Kang Zuo (左康)

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Stability, Geometry and Arithmetic

Professor Shing-Tung Yau has made many fundamental contributions to modern mathematics. As an algebraic geometer, I shall concentrate on one of them in this note, namely his extremely important theorem on Kähler-Einstein metrics.

Around 1976 Yau solved the Calabi conjecture on the existence of Kähler-Einstein metric on projective manifolds whose canonical line bundles are ample or trivial. This theorem has seen immediately several consequences of great depth in algebraic geometry. For instance the Miyaoka-Yau inequality affirms that for a complex projective surface X of general type holds the inequality $c_1^2 \leq 3c_2$ with c_1 resp. c_2 the first resp. second Chern numbers of X. In fact, Yau's original theorem is much stronger: the equality holds if and only if X is uniformized by the two-dimensional complex ball. Moreover, the general form of Yau's theorem is the so-called uniformization theorem of Yau, which says that a complex projective manifold of irreducible universal cover is uniformized by a bounded symmetric domain of rank ≥ 2, as long as its canonical line bundle is ample and some symmetric tensor of its tangent bundle contains a subbundle of maximal slope.

During my stay in the Chinese University of Hong Kong from 2000 to 2004, I learned this theorem from Professor Yau. My collaboration with Eckart Viehweg on the characterization of Shimura subvarieties in the moduli space of abelian varieties was highly inspired by this theorem. Very recently, a joint of work of myself with Chen and Lu made some progress on Coleman-Oort conjecture: for $g \geq 12$ the exists no positive-dimensional Shimura subvariety in $\mathcal{A}_g$ contained in the Torelli locus generically. Our approach is crucially based Yau's uniformization theorem. Together with the André-Oort conjecture solved by Tsimerman, we have also proved the Coleman conjecture for the moduli space of super-elliptic curves, namely there exists only finitely many curves of CM Jacobians which are meanwhile defined by equations of the form $y^n = f(x)$ and of fixed genus $g \geq 8$. It is indeed surprising that Yau's theorem from the transcendental world has such strong consequences in arithmetic. We are highly convinced that the study of Yau's type Chern number inequality on higher dimensional varieties combined with Hodge theory is the right way solving the Coleman-Oort conjecture completely.

In the p-adic world, my joint work with Lan and Sheng we introduced the notion of Higgs-de Rham flows on semi-stable Higgs bundles on smooth varieties over finite fields which are liftable to modulo p^2. Which plays the analogue role as Yang-Mills-Higgs metric for complex manifolds in the construction of p-adic representations of étale fundamental groups. Andreas Langer has been successful in applying the theory of Higgs-de Rham flow and establishing the Miyaoka-Yau inequality for projective surfaces over finite fields which are liftable modulo p^2. Nevertheless, I should say, so far the present p-adic method can not recover Yau's uniformization theorem yet. It is really a big challenge for us in developing Yau's uniformization theorem for p-adic manifolds to the full generality. There is a still ongoing research project (with J. Lu, X. Lu, R. R. Sun and J. B. Yang) on the arithmetic uniformization theorem, and we strongly believe that it will be highly inspired by Yau's original theorem of uniformization.