are independent of discretization parameters. Numerical examples are reported to demonstrate that the performance of the proposed preconditioner is efficient, and is better than other approaches. This thesis ends with a chapter of a brief conclusion.

Advisor: Sun Haiwei

Bachelor Thesis Awards, Gold Prize

Bingyi Chen, Tsinghua University

Thesis Title: Classification of 3 Dimensional Rational Complete Intersection Singularities

Abstract: In this paper, we classify three dimensional isolated weighted homogeneous rational complete intersection singularities according to the weight type and calculate the Milnor numbers and basis of miniversal deformation of these singularities.

Advisor: Stephen S. T. Yau

Ziyu Li, Ningbo University

Thesis Title: Infinite-Dimensional Geometric Structure of Stock Market

Abstract: This paper first reviews and provides a positive analysis on small-world, scale-free and self-similar properties of the stock market. Then, based on the stock market time series and geodesic structure, we construct metric structures on the stock markets in China and NASDAQ. Applying the improved covering and packing algorithms to the above metric structure, we obtain an experimental result that the overall behaviors of the stock markets in China and NASDAQ are characterized by "infinite dimension". Therefore, any forecast of stock market behaviors through finite parameters contains not only white noise but system error.

Advisor: Lifeng Xi

Bachelor Thesis Awards, Silver Prize

Bai Chenyu, University of Chinese Academy of Science

Thesis Title: Studies on Hyperplane Sections of Rational Homogeneous Varieties

Abstract: In this paper we study some properties of the hyperplane section of rational homogeneous varieties with Picard number one. We calculate the dimension of their automorphic groups, compute the cohomology and Euler number of their tangent bundles. We prove a vanishing theorem and use it to classify all such algebraic varieties with local rigidity. The main tool is Borel-Weil-Bott theorem.

Advisor: Fu Baohua

He Yiqin, Xiangtan University

Thesis Title: On the Directed Strongly Regular Cayley Graphs

Abstract: The concept of directed strongly regular graphs (DSRG) was introduced by Duval in 1988 [6].

In the present paper, we construct some directed strongly regular Cayley graphs. By using induced representation, we have a look at the Cayley digraphs $\mathbf{Cay}(N \rtimes_\theta H, N_1 \times H)$ with $N_1 \subset N$, and determine its characteristic polynomial and its minimal polynomial. Based on this result, we generalize the semidirect product construction method of Art M. Duval and Dmitri Iourinski in [7] and obtain a larger family of directed strongly regular graphs. We also construct some directed strongly regular Cayley graphs on dihedral groups, which partially generalizes the earlier result of Mikhail Klin, Akihiro Munemasa, Mikhail Muzychuk, and Paul Hermann Zieschang in [17]. This paper also characterizes some certain directed strongly regular Cayley graphs on dihedral groups D_{p^α}, where p is a prime and $\alpha \geq 1$ is a positive integer.

We also show that a Cayley digraph $\mathbf{Cay}(G,S)$ is not directed strongly regular graph if S is a union of some conjugate classes of G. This generalizes an earlier result of L. K. Jørgensen [15] on abelian groups. Finally, we discuss the vertices which have the same out-neighbour set (or in-neighbours set) in a DSRG.

Advisor: Zhang Bicheng

Doctor Thesis Awards, Honorable Mention

Cao Yalong, The Chinese University of Hong Kong

Thesis Title: Gauge Theory and Calibrated Geometry for Calabi-Yau 4-folds

Advisor: Naichung Conan Leung

Kwok-Wing Tsoi, King's College London

Thesis Title: On Special Elements for p-adic Representations and Higher Rank Iwasawa

Advisor: David Burns

Chen Xiuqiong, Tsinghua University

Thesis Title: On the Study of the Direct Method and Suboptimal Method in Nonlinear Filtering Problems

Advisor: Stephen S. T. Yau

Dong Xin, Nagoya University

Thesis Title: Bergman Kernel and Its Boundary Asymptotics

Advisor: Takeo Ohsawa

Hu Kaibo, Peking University

Thesis Title: Finite Element Exterior Calculus for Multiphysics Problems

Advisor: Xu Jinchao

Huang Min, Zhejiang University

Thesis Title: On Cluster Algebras: Structure, Methods, Open Problems

Advisor: Li Fang

Huang Ruizhi, University of Chinese Academy of Science

Thesis Title: Algebraic Structures in Unstable Homotopy Theory

Advisor: Wu Jie

Lei Li, Zhejiang University

Thesis Title: Mean Curvature Flow and Differentiable Sphere Theorems for Submanifolds in Space Forms

Advisor: Xu Hongwei

Li Shihao, University of Chinese Academy of Science

Thesis Title: Orthogonal Polynomials, Toda Lattices and Peakon Equations

Advisor: Hu Xingbiao

Li Yingzhou, Stanford University

Thesis Title: Sparse Factorizations and Scalable Algorithms for Differential and Integral Operators

Advisor: Ying Lexing

Lin Chien, Taiwan University

Thesis Title: Geometric Analysis in Pseudohermitian Manifolds

Advisor: Shu-Cheng Chang

Wai Yeung Lam, Brown University

Thesis Title: Infinitesimal Deformations of Discrete Surfaces

Advisor: Ulrich Pinkall

Ling Shuyang, New York University

Thesis Title: Bilinear Inverse Problems: Theory, Algorithms, and Applications

Advisor: Thomas Strohmer

Pan Yu, Duke University

Thesis Title: Augmentations and Exact Lagrangian Cobordisms

Advisor: Lenhard Ng

Ren Jinbo, University of Virginia

Thesis Title: Autourde la Conjecture de Zilbert-Pink pour les VariÉTÉS de Shimura (Around the Zilber-Pink conjecture for Shimura Varieties)

Advisor: Emmanuel Ullmo

Shi Quan, University of Zurich

Thesis Title: Fragmentations, Growth-Fragmentations, and Random Structures

Advisor: Jean Bertoin

Sun Ju, Columbia University

Thesis Title: When Are Nonconvex Optimization Problems Not Scary?

Advisor: John Wright

Yat-Hin Suen, The Chinese University of Hong Kong

Thesis Title: SYZ Mirror Symmetry for Immersed Lagrangian Multi-Sesctions

Advisor: Kwokwai Chan

Shu Ruiwen, University of Wisconsin-Madison

Thesis Title: Uncertainty Quantification and Sensitivity Analysis for Multiscale Kinetic Equations with Random Inputs

Advisor: Jin Shi

Tang Xinxing, Peking University

Thesis Title: The Genus 1 Theory of LG B-Model

Advisor: Huijun Fan

Tang Xiudi, University of California San Diego

Thesis Title: Symplectic Stability and New Symplectic Invariants of Integrable Systems

Advisor: Álvaro Pelayo

Tian Kun, Tsinghua University

Thesis Title: Classification of Gene and Protein Sequences

Advisor: Stephen S.-T. Yau

Tian Xiaochuan, Columbia University

Thesis Title: Nonlocal Models with a Finite Range of Nonlocal Interactions

Advisor: Du Qiang

Wang Dong, The Hong Kong University of Science and Technology

Thesis Title: Threshold Dynamics Method: Theories, Algorithms, and Applications

Advisor: Wang XiaoPing

Wang Zhiwei, Université de Lorraine

Thesis Title: Les Plus Grands Facteurs Premiers d'entiersconsécutifs (The Largest Prime Factors of Consecutive Integers)

Advisor: Cécile Dartyge, Jei Wu

Xu Daxin, Université Paris-Saclay

Thesis Title: P-Adic and Modulo P^N Simpson's Correspondences

Advisor: Ahmed Abbes

Yang Yilong, University of California Los Angeles

Thesis Title: Shapes of Finite Groups through Covering Properties and Cayley Graphs

Advisor: Terence Tao

Zhang Tao, Zhejiang University

Thesis Title: Combinatorial Configurations, Lattice Tilings and Their Applications In Information Science

Advisor: Ge Gennian

Man Wai Cheung, Harvard University

Thesis Title: Tropical Techniques in Cluster Theory and Enumerative Geometry

Advisor: Mark Gross

Zhou Shenglong, Univeristy of Southampton

Thesis Title: Multidimensional Scaling via Euclidean Distance Matrix Optimization

Advisor: Qi Houduo

Bachelor Thesis Awards, Honorable Mention

Cao Huabin, Xiangtan Univerisity

Thesis Title: One-to-one Disjoint Path Covers in Digraphs

Advisor: Zhang Bicheng