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Dr. Michael Douglas received his PhD in Physics from Caltech under the supervision of John Schwarz. After spending one year as a postdoc at University of Chicago, he moved to Rutgers University. He became an associate professor at Rutgers University in 1995, and left for a year in 1997–1998 to take up a permanent position at the Institut des Hautes Études Scientifiques. He then returned to Rutgers University and in 2000 became the director of the NHETC. In 2008, Douglas moved from Rutgers to become the first permanent member of the Simons Center for Geometry and Physics at Stony Brook University. His research interests include matrix model, Dirichlet branes, noncommutative geometry in string theory and the statistical approach to string phenomenology.

Douglas received the 2000 Sackler Prize in theoretical physics, held a Louis Michel Visiting Professorship at the Institut des Hautes Études Scientifiques, and was a Gordon Moore Visiting Scholar at Caltech, and a Clay Mathematics Institute Mathematical Emissary. In 2012 he became a Fellow of the American Mathematical Society.

On a Few of Shing-Tung Yau's Contributions to String Theory

Let me begin by thanking the editors of this volume for inviting me to contribute. At first they asked me to write an article on the SYZ conjecture [1]. However, despite being a lead editor of a book almost half of which is devoted to the SYZ conjecture [2], I felt that my expertise in this area was not really up to it (in fact, that part of the book was edited by Mark Gross). Another natural topic, the importance of Yau's proof of the Calabi conjecture for string theory, I had already written about in [3]. Still, I appreciated the offer and I felt that it really could not be hard to find plenty more to write about Yau's contributions to string theory, so I accepted the invitation.

I began by doing what particle physicists often do when beginning to write a paper, namely, to do a search on Inspire [4], in this case for `find author S.-T. Yau`. Nowadays there is also an author profile page which makes this easier. Somewhat to my surprise, I found 207 papers in their database, more than my own 128. This could make for quite a long article!

To narrow down the scope, I restricted myself to the "famous" papers with at least 250 citations, of which there turned out to be 5 (remember, this only covers physics papers). Number 1 on this list was indeed SYZ, while numbers 2 and 4 are the proof of the positive mass theorem with Richard Schoen [5, 7].

The positive mass and energy theorems are arguably some of the deepest results in mathematical physics, as it is a basic consistency condition for general relativity that localized solutions must have positive mass, since otherwise Minkowski space-time would be unstable to decay to negative mass objects. Furthermore the claim is not at all obvious, as gravitational attraction contributes negatively to the total mass of a bound system. Schoen and Yau developed a geometric proof, originally in four space-time dimensions, and later generalized to up to eight space-time dimensions [6]. Soon after, Witten contributed a noteworthy proof using spinors [8].

The same physical consistency condition applies in string theory, and it would be nice to have a proof for ten and eleven space-time dimensions. Apparently there are technical barriers to extending the geometric proof (though see [9]). Witten's proof works in any number of dimensions, but assumes that space is a spin manifold. Now since superstring theory has fermions, one might think that this would be required anyways, but this is not obvious: the spinors in supergravity couple to other bundles besides the spinor bundle, there are quantum global anomalies, and the IIb superstring has a self-dual five-form field which also affects the topological constraints. I do not think this is fully understood in string/M theory, see [10], §4.6 for some of the many issues.

More seriously, the proof assumes that space-time is a manifold, but space-time in string/M theory can have singularities. Even worse, the dominant energy condition the proof assumes as a hypothesis, can fail both in quantum gravity and in string theory. Thinking about the problem in quantum gravity seems to be pointing towards a radical new connection between the geometry of space-time and quantum concepts such as entanglement entropy, see for example [11, 12]. Leaving quantum gravity for the future, one can think about the semiclassical limit of string theory and M theory as described by ten and eleven-dimensional supergravity theories with additional terms in the action beyond the minimal two derivative terms, and allowing certain singular solutions. These higher derivative terms include the Chern-Simons terms which play an important role in the original Candelas, Horowitz, Strominger and Witten construction of Calabi-Yau compactifications of the heterotic string [13]. These terms can lead to violations of the dominant energy condition, so it would be interesting to know if they can lead to violations of the positive energy theorem.

Another example of higher derivative terms in string theory is the Dirac-Born-Infeld action on a Dirichlet brane world-volume. This can be related to modified stability conditions in string theory, as discussed in [14, 15, 2]. This is a good place to mention my own work with Reinbacher and Yau [15], where we made the so-called "DRY conjecture" about stable bundles on Calabi-Yau threefolds, giving sufficient conditions on the Chern classes for their existence, including an upper bound on the third Chern class. These bounds were motivated by the attractor mechanism in string theory, according to which stable bundles can be used to produce D-branes which are equivalent to extremal black holes in string theory. Following a conjecture of Moore [16] about these extremal black holes leads to our conjecture. Some recent works on the conjecture are [17, 18].

Paper number 3 is the famous "Stringy Cosmic Strings" paper with Greene, Shapere and Vafa. [19] In physics, a cosmic string is a codimension two solution of general relativity coupled to a scalar field. It is characterized by its behavior at infinity: the scalar has a winding number, and the metric is asymptotically flat but with a deficit angle. It had been well studied in cosmology but what had not been much appreciated was the wealth of possibilities if the scalar field takes values in a nontrivial target manifold, in physics terms defining a nonlinear sigma model. This comes up naturally in string compactification, as the choice of metric on the compact manifold is parameterized by scalar fields in the lower dimensional theory, usually called moduli fields. For example, if we consider compactification on T^2, the flat metric is parameterized by a real field (the volume) and a complex structure modulus which lives on the upper half plane modulo $SL(2,\mathbb{Z})$. In this case, the winding number of the simple solution generalizes to a monodromy which can be a general element of $SL(2,\mathbb{Z})$.[1]

Now, if the scalar field theory comes from a reduction of Einstein theory in the higher dimension, then it is natural to expect that solutions of the lower dimensional theory, including the stringy cosmic strings, will be Ricci flat manifolds from the higher dimensional point of view. This is true and in the T^2 example, if we use the complex structure modulus of the T^2 as our scalar, in the cosmic string solutions it is a holomorphic function of the two spatial dimensions. In mathematical terms, the solutions are elliptic fibrations. The authors then show that all of these stringy cosmic string solutions correspond to two complex dimensional Ricci-flat Kähler manifolds. Thus, there is a single natural family of solutions in which space is compact: the K3 manifold regarded as an elliptic fibration over $\mathbb{P}^1$. But there is a wide class of solutions in which space is noncompact, and the authors devote much discussion to argue that these noncompact Calabi-Yau manifolds are just as interesting for string theory.

This was very prescient, as the study of noncompact Calabi-Yau manifolds turned out to be more important than the compact case for much of the development of superstring duality in the mid-90's. This was because the natural string theory duals to $N=1$ and $N=2$ supersymmetric quantum field theories are strings on noncompact Calabi-Yau manifolds. Although there are other arguments for this, one of the simplest is to consider theories of Dirichlet branes which fill the Minkowski space-time dimensions and which sit at points in the extra dimensions. In these theories, the energy scale of the open string degrees of freedom is proportional to the distance between the branes. Thus, to take the low energy limit, one focuses on a small region in the extra dimensions, which is noncompact. To get nontrivial theories with less than $N=4$ supersymmetry, the small region must have reduced holonomy and be topologically nontrivial. A prototypical example is to consider the resolution of an orbifold singularity $\mathbb{C}^2/\Gamma$ or $\mathbb{C}^3/\Gamma$, as first studied in [20, 21]. By now there are literally thousands of papers which study string theory on noncompact Calabi-Yau manifolds.

[1] String compactifications have another parameter, the integral of the two-form B over the T^2. This combines with the volume to produce another complex variable, and the string symmetry T-duality acts on this variable by another $SL(2,\mathbb{Z})$. In fact the full duality group is $SO(2,2;\mathbb{Z})$.

Equally prescient was the description of elliptically fibered Calabi-Yau manifolds in terms of a modulus field varying holomorphically on the base. This modulus need not have a geometric origin, and another example of a modulus admitting a duality action of $SL(2,\mathbb{Z})$ is the dilaton-axion field of type IIb superstring theory. The cosmic string in this case is a seven-brane, a Dirichlet brane in the case in which the monodromy is $\tau \to \tau + 1$, but more generally a duality image or (p,q)-seven-brane. String compactifications which incorporate these seven-branes are now known as F theory compactifications [22], and this is the most general class of string compactifications currently known.

Finally, paper number 5 is a joint work with Hosono, Klemm and Theisen on mirror symmetry for hypersurfaces in toric varieties [23]. From the point of view of physics and string theory, this is a technical paper, explaining how to derive Picard-Fuchs equations and the mirror map for Calabi-Yau hypersurfaces in toric varieties. Still, as technical results go, these are very important. One famous application was the work [24] of Kachru and Vafa on type II-heterotic duality, which first showed how to get exact results for $N = 2$ four-dimensional gauge theories from these results of mirror symmetry. This was also the starting point for the later work on flux compactifications and moduli stabilization, so it plays a central role in current discussions of quasi-realistic string compactifications [25].

To conclude, Shing-Tung Yau's contributions to string theory have been very broad and very deep, and here we saw this just from looking at the first five papers in this list of physics publications. Best wishes on this occasion of his seventieth birthday, and the string theory community looks forward to many more.

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