Contributions of Stephen Shing-Toung Yau in Non-Linear Filtering and Control Theory

by Tyrone Duncan

Editor's Note: Tyrone Duncan is a professor of mathematics at the University of Kansas, his fundamental contributions focus on nonlinear filtering, stochastic control, and the relation between probability and geometry. He is one of the founders of nonlinear filtering theory. His research interests are in differential geometry, probability, stochastic control, and statistics. He was elected as IEEE Fellow in 1999 and IFAC Fellow in 2000. He won the W. T. and Idalia Reid Prize from SIAM in 2013. In 2015, he was elected as Simons Fellow and SIAM Fellow.

Stephen Shing-Toung Yau has studied nonlinear filtering (NLF) and control theory for more than thirty years. The NLF problem is to determine an estimate of the signal from these noisy observations. A complete solution to the NLF problems is often described by a stochastic partial differential equation (SDE) that is driven by the observations which is called the Duncan-Mortensen-Zakai (DMZ) equation. It is the unnormalized probability density for this estimation problem.

Ever since the technique of Kalman-Bucy filter popularized, there has been an intense interest in finding new classes of the finite dimensional recursive filters. In [1], Yau for the first time established a simple algebraic necessary and sufficient condition for an estimation algebra (EA) of a general class of filtering systems to be finite dimensional. After a decade of efforts, Yau finally summarized his fruitful results on classification of the EA in [2], where he completely classified the EA with maximal rank. The classification question was proposed by Brockett in his famous talk in International Congress of Mathematics, 1983. The classification of the EA with non-maximal rank is far from being completed. He is still actively working in this direction, for example [3] and so on. This topic has been a major area of Yau's research in his early works. Yau's classification methods require important techniques from algebra and algebraic geometry. He is the world leader in this study.

In 2008, Yau and his co-workers proposed the so-called Yau-Yau algorithm, which can solve the pathwise DMZ equation efficiently [4]. In this algorithm, Yau applied the idea of splitting the heavy computations into two parts: solving partial differential equation (PDE) off-line and synchronizing the observations on-line. It has been shown that the approximate solution converged to the true solution of DMZ equation by very delicate PDE techniques. Later, Yau and his co-worker generalized the Yau-Yau algorithm to the time-varying system [5]. This algorithm was first implemented in scalar state NLF problem with the help of Hermite spectral method and verified its real-time manner in [6]. To overcome the curse of dimensionality of high-dimensional states' NLF problems, Yau and his collaborator investigated the sparse grid algorithm combined with the Hermite spectral method in solving parabolic PDEs [7]. Besides the splitting-up idea in Yau-Yau algorithm, Yau and his co-worker is still working on the direct method in solving the DMZ method [8]. These works brought the methods of solving the DMZ equation to the practical tools in NLF problems. These works are original and fascinating.

Nevertheless, the computational load of the above algorithm is extremely high even with advanced strategies, compared with the local approaches, such as the Kalman filter and its derivatives. Taking the Carleman linearization technique in mind, Yau and his co-worker proposed novel algorithms to the polynomial systems, so to the NLF problems [9], [10]. In all, Yau has made tremendous contributions in control community.

In fact, the real time filters constructed by Yau-Yau algorithm are the only filters that can be proven mathematically to track the state accurately in real time [5]–[8]. It is clear that he is the leading expert in the NLF problems in the world.

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