Felix Finster

Dr. Felix Finster received his PhD from ETH Zürich in 1995. Between 1996 to 1998, he was a postdoc at Harvard University. After that, he spent several years at Leipzig University. He has been a full professor of mathematics at the University of Regensburg since 2002. He works in the field of mathematical side of general relativity and quantum theory including the study of the dynamics waves in a blackhole joint with Niky Kamran, Joel Smoller and Shing-Tung Yau.

Linear Stability of Rotating Black Holes

It is a great pleasure and honor to contribute to the booklet dedicated to Professor Yau's 70th birthday. First of all, happy birthday and all the best! I wish him good health and many more happy and productive years. Since our joint results have been published and survey articles are available, there seems no point in entering mathematics. Instead, I would like to make a few personal remarks. When I came to Harvard as a post-doc in 1996, I immediately felt warmly welcome by Professor Yau and by his work group. I was pleasantly surprised that, despite the fact that he was extremely busy, he took the time to discuss with me and to give me valuable advice. Sometimes, our discussions were very brief, but even a short hint gave me a new direction for thinking about the mathematics problem which I was working on. I also would like to point out that I constantly felt his encouragement and support, even in situations which at the time seemed very difficult to me. Finally, I am very grateful that he brought me together with the right people. When Joel Smoller gave a seminar talk at Harvard in April 1997 (a talk which I even missed!), he asked me to talk to him, and he even came up with a concrete problem for us three to work on together. This led to our joint collaboration (partly also with Niky Kamran, who teamed up in September 2018) which went on for more than ten years. After that, Joel Smoller and I continued the taken path. This led to the proof of linear stability of rotating black holes under perturbations of general spin (see [1] and the survey article [2]). I would like to conclude by stating our final result, just to give an example for research which, although Professor Yau is not a co-author, nevertheless was incited by him, is based on his work and was inspired by his ideas.

Rotating black holes are modelled by the Kerr geometry. Linear perturbations of general spin are described by the Teukolsky master equation, a wave equation for a complex scalar field ϕ, in which the spin enters as a parameter $s \in \{0, \frac{1}{2}, 1, \frac{3}{2}, \ldots\}$. We consider the Cauchy problem with initial data at time t_0

$$\phi|_{t=0} = \phi_0 \quad \text{and} \quad \partial_t \phi|_{t=0} = \phi_1 \,,$$

which for simplicity is assumed to be smooth and compactly supported outside the event horizon r_1,

$$\phi_0, \phi_1 \in C_0^\infty\big((r_1, \infty) \times S^2\big) \,. \tag{1}$$

Since the Kerr geometry is axisymmetric, the Teukolsky equation decouples into separate equations for each azimuthal mode. Therefore, the solution of the Cauchy problem is obtained by solving the Cauchy problem for each azimuthal mode and taking the sum of the resulting solutions. With this in mind, we restrict attention to a single azimuthal mode, i.e.

$$\begin{aligned} \phi_0(r, \vartheta, \varphi) &= e^{-ik\varphi}\, \phi_0^{(k)}(r, \vartheta) \,, \\ \phi_1(r, \vartheta, \varphi) &= e^{-ik\varphi}\, \phi_1^{(k)}(r, \vartheta) \end{aligned} \tag{2}$$

for given $k \in \mathbb{Z}/2$ (if s is half integer, then so is k).

Theorem 1. *Consider a non-extreme Kerr black hole of mass M and angular momentum aM with $M^2 > a^2 > 0$. Then for any $s \geq \frac{1}{2}$ and any $k \in \mathbb{Z}/2$, the solution of the Teukolsky equation with initial data of the form (1) and (2) decays to zero in $L^\infty_{\text{loc}}((r_1, \infty) \times S^2)$.*

This theorem establishes in the dynamical setting that the non-extreme Kerr black hole is linearly stable.

References

[1] F. Finster and J. Smoller, *Linear stability of the non-extreme Kerr black hole*, Adv. Theor. Math. Phys. **21** (2017), no. 8, 1991–2085.

[2] F. Finster and J. Smoller, *Linear stability of rotating black holes: Outline of the proof*, Nonlinear Analysis in Geometry and Applied Mathematics (L. Bieri, P. T. Chruściel, and S.-T. Yau, eds.), Harvard University Center of Mathematical Sciences and Applications (CMSA) Series in Mathematics, no. 1, International Press, Somerville, MA, 2017, pp. 77–90.