Stephen Yau, a Teacher and a Friend

by Xue Luo, Guowei Wei, Changchuan Yin and Huaiqing Zuo

Editor's Note: At the 2019 International Congress of Chinese Mathematicians (ICCM), Stephen Yau received the Chern Award in recognition of his lifetime of contributions to mathematics, in such areas as Several Complex Variables, Control Theory, and Bioinformatics. In this article, the authors discuss some of these contributions.

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Stephen S.-T. Yau is a devoted researcher and educator. He spends most of his time with his students. Days with Stephen are full of excitement as well as warmth. He encourages his students working hard, because he himself is a hard worker. He treats his students like his friends; he is never highbrow. It fits him so well that he was awarded the life achievement Chern Award in the International Congress of Chinese Mathematicians this summer.

Stephen S.-T. Yau is a leading mathematician, whose work involves a variety of subjects in both pure and applied mathematics. He is a professor of mathematics at Tsinghua University since 2011. Previously, he held a Distinguished Professorship at the University of Illinois at Chicago, where he had taught mathematics and conducted research for more than 30 years. He earned his Ph.D. from the State University of New York at Stony Brook, was a postdoctoral scholar at Institute for Advanced Study at Princeton, and became a Benjamin Pierre Assistant Professor at Harvard University.

Yau is a gifted and prolific researcher. He authored or coauthored more than 360 scientific papers. Yau pioneered the study of Cauchy-Riemann (CR) geometry. In particular, Joseph Kohn and Hugo Rossi conjectured that Kohn-Rossi cohomology on a strongly pseudoconvex CR manifold X depends only on the interior singularities of the variety bounded by X. Yau's theorem published in the Annals of Mathematics in 1980 answered the conjecture affirmatively. As a result, he solved the famous complex Plateau problem in some important cases and opened a new direction in CR geometry.

Yau is a well-known expert in singularity theory. He invented the notion of elliptic sequence and introduced a new theory of maximally elliptic singularities. He successfully classified topologically elliptic hypersurface singularities. This research opens a new mathematics domain in surface singularities. Yau sequence and Yau cycle play a fundamental role in studying normal surface singularities. His other significant work includes a joint work with John Mather published in Inventiones in 1982 where a famous theorem states that two isolated hypersurface singularities are biholomorphically equivalent if and only if their moduli algebras are isomorphic. Thus, via their result, the biholomorphic classification of isolated hypersurface singularities is contained in the algebraic classification of commutative Artinian local algebras. In another important contribution, Yau was the first to systematically study the Lie algebras of derivations of these moduli algebras. He proved that these Lie algebras are solvable and showed in many important cases how they change as the complex structures of the singularity vary. Therefore, Yau has established a natural connection between the theory of singularities and the theory of finite dimensional solvable Lie algebras. These Lie algebras are

called Yau algebras by Elashvili and Khimshiashvili. They defined the dimension of these Lie algebras as the Yau number.

Since 1990, Yau has been interested in many branches of applied mathematics, including filtering, control theory, and bioinformatics. In 1990, Yau introduced a general class of finite dimensional filters, which were later named as Yau filters. The Yau filters include all previously well-known filters, such as Kalman-Bucy filters and Benes filters, as special cases. Yau filters are unique in the sense that they work for arbitrary initial distribution. Essentially, all finite dimensional filters are Yau filters. Furthermore, the classification of finite-dimensional estimation algebras with maximal rank proposed by Roger Brockett in his 1983 ICM lecture was completed by Yau. A challenge proposed by the Naval Research Office is that with enough computational resources, how to solve the nonlinear filtering (NLF) problem efficiently. Using delicate PDE techniques and ideas from quantum mechanics, Yau and S.T.Yau solved this problem by their famous Yau-Yau algorithm. They constructed an explicit integral solution to the Kolmogorov equation, which plays an important role in NLF problem and has a significant impact on both engineering and mathematics. In fact, his solution to the Kolmogorov equation, a half-century old open problem, puts him on par with world's top mathematicians. In 2013, Yau and his co-worker implemented Yau-Yau algorithm by Hermite spectral method for NLF problems with low-dimensional state. Also the convergence rate has been analyzed. To overcome "the curse of dimensionality" in NLF problems with high-dimensional state, Yau and his co-worker combined the sparse-grid algorithm with spectral method in numerically solving the Kolmogorov equation.

Recently, Yau has paid attention to bioinformatics for its obvious importance in the 21st Century. His work focuses on mathematical methods and algorithms for contemporary biology, specifically, for DNA and protein sequence data analysis. Yau has pioneered a number of mathematical approaches which may potentially revolutionize the field. In his work published Nucleic Acids Research, Yau proposed a two-dimensional DNA graphical representation without degeneracy, offering a unique approach for genome visualization. Additionally, Yau introduced a natural vector for the numerical characterization of DNA and protein sequences. The resulting protein map is able to efficiently analyze protein sequences, structural domains, and arbitrary amino acid sequences. This work provides a new powerful tool for protein functional classification. Furthermore, Yau proposed a novel fast Fourier algorithm for genome analysis, revealing genome structure, periodicity, and similarity. Finally, Yau also proposed the Yau-Hausdorff distance for analyzing genome sequences, providing an efficient mathematical metric for analyzing the similarity of biological species. This approach has also been used to efficiently measure the similarity of protein structures.

Yau has a strong record in community service. He is the founding editor of Journal of Algebraic Geometry and a co-founding editor of Communications in Information and Systems. He co-organized many international conferences. Yau is also a very dedicated educator and has mentored 45 Ph.D. students.

Yau holds a long list prestigious titles, including University Distinguished Professor at the University of Illinois at Chicago (2005–2011), IEEE Fellow (2003), Guggenheim Fellowship (2000–2001), and Alfred P. Sloan Research Fellowship (1980–1982).