A Brief Chronicle of the Levi (Hartogs' Inverse) Problem, Coherence and Open Problem

To the Memory of Professor Akira Takeuchi

by Junjiro Noguchi*†

Abstract. Here we chronologically summarize briefly the developments of the Levi (Hartogs' Inverse) Problem together with the notion of coherence and its solution, shedding light on some records which have not been discussed in the past references. In particular, we will discuss K. Oka's unpublished papers *1943* which solved the Levi (Hartogs' Inverse) Problem for unramified Riemann domains of arbitrary dimension $n \geq 2$, usually referred as it was solved by Oka IX in *1953*, H.J. Bremermann and F. Norguet in *1954* for univalent domains, independently.

At the end we emphasize an open problem in a ramified case.

MSC 2010 subject classifications: 32A99, 32E30.

Keywords and phrases: Coherence, Oka, Levi problem, several complex variables.

1. Introduction

There are now a number of interesting and invaluable comments/surveys on the developments of the titled Problem and Coherence in complex analysis of several variables such as, e.g., H. Cartan's comments in [39], H. Grauert's Commentary of [11] Part II, I. Lieb [21]. The purpose of the present article is to recall briefly the developments of the problem and the solution together with the notion of coherence, shedding light on some records and unpublished manuscripts of K. Oka that have not been discussed very much in the former references. We will see that the original Levi (Hartogs' Inverse) Problem itself was historically solved for unramified Riemann domains over $\mathbf{C}^n$ in Oka's unpublished papers 1943 (cf. [32] **Theorem I** at p. 27), and then observe how the notion of "*Coherence*" ("*Idéaux de domaines indéterminés*" in Oka's terms) evolved from the problem: Here there is a new point, for those two issues have been discussed independently in the past references (cf., e.g., [13] Introduction, [21]).

As we will see in §2, the turn of years "*1943/44*" was indeed a watershed in the study of analytic function theory of several variables. In 1943 K. Oka finished the Three Big Problems in the survey monograph of Behnke–Thullen 1934 (see items 5, 10 in §2). In the next year 1944 K. Oka began to study the arithmetic property of analytic functions of several variables by investigating Weierstrass' Preparation Theorem ([40]) which later led to the notion of "coherence" in 1948 ([34]), and in the same year 1944 H. Cartan wrote an experimental paper [4] (cf. item 11 in §2).

We will employ commonly used notion and terminologies in analytic function theory of several variables without definitions (cf., e.g., [14], [18], [12], [22], [23]) except for a *Riemann domain* X over $\mathbf{C}^n$ (resp.

* Graduate School of Mathematical Sciences, University of Tokyo (Emeritus), Komaba, Meguro-ku, Tokyo 153-8914, Japan

E-mail: noguchi@ms.u-tokyo.ac.jp

† Research supported in part by Grant-in-Aid for Scientific Research (C) 15K04917.

$\mathbf{P}^n(\mathbf{C})$), which in the present note is a possibly singular reduced complex space X together with a holomorphic map $\pi : X \to \mathbf{C}^n$ (resp. $\mathbf{P}^n(\mathbf{C})$) such that the fibers $\pi^{-1}\{z\}$ are discrete for all $z \in \mathbf{C}^n$ (resp. $\mathbf{P}^n(\mathbf{C})$): If π is locally biholomorphic, then X is called an *unramified* Riemann domain over $\mathbf{C}^n$ (resp. $\mathbf{P}^n(\mathbf{C})$) (X is necessarily non-singular in this case).

2. Levi (Hartogs' Inverse) Problem and Coherence

Karl Weierstrass proved his famous Preparation Theorem about *1860* (cf. [13] p. 38). According to K. Oka, K. Weierstrass considered that the theory of analytic functions of two or more variables would be quite similar to that of one variable, and in particular that the shape of singularities of those functions should be arbitrary; this observation had lasted for quite a while. Then, however, different phenomena had been found, as the subject had been studied more.

It is noted that the following list is far from being complete:

1. Friedrich Hartogs [15], *1906*: He found a phenomenon of simultaneous analytic continuation of complex analytic functions of two or more variables (Hartogs' phenomenon).
2. Eugenio Elia Levi [19]/[20], *1910/11*: With the boundary regularity he made clear the pseudoconvexity property of the boundary of a domain of holomorphy.
3. Henri Cartan–Peter Thullen [7], *1932*: They proved the equivalence of domains of holomorphy and holomorphically convex ones. Then, K. Oka systematically used the property of holomorphic convexity.
4. Wahlter Rückert [41], *1933*: Here, Rückert's Nullstellensatz, which is sometimes called the Hilbert–Rückert Nullstellensatz, was proved. This result played later a fundamental role in the study of singular complex analytic spaces and the coherence, but at the beginning the importance was not recognized very much.
5. Heinrich Behnke–P. Thullen [1], *1934*: In this monograph they surveyed the research state of the theory of several complex variables and raised the *Three Big Problems* in several complex variables, on which they put a special importance:
   (a) Levi (Hartogs' Inverse) Problem[1] ([1] Chap. IV).
   (b) Cousin I/II Problems ([1] Chap. V).
   (c) Problem of developments (Approximation problem of Runge type) ([1] Chap. VI).

   The monograph was of a special importance for K. Oka to change his research direction to these problems[2].
6. Kiyoshi Oka I–III [27], [28], [29], *1936–1939*: He solved Problems (b) and (c) above, introducing a principle (method) termed "Jôku-Ikô"[3]. The well-known "Oka Principle" is in Oka III.
7. Henri Cartan [3], *1940*: H. Cartan introduced the algebraic notion of ideals, congruence, etc. into the theory of analytic function theory, and proved his matrix decomposition theorem.[4]
8. K. Oka [30], *1941*: This is an announcement of the affirmative solution to the Levi (Hartogs' Inverse) Problem for univalent domains (subdomains) of $\mathbf{C}^2$.

   Although the communication between K. Oka and H. Cartan was suspended by the war, K. Oka tried to communicate with H. Cartan through H. Behnke. We quote a part of H. Cartan [6], which represents the mode of relationship among K. Oka, H. Behnke and H. Cartan even under the war:

   > 1939, 1940. C'est la guerre. Dès rétablissment des relations postales entre la France occupée et l'Allemagne, en février 1941, je reçois une lettre de mon ami Behnke. Il me fait part d'une lettre de Oka (datée de décembre 1940) qui annonce qu'il a résolu le problème de la pseudo-convexité globale (problème de Levi). Oka avait joint à sa lettre deux exemplaires de son manuscrit, dont l'un m'est destiné (mais ne me parviendra pas). Heinrich Behnke prend la peine de recopier de sa main la lettre de Oka, écrite en français. Je ne résiste pas à l'envie d'en citer un extrait: ≪Comme vous le connaîtrez bien, j'étudie la théorie d'après votre point de vue. Et, pour le "Hauptproblem der Theorie der Singularitäten" formulé dans votre ouvrage, je viens heureusement à vous faire la réponse affirmative. Je voudrais présenter le manuscrit de ma Note sur le problème à vous et à votre ami, M. Henri Cartan, mais je n'ai rien de nouvelle de lui. Et je vous envoie ici deux manuscrits. Auriez-vous la bonté de lui remettre un à quelque occasion favorable?≫ L'ouvrage dont parle Oka est naturellement le "Bericht Behnke-Thullen" qui a servi de point de départ à toutes les recherches d'Oka.

[1] K. Oka termed the problem as Hartogs' Inverse Problem (cf. [30], [31], [36]). Hartogs' Inverse Problem is of a more primitive or more general form than the Levi Problem in the sense that the latter assumes a C^2 boundary regularity of a given domain, whereas the first does not.

[2] At that time he had been writing an unpublished paper titled "Fonctions algébriques permutables avec une fonction rationnelle non-linéaire", pp. 97, which was typed in French (cf. [40]).
It is also interesting to note that K. Oka solved these problems in the reversed order in time.

[3] "上空移行" (in Japanese, Kanji letters): This is a method or a principle of K. Oka all through his series of papers [27]–[36] such that to solve a problem on a difficult domain one embeds the domain into a higher dimensional polydisk, extends the problem on the polydisk, and then solves it by making use of the simple shape of the polydisk (cf. [23]).

[4] This paper was the last one of the research informations from the outside of Japan that K. Oka had until the end of the war (1940–1945).

Nouvelle lettre de Behnke du 8 août 1941: avec l'aide de Wilhelm Süss, il s'est réfugié à Fribourg avec sa femme qui va avoir un bébé. Entre temps, Oka a publié sa Note au Japon.

1939, 1940. It is the war. By the reestablishment of the postal relations between occupied France and Germany, on February 1941, I received a letter from my friend Behnke. He tells me a letter of Oka (dated December 1940) which announces that he has solved the problem of the pseudo-convexity in global (problem of Levi). Oka has added to his letter two copies of his manuscript, one of which is addressed to me (but not reached me). Heinrich Behnke took the effort to recopy by his own hand the letter of Oka written in French. I cannot resist the desire to cite an extract: ≪As you know well, I study the theory from your viewpoint. And, for the "Main Problem of the theory of singularities" formulated in your book, I happily get to show you the affirmative answer. I would like to present the manuscript of my Note on the problem to you and to your friend M. Henri Cartan, but I have no news from him. And I send here two manuscripts. Would you kindly give one to him on any your favorite occasion?≫ The book mentioned there by Oka is naturally the "Bericht Behnke–Thullen" which has served from the point of beginning to all researches of Oka.

New letter from Behnke on 8 August 1941: with the aid of Wilhelm Süss, he escaped to Fribourg with his wife who was going to have a baby. During the time, Oka has published his Note at Japan.

(transl. by the author)

9. K. Oka VI [31], *1942*: This is the full paper of the former one with a remark on the validity of the result in all dimensions $n \geq 2$. The key of the proof was the so-called Oka's Heftungslemma which was proved by means of Weil's integral formula in two dimensional case. Here he dealt with the Hartogs pseudoconvexity, so that he put no condition on the regularity of the boundary of the domain, while the notion of Levi pseudoconvexity needs at least C^2-regularity.
In the course of the proof he modified the Levi pseudoconvexity by introducing a new class of real-valued functions called "fonctions pseudoconvexes" in Oka VI §11, which were also called "fonctions plurisousharmoniques" by Pierre Lelong around the same time.
10. K. Oka [40], [32], *1943*, Research reports to Teiji Takagi (in Japanese, unpublished) (cf. §3): In this year just after Oka VI, *Oka proved the Levi (Hartogs' Inverse) Problem for unramified Riemann domains of general dimension* ≥ 2 in a series of five research reports of pp. 109 in total, sent to Teiji Takagi (Tokyo, well known as the founder of class field theory). The reports were written in Japanese and unpublished; they are now available in [40] (Japanese). For an unramified Riemann domain over $\mathbf{C}^n$ with $n \geq 2$ he first proved the *Jôku-Ikô*, and then by making use of it he proceeded to the *Approximation Problem* and *Cousin I/II Problems*, and finally solved *Levi's (Hartogs' Inverse) Problem*. Let us quote the most main result of the papers (see [32][5], p. 27):

> **Theorem I.** *A pseudoconvex finite domain*[6] *with no interior ramification point is a domain of holomorphy.*

He remarked this fact three times in his published papers, first in his survey note [33] (1949), in VIII [35] (1951), and in IX [36] (1953).
Comparison to Oka VI (1942): The method of the proof was very different from the previous one of Oka VI. In these reports 1943 he proved Heftungslemma by the combination of Jôku-Ikô and Cauchy's integral formula (in fact, it is a half of Cauchy integral called the *Cousin integral*) in place of Weil's integral formula, which was not obtained on an unramified Riemann domain over $\mathbf{C}^n$ (cf. [32], [36] §24).
Comparison to Oka IX (1953): It is the same in both solutions to construct a continuous plurisubharmonic exhaustion function on a pseudoconvex unramified Riemann domain, but in 1943 the coherence theorems of K. Oka VII/VIII (see item 13) used in Oka IX was not yet invented, and hence not used. But, it is noted that in the course he proved a *sort of coherence theorem* in a special case (cf. §3 below).
N.B. As H. Cartan wrote in Oka [39], p. XII,

> "Mais, il faut avouer que les aspects techniques de ses démonstrations et le mode de présentation de ses résultats rendent difficile la tâche du lecteur, et que ce n'est qu'au prix d'un réel effort que l'on parvient à saisir la portée de ses résultats, qui est considérable."
>
> "But, one must admit that the technical aspects of his demonstrations and the mode of presentation of his results make it difficult for the reader, and that it is only at the price of a real effort that one can grasp the extent of his results, which is considerable."
>
> (transl. by the author)

it is yet not easy to read these unpublished papers. But, in fact, it is possible to complete the proofs of the Three Big Problems without Weierstrass' Preparation Theorem (essential in the proof of coherence), or the theory of sheaf cohomologies of Cartan–Serre, nor $L^2 - \bar{\partial}$ method of Hörmander (if interested, cf. [25]).
11. H. Cartan [4], *1944*: Let us quote from Grauert–Remmert [13] Introduction, 2.

> Of greatest importance in Complex Analysis is the concept of a coherent analytic sheaf. Already in 1944 Cartan had experimented with the notion of a coherent system of punctual modules. He posed the fundamental problem, whether for any finite system of holomorphic functions the derived module system of punctual relations is coherent. This is exactly the problem, whether the sheaf $\mathcal{O}_{\mathbf{C}^n}$ of germs of holomorphic

[5] For convenience, the present author translated the most important last one among the five into English.
[6] Here "finite domain" means "domain over $\mathbf{C}^n$".

functions on complex n-space is coherent. In 1948 Oka gave an affirmative answer; in 1950 Cartan simplified Oka's proof, introducing the terminology "faisceau cohérent". This paper was not known in Japan, in particular to K. Oka by the interruption caused by the war.

12. Shin Hitotsumatsu [16], *1949*: He generalized Oka's Heftungslemma to the n-dimensional case with arbitrary $n \geq 2$, so that *he solved the Levi (Hartogs' Inverse) Problem in the case of univalent domains of* $\mathbf{C}^n$ *with* $n \geq 2$. The proof relied on Weil's integral formula in n-variables. This was published in Japanese and has not been referred in the former references.
13. K. Oka VII/VIII [34]/[35], *1948*[7]*/51*: He proved his three coherence theorems (1'st, $\mathscr{O}_{\mathbf{C}^n}$; 2'nd, $\mathscr{I}\langle A\rangle$, ideal sheaves of analytic subsets A, also proved by H. Cartan [5]; 3'rd, Normalization Theorem). Here, *Oka's aim of "coherence" was to prove the Levi (Hartogs' Inverse) Problem obtained in 1943 (item 10) for singular ramified Riemann domains over* $\mathbf{C}^n$.

    This intention of K. Oka, which later countered by J.E. Fornæss' example (see item 20), might have two aspects: One was the ten years delay of the publication of the solution to the Levi (Hartogs' Inverse) Problem for unramified Riemann domains over $\mathbf{C}^n (n \geqq 2)$, and the other was the motive locomotive of the study that led to the completely new concept of "Coherence" (Idéaux de domaines indéterminés) in 1948/51.

    Nowadays we can find *two versions of Oka VII*; one is [34], and the other original is in [38]. The English translation of Oka VII in [39] is based on the original in [38].[8]

    Probably, the most notable part of the difference in the two versions is the last part of the introduction, where in the original [38] VII he wrote:

    > Or, nous, devant le beau système de problèmes à F. Hartogs et aux successeurs, voulons léguer des nouveaux problèmes à ceux qui nous suivront; or, comme le champ de fonctions analytiques de plusieurs variables s'étend heureusement aux divers branches de mathématiques, nous serons permis de rêver divers types de nouveaux problèmes y préparant.

    English translation from [39] (only added "Now" at the beginning):

    > Now, having found ourselves face to face with the beautiful problems introduced by F. Hartogs and his successors, we should like, in turn, to bequeath new problems to those who will follow us. The field of analytic functions of several variables happily extends into diverse branches of mathematics, and we might be permitted to dream of the many types of new problems in store for us.[9]

    The paragraph above was completely deleted from the published paper without notification to K. Oka. As K. Oka knew the differences between the published [34] and the original [38] VII, he thought it would be necessary to publish the original VII, once again in a journal, recognizing that it is of an extremely exceptional case. Thus, he wrote an article, "*Propos postérieur*" [37], which he would have wished to put as an "Appendix" to his original VII (however, in [38] VII there is no "Appendice").

    It is really interesting to learn how deeply he was concerned with this problem of the motivation and how he developed his innovative study. (Cf. [23] Chap. 9 "On Coherence" for more comparisons.)
14. K. Oka, IX [36], *1953*: He solved affirmatively the Levi (Hartogs' Inverse) Problem for unramified Riemann domains over $\mathbf{C}^n$ ($n \geq 2$). As mentioned in the paper, the proof was essentially the same as in his 1943 unpublished papers (item 10), and so it was very different from that in Oka VI. The most essential part, his Heftungslemma, was here proved by making use of his First and Second Coherence Theorems, Jôku-Ikô, and the Cousin integral (cf. item 10 and [36]) in place of Weil's integral formula which was used in Oka VI but not available on unramified Riemann domains over $\mathbf{C}^n$ (see [36] §23).

    Here he called the problem "*Hartogs' Inverse Problem*"; in fact, he put no condition on the boundary regularity of the domain. In the course, he proved (b) and (c) of item 5 on a holomorphically convex unramified Riemann domain over $\mathbf{C}^n$.

    He left two open problems ([36] §23): 1) the Levi (Hartogs' Inverse) Problem for unramified Riemann domains over $\mathbf{P}^n(\mathbf{C})$; 2) the Levi (Hartogs' Inverse) Problem for *ramified* Riemann domains over $\mathbf{C}^n$ or over $\mathbf{P}^n(\mathbf{C})$.
15. Hans J. Bremermann [2]; François Norguet [26], *1954*: They proved independently the Levi (Hartogs' Inverse) Problem for univalent domains of $\mathbf{C}^n$ with arbitrary $n \geq 2$ by proving Oka's Heftungslemma in the n-dimensional case with Weil's integral formula, as in Hitotsumatsu [16] 1949 (see item 12).

[7] This is the year of the received date of Oka VII [34] which was in fact published in 1950; it took rather long time for publication. In a number of references it is referred so that K. Oka proved his First Coherence Theorem (the coherence of $\mathscr{O}_{\mathbf{C}^n}$) in *1948* just as in item 11. Here we followed it.

[8] Probably because of these complexities of "dates", all records of the received dates of the papers in [39] are deleted.

[9] One should notice that when K. Oka wrote these words, he had already finished the Three Big Problems of Behnke–Thullen five years before (cf. item 10).

16. Hans Grauert [10], *1958*: He gave a considerably simplified proof of Oka's Theorem (IX) (item 14) by his well-known "Bumping Method" combined with L. Schwartz's finite dimensionality theorem[10].
17. H. Grauert, about *1960*: A counter-example to the Levi (Hartogs' Inverse) Problem for a ramified Riemann domain over $\mathbf{P}^n(\mathbf{C})$.
18. Reiko Fujita [9], *1963*; Akira Takeuchi [42], *1964*: Independently, they affirmatively proved the Levi (Hartogs' Inverse) Problem for unramified Riemann domains over $\mathbf{P}^n(\mathbf{C})$ with at least one boundary point.
19. Lars Hörmander [17], *1965*: He proved the Levi (Hartogs' Inverse) Problem by solving directly the $\bar{\partial}$-equations with an L^2-method. At the beginning of Chap. IV of his well-known book [18] L. Hörmander wrote: "In this chapter we abandon the classical methods ..... Instead, ... the Cauchy–Riemann equations where the main point is an L^2 estimate ....."

   ⋮

20. John Eric Fornæss [8], *1978*: He gave a counter-example to the Levi (Hartogs' Inverse) Problem by constructing a smooth 2-sheeted ramified Riemann domain $X \xrightarrow{\pi} \mathbf{C}^2$, which is locally Stein but not globally: Here being locally Stein is defined as for every $z \in \mathbf{C}^2$ there is a neighborhood U of z such that $\pi^{-1}U$ is Stein.

   ⋮

3. Oka's Unpublished Papers 1943 and Ramified Riemann Domains

K. Oka solved affirmatively the Levi (Hartogs' Inverse) Problem for univalent domains of $\mathbf{C}^2$ in 1942 ([31]) and for unramified Riemann domains over $\mathbf{C}^n$ (arbitrary $n \geq 2$) in 1943 by writing five research reports to T. Takagi (Tokyo) (cf. §2, item 10):

1) On analytic functions of several variables: VII – Subproblem on congruence of holomorphic functions, pp. 28.
2) On analytic functions of several variables: VIII – The first fundamental lemma on finite domains without ramification points, pp. 11.
3) On analytic functions of several variables: IX – Pseudoconvex functions, pp. 30.
4) On analytic functions of several variables: X – The second fundamental lemma, pp. 11.
5) On analytic functions of several variables: XI – Pseudoconvex domains and finite domains of holomorphy, Some theorems on finite domains of holomorphy, pp. 29.

[10] The proof of this theorem has been known to be rather long and involved by making use of the dual spaces. Now a very simple proof of it in a slightly generalized form is available (see [23] §7.3.4.).

It is noteworthy that in the above VII he proved a special case of *coherence property* (for the so-called Oka maps used in Jôku-Ikô) already in 1943.

He did not translated these handwritten manuscripts into French for publications, but immediately began to study the Levi (Hartogs' Inverse) Problem for Riemann domains with ramifications. He subsequently wrote the following in the same series as above:

6) On analytic functions of several variables: XII – Representation of analytic subsets, pp. 24, 1944.
7) On analytic functions of several variables: XII – Extension of the Cousin II problem, pp. 16, 1945.
8) XIII – On a condition in Weierstrass' preparation theorem, pp. 67, 1945.

Here in 6) XII above, he first used Weierstrass' Preparation Theorem for the study of the congruence problem of holomorphic functions: The purpose was to deal with singular Riemann domains with ramifications, and this study motivated and led him to invent the "*Coherence*" (*"Idéaux de domaines indéterminés"* in Oka's terms) of holomorphic functions ([34], [35]).

In a talk titled "On analytic functions of several variables" at Yukawa Institute for Theoretical Physics, Kyoto University 1964, K. Oka put a special emphasis again on the problem of ramified Riemann domains (cf. [40], Unpublished manuscripts, No. 19), telling that:

> As for Hartogs' Inverse Problem .... And the problem to allow ramification points remains completely unsolved. I have worked on this for a rather long time, but I am obstinately keeping the position to prove it unconditionally. For I have been doing so up to the present, so otherwise, it is a pity ..... H. Grauert wrote a paper such that there is an algebraic ramified domain which is a domain of holomorphy but not pseudoconvex ....

H. Grauert also emphasized the Levi (Hartogs' Inverse) Problem for ramified Riemann domains in the talk at Memorial Conference of Kiyoshi Oka's Centennial Birthday, Kyoto/Nara 2001, which we now formulate as follows.

Problem (*Oka's Dream*, cf. item 13). Let $\pi : X \to \mathbf{C}^n$ be a ramified Riemann domain (cf. the beginning of §1). Assume that for every point $a \in \mathbf{C}^n$ there is a neighborhood U of a in $\mathbf{C}^n$ such that $\pi^{-}U$ is Stein (locally Stein). Find sufficient or necessary conditions for X to be Stein.

The problem above is open even for non-singular X (cf. [24] for some affirmative result).

References

[1] H. Behnke and P. Thullen, Theorie der Funktionen mehrerer komplexer Veränderlichen, Ergebnisse der Mathematik und ihrer Grenzgebiete Bd. 3, Springer-Verlag, Heidelberg, 1934.

[2] H.J. Bremermann, Über die Äquivalenz der pseudokonvexen Gebiete und der Holomorphiegebiete im Raum von n komplexen Veränderlichen, Math. Ann. **128** (1954), 63–91.

[3] H. Cartan, Sur les matrices holomorphes de n variables complexes, J. Math. pure appl. **19** (1940), 1–26.

[4] H. Cartan, Idéaux de fonctions analytiques de n variables complexes, Ann. Sci. École Norm. Sup. **61** (1944), 149–197.

[5] H. Cartan, Idéaux et modules de fonctions analytiques de variables complexes, Bull. Soc. Math. France **78** (1950), 29–64.

[6] H. Cartan, "Quelques Souvenirs" presented to H. Behnke's 80th birthday in October 1978 at Münster, Springer-Verlag, Berlin-Heidelberg-New York.

[7] H. Cartan und P. Thullen, Regularitäts- und Konvergenzbereiche, Math. Ann. **106** (1932), 617–647.

[8] J.E. Fornæss, A counterexample for the Levi problem for branched Riemann domains over $\mathbf{C}^n$, Math. Ann. **234** (1978), 275–277.

[9] R. Fujita, Domaines sans point critique intérieur sur l'espace projectif complexe, J. Math. Soc. Jpn. **15** (1963), 443–473.

[10] H. Grauert, On Levi's problem and the imbedding of real-analytic manifolds, Ann. Math. **68** (1958), 460–472.

[11] H. Grauert, Selected Papers, Vol. 1, Springer-Verlag, Berlin-Heidelberg-New York-London-Paris-Tokyo-Hong Kong-Barcelona-Budapest, 1994.

[12] H. Grauert and R. Remmert, Theorie der Steinschen Räume, Grundl. Math. Wiss. 227, Springer-Verlag, Berlin, 1977: Translated to English by A. Huckleberry, Theory of Stein Spaces, Springer-Verlag, Berlin, 1979: Translated to Japanese by K. Miyajima, Stein Kukan Ron, Springer, Tokyo, 2009.

[13] H. Grauert and R. Remmert, Coherent Analytic Sheaves, Grundl. der Math. Wissen. vol. 265, Springer-Verlag, Berlin, 1984.

[14] R.C. Gunning and H. Rossi, Analytic Functions of Several Complex Variables, Prentice-Hall (AMS Chelsea Publishing), 1965.

[15] F. Hartogs, Einige Folgerungen aus der Cauchyschen Integralformel bei Funktionen mehrerer Veränderlichen, Münchener Berichte **36** (1906), 223–242.

[16] S. Hitotsumatsu, On Oka's Heftungs Theorem (Japanese), Sugaku **1** (4) (1949), 304–307, Math. Soc. Jpn.

[17] L. Hörmander, L^2 estimates and existence theorems for the $\bar{\partial}$ operator, Acta Math. **113** (1965), 89–152.

[18] L. Hörmander, Introduction to Complex Analysis in Several Variables, First Edition 1966 (Third Edition 1990), North-Holland.

[19] E.E. Levi, Studii sui punti singolari essenziali delle funzioni analitiche di due o più variabili complesse, Ann. Math. Pure e Appl. Ser. III **17** (1910), 61–87.

[20] E.E. Levi, Sulle ipersuperficie delle spazi a 4 dimensioni che possone essere frontiera del campo de esistenza di una funzione analitica di due variabili complesse, Ann. Mat. Pura Appl. **18** (1911), 69–79.

[21] I. Lieb, Das Levische Problem, Bonn. Math. Schr. No. 387 (2007), 1–34; Translation into French, Le problème de Levi, Gaz. Math. Soc. Math. Fr. **115** (2008), 9–34.

[22] T. Nishino, Function Theory in Several Complex Variables (in Japanese), The University of Tokyo Press, Tokyo, 1996; Translation into English by N. Levenberg and H. Yamaguchi, Amer. Math. Soc. Providence, R.I., 2001.

[23] J. Noguchi, Analytic Function Theory of Several Variables – Elements of Oka's Coherence, Springer, Singapore, 2016; translated from Analytic Function Theory of Several Variables (in Japanese), Asakura-Shoten, Tokyo, 2013, 2nd Edition (Revised), 2019.

[24] J. Noguchi, Inverse of Abelian integrals and ramified Riemann domains, Math. Ann. **367**, No. 1 (2017), 229–249.

[25] J. Noguchi, A weak coherence theorem and remarks to the Oka theory, preprint 2018, arXiv:1704.07726v2, URL http://www.ms.u-tokyo.ac.jp/~noguchi/ (B3), to appear in Kodai Math. J.

[26] F. Norguet, Sur les domains d'holomorphie des fonctions uniformes de plusieurs variables complexes (Passage du local au global), Bull. Soc. Math. France **82** (1954), 137–159.

[27] K. Oka, Sur les fonctions analytiques de plusieurs variables – I – Domaines convexes par rapport aux fonctions rationnelles, J. Sci. Hiroshima Univ. Ser. A **6** (1936), 245–255.

[28] K. Oka, Sur les fonctions analytiques de plusieurs variables – II – Domaines d'holomorphie, J. Sci. Hiroshima Univ. Ser. A **7** (1937), 115–130.

[29] K. Oka, Sur les fonctions analytiques de plusieurs variables – III – Deuxième problème de Cousin, J. Sci. Hiroshima Univ. **9** (1939), 7–19.

[30] K. Oka, Sur les domaines pseudoconvexes, Proc. Imperial Acad. Tokyo **17** (1941), 7–10.

[31] K. Oka, Sur les fonctions analytiques de plusieurs variables – VI Domaines pseudoconvexes, Tôhoku Math. J. **49** (1942), 15–52.

[32] K. Oka, On analytic functions of several variables: XI – Pseudoconvex domains and finite domains of holomorphy, Some theorems on finite domains of holomorphy, 1943: URL http://www.ms.u-tokyo.ac.jp/~noguchi/oka/.

[33] K. Oka, Note sur les fonctions analytiques de plusieurs variables, Kōdai Math. Sem. Rep. (1949), no. 5–6, 15–18.

[34] K. Oka, Sur les fonctions analytiques de plusieurs variables – VII Sur quelques notions arithmétiques, Bull. Soc. Math. France **78** (1950), 1–27.

[35] K. Oka, Sur les fonctions analytiques de plusieurs variables – VIII Lemme fondamental, J. Math. Soc. Jpn. **3** (1951), No. 1, 204–214, No. 2, 259–278.

[36] K. Oka, Sur les fonctions analytiques de plusieurs variables – IX Domaines finis sans point critique intérieur, Jpn. J. Math. **23** (1953), 97–155.

[37] K. Oka, Appendice – Sur les formes objectives et les contenus subjectifs dans les sciences mathématiques; Propos postérieur – Pourquoi le présent mémoire est publié de nouveau, 1953: URL http://www.ms.u-tokyo.ac.jp/~noguchi/oka/.

[38] K. Oka, Sur les fonctions analytiques de plusieurs variables, Iwanami Shoten, Tokyo, 1961.

[39] K. Oka, Collected Works, Translated by R. Narasimhan, Ed. R. Remmert, Springer-Verlag, Berlin-Heidelberg-New York-Tokyo, 1984.

[40] K. Oka, Posthumous Papers of Kiyoshi Oka, Eds. T. Nishino and A. Takeuchi, Kyoto, 1980–1983: Oka Kiyoshi Collection, Library of Nara Women's University, URL http://www.lib.nara-wu.ac.jp/oka/index_eng.html.

[41] W. Rückert, Zum Eliminationsproblem der Potenzreihenideale, Math. Ann. **107** (1933), 259–281.

[42] A. Takeuchi, Domaines pseudoconvexes infinis et la métrique riemannienne dans un espace projectif, J. Math. Soc. Jpn. **16** (1964), 159–181.