David Jerison

Dr. David Jerison received his PhD from Princeton University. After postdoctoral research at the University of Chicago, he came to MIT in 1981. He is currently a professor of Mathematics in MIT. He is an expert in partial differential equations and Fourier analysis.

In 1999, Jerison was elected as a Fellow of the American Academy of Arts and Sciences. He became a MacVicar Fellow in 2004. In 2012, he became a Fellow of the American Mathematical Society. In 2012, he received the Stefan Bergman Prize from the American Mathematical Society.

Action at a Distance

We live in a golden age of mathematical discovery, and S.-T. Yau is one of the principal actors in that golden age. It's a great privilege to be able to offer my appreciation of him on the occasion of his 70th birthday. I will discuss a few papers Yau wrote that made a deep impression on me and then talk about his role in supporting the community of mathematicians. He has a breadth of interests that is breathtaking and a supreme dedication to our community. My perspective will only reflect fragments of the whole, as others will undoubtedly make clear.

Many of the difficulties and challenges presented by elliptic partial differential equations are associated with their nonlocal character, their *action at a distance*. The most important tools in the theory of elliptic equations are Harnack inequalities. These inequalities are quantitative forms of the maximum principle that control the nonlocal behavior and govern the interactions at a distance. They can also be viewed as quantitative forms of connectivity.

In 1975 in [Y75], Yau proved a form of the maximum principle for the Laplacian on non-compact Riemannian manifolds that revealed the role of Ricci curvature. In the paper, he points out that his estimates[1] are infinitesimal versions of the Harnack inequality. The techniques in this paper, the systematic exploitation of differential inequalities, are of great importance and constitute a central theme of Yau's subsequent work. This is evident in spectacular form in his resolution of the Calabi Conjectures in complex geometry and also in later developments in Riemannian geometry.

In [CY75], Yau and S.-Y. Cheng extended Yau's estimates and gave several applications to global geometry. In 1986, Yau and P. Li [LY86] proved a sharp infinitesimal parabolic Harnack inequality, and there is a straight line from that paper to the Harnack inequality for Ricci flow due to Richard Hamilton. The work of Grisha Perelman resolving the Poincaré Conjecture in dimension 3 and Thurston's Geometrization Conjecture makes crucial use not only of the results of Hamilton, but also of ideas of the Li-Yau paper. There is also Yau's direct suggestion to Hamilton that his Ricci flow had the potential to prove geometrization. When describing these exciting developments, I like to make an analogy with the story of Exodus. Perelman, who reached the Promised Land, represents Joshua; Hamilton played the role of Moses, and Yau the role of the Burning Bush.

The work of Yau that helped me the most in my own research is his 1976 paper with S.-Y. Cheng [CY76] resolving the Minkowski problem in the C^∞ category. The Minkowski problem is the problem of finding a convex polyhedron given the normals and areas of its faces. The continuous version is the problem of finding a convex body given its Gauss curvature on the Gauss sphere. This problem has a long history involving Minkowski, Brunn, Hilbert, Weyl, Alexandrov,

[1] It is amusing to note that on the very first page of [Y75], there is a misprint (d in place of f) in the statement of the main result. Ah, the "old days" when the important literature of many parts of analysis and geometry was plagued with misprints and typesetting errors that kept researchers forever on their guard.

Pogorelov, and Nirenberg. From the definitive paper by Cheng and Yau, I learned not only local and global estimates for the fully nonlinear real Monge-Ampère equation, but also how to compute with orthonormal frames in the Cartan formalism.

The proof in 1979 by Yau and R. Schoen [SY79] of the positive mass theorem was inspiring. It showed that even major conjectures with ramifications in physics are accessible in our era. Around this time, perhaps encouraged by his own great success resolving famous unsolved problems, Yau embarked on a project of assembling usable lists of good unsolved problems. These problems have motivated and enlightened mathematicians for several decades, starting in the early 1980s.

In 1992, Yau created a publishing house, International Press of Boston, that publishes several leading journals and book series in mathematics. I am proud to have assisted Yau in his efforts to help communicate mathematical ideas through his publications. I served with him for many years on the editorial board of the Current Developments in Mathematics (CDM) Lecture Series. I also serve as an editor on the Cambridge Journal of Mathematics. These publications exist solely because of Yau's enormous energy and effort.

In 1997, Yau suggested that we assemble some unsolved problems for a CDM volume. I handled the conjectures in analysis. One of the problems, global well-posedness for the energy critical nonlinear defocussing Schrödinger equation, posed by Jean Bourgain, was solved in 3-dimensional Euclidean space in an important 2008 paper by J. Colliander, M. Keel, G. Staffilani, H. Takaoka and T. Tao. Tao gave a lecture on this and other work at the 2006 CDM meeting. Another problem, submitted by Nikolai Nadirashvili, reinterprets a question about bounds on the length of nodal sets on 2-manifolds (already proposed by Yau) in terms of a simpler question about lower bounds on the area of zero sets of harmonic functions in dimension 3. This problem was resolved in 2016 in celebrated work by A. Logunov and E. Malinnikova. They spoke at the 2018 CDM meeting. These successes show the wisdom of Yau in creating these resources. They also demonstrate one of the ways in which his own "action at a distance" works.

Yau has been kind and generous to me at every stage of my career. We saw each other regularly when I attended his lecture class on Monge-Ampère equations during my visit to Harvard in 2004–05. He attended the lectures I gave at that time on the work of Perelman. Some time in 2005, Yau gave a lecture at Harvard on the legacy of S.-S. Chern, who died at the end of 2004. I highly recommend the masterful text [Y06], which puts Chern's work in historical context and explains its essence via Chern's proof of the Gauss-Bonnet formula. In person, the lecture was even more revealing, not only because of the photographs and further details, but also because of the great care that Yau brought to it. I wonder whether anyone is able to report on Yau's legacy with as much authority, clarity, and insight. Perhaps this document can contribute a small part, but a large variety of perspectives will be required to encompass his contributions.

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