
Niky Kamran

Dr. Niky Kamran received his PhD from the University of Waterloo in 1984. In 1986, he became an assistant professor at Waterloo and then, after spending a year as a member of the Institute for Advanced Study in Princeton, he moved to McGill University in 1989. He has been a full professor since 1995. His research interests include geometric analysis, differential geometry and mathematical physics.

Kamran won the Aisenstadt Prize of the Centre de Recherches Mathématiques in 1992 and was elected a Fellow of the Royal Society of Canada in 2002. He was a Killam Fellow from 2006 to 2008. In 2012, Kamran became one of the inaugural Fellows of the American Mathematical Society. In 2014, he was the winner of the CRM-Fields-PIMS prize.

Impressions of Shing-Tung Yau and His Mathematical World

It is a great pleasure and honour to have this opportunity to make a contribution to the volume of testimonials being presented to Professor Shing-Tung Yau on the occasion of his seventieth birthday.

Shing-Tung Yau is universally recognized as one of the mathematical giants of our time. His epoch-making achievements have had and continue to have an immense and transformative impact across the astonishingly broad range of areas of mathematics to which he has contributed. His complete dedication to research, the exceptional power and brilliance of his mind, his tenacity in pursuing the most fundamental questions, the unparalleled generosity with which he openly shares his immense knowledge and deep insights for the benefit of the current and future generations of researchers by carefully compiling large lists of open problems augmented by detailed comments, his unique capacity to communicate effectively with physicists, engineers and mathematical scientists from all disciplines, and his keen interest in

bringing together students, post-docs and seasoned researchers for the benefit of scientific progress are all second to none. Yau's mathematical world is thus incredibly rich, broad and multifaceted, and knows no boundaries other than the pursuit of knowledge and truth at the highest level.

Starting in 1998 and for a period of about ten years, I had the privilege of being a research collaborator of Shing-Tung Yau in an extended joint project with Felix Finster and Joel Smoller, the goal of which was to understand the long term behaviour of Dirac spinors and scalar fields in the space-time geometry of the Kerr rotating black hole. My involvement came after Finster, Smoller and Yau had already produced a series of extremely important works on the interaction of gravity with non-Abelian gauge fields and spinor fields, in which they had analyzed in depth the delicate problem of the existence of either regular or black hole solutions of the Einstein-Dirac-Yang Mills equations in a spherically symmetric space-time geometry. In May of 1998, they posted a paper on the arXiv in which they proved a remarkable result to the effect that the Dirac equation had no normalizable time-periodic solutions in the non-extreme Reissner-Nordström geometry. Their proof made use of a very ingenious and beautiful argument based on the radial flux induced across the event horizon by the conserved Dirac current. I read their paper and was immediately taken by its originality and the elegance of its methods. I was also fascinated by its main result, because it stood in apparent contrast with what I knew about the existence of stable circular orbits in the Schwarzschild geometry. Reflecting on their paper made me wonder if their theorem could be generalized to the axisymmetric setting of the Kerr-Newman geometry, by constructing a radial flux through the use of Chandrasekhar's ingenious separation of variables result for the Dirac equation in this geometry. It is thus with some trepidation that I wrote to Yau to ask if he and his collaborators had considered this possibility. An encouraging response

soon followed which is how my involvement in the collaboration began. We had a close, intense and very pleasant four-way collaboration that lasted for about ten years, during which we analyzed in depth the long term dynamics of Dirac and massless scalar fields in Kerr. Our work was concluded with the proof of decay for scalar waves in Kerr and in the first rigorous treatment of the Penrose process of energy extraction, by means of super-radiance for initial data corresponding to scalar wave packets. An expository paper summarizing our results and putting them in context was published in the Bulletin of the AMS in 2009. After this, Felix and Joel focused their efforts on the Teukolsky equation and proved a very important decay theorem for higher spin fields in Kerr geometry, thus establishing the long-awaited linear stability theorem for the Kerr metric under gravitational perturbations.

Yau very kindly invited me for working visits throughout our collaboration and this is how I got exposed to at least a small part of his mathematical world. Our working sessions never failed to leave an indelible impression on me. I thus vividly remember the first of these visits, during which the insightfulness of a remark that he made, while we were working in his Harvard office, on the sensitivity of our decay results for Dirac waves to the algebraic type of the Weyl tensor, took me by complete surprise, and forced me to rethink my own understanding of results which I had thought about for a long time. This visit, as well as other visits to Yau during which Felix was also visiting from Germany, were thus immensely stimulating, and provided constant learning experiences for me. One of the times when Felix was present also remains engraved in my memory. Yau was visiting Columbia as the Eilenberg Chair holder at the time, and Felix and I went to New York at his invitation to work with him. It is during that visit that Yau suggested that we should use our integral spectral representation of the solution of the Cauchy problem for the scalar wave equation to carry out a rigorous treatment of the Penrose process. I remember feeling electrified by this idea, and wanting to understand

without delay how to set up the appropriate Cauchy data for wave packets. The actual work turned out to be quite involved and took a lot of time, patience and careful effort from all four of us in order to come to fruition, but the initial impetus given by Yau's vision of how this should be done was of paramount importance to our success. I have felt similarly energized listening to many of Yau's lectures, whether it was on his extremely original work with Melissa Liu and Mu-Tao Wang on quasi-local mass in General Relativity, which he gave as the Aisenstadt Chair holder at the Centre de Recherches Mathématiques here in Montreal during a program that Felix and I had organized, or on his breakthrough results with Ji-Xiang Fu on the Strominger system, which he lectured on at Joel Smoller's 70th birthday conference at Stanford. I couldn't wait to go back to my office every time after hearing him, and get back to working on my research, feeling inspired and energized by the experience of Yau's lectures. Even though we are not collaborating on a joint project at present, Yau has very generously continued to give his encouragement, guidance and suggestions, this most recently when I visited him at CMSA during the thematic program on PDEs, and lectured on non-uniqueness results for the Calderón problem.

I have only written in the above about the aspects of Yau's mathematical world to which I have been directly exposed, and I realize that this covers only a very small portion of the astoundingly broad range of subjects to which he has made fundamental contributions. I hope though to have been able to convey a sense of my feelings of deep admiration and profound gratitude towards him. I feel that Yau carries the same spirit as the great scientists of the past, who did not put artificial barriers between pure mathematics, applied mathematics and physics, and who were solely motivated by their search for the fundamental truths in science. Their success was the result of the openness of their minds, their tremendous hard work and their genius, as is the case for Yau.

I wish him a most wonderful birthday, and many many happy returns!