John Coates

Dr. John Coates received his PhD from University of Cambridge in 1969. During 1986 to 2012, he has been the Sadleirian Professor of Pure Mathematics at the University of Cambridge. He works on number theory. His research interests include Iwasawa theory.

Coates was elected a Fellow of the Royal Society of London in 1985, and was president of the London Mathematical Society from 1988 to 1990. In 1997, he was awarded the Senior Whitehead Prize.

Shing-Tung Yau and the Rebirth of Chinese Number Theory

1. Introduction

The origins of number theory lie deep in the ancient history of China, India and Persia, with most discoveries being primarily motivated by numerical calculations. This Asian dominance of the subject continued at least until the 13th century, with the "Yang Hui Suan Fa" being probably the most important earliest surviving printed mathematical text (no early Chinese printing survives, but a Korean printing from 1433 exists in the National Palace Museum in Taipei). However, it was only in the first 40 years of the 20th century that Chinese mathematicians became aware of the great advances in number theory made in Europe in the 19th and early 20th century, especially in Germany. The most notable figure in the development of number theory in China in this period was Luogeng Hua of Tsinghua University and the Chinese Academy of Sciences. However, the ravages of the Cultural Revolution had a very adverse effect on mathematics in general within China, and especially on research in number theory. Happily today research in number theory is flourishing within both China itself, and also in the broad community of Chinese mathematicians working in Asia and North America. In particular, it seems no exaggeration to say that the future development of arithmetic geometry is now largely in Chinese hands. In this brief article, I want to explain how my old friend Shing-Tung Yau has played a key role in this happy evolution of Chinese number theory, and indeed Chinese mathematics as a whole.

2. Early Years

I first met Yau when he came to Stanford University in 1974, a year after I had arrived at Stanford. I was rapidly struck by two qualities of Yau at this time. Firstly, he had very broad mathematical interests (for example, I was amazed that he sat patiently though a rather technical graduate course which I gave on Iwasawa theory). Secondly, he was remarkably hard working. Indeed, I would occasionally come into the Stanford Mathematics Department towards midnight, and on every occasion I remember finding Yau there either doing his own work or talking to the keen group of graduate students which he quickly gathered around him. At the end of the academic year 1974–1975, I decided to move to Cambridge University, thus ending the only time in my life when Yau and I were members of the same Department. However, I sensed very well what a deep and original mathematician Yau was from seeing him on an almost daily basis for about a year. Over the next fifteen years, I watched in admiration as I saw Yau rise to the highest international prominence for his deep work in geometry and some related areas of theoretical physics, but I will have to leave it to other colleagues, more qualified than I, to write about his great achievements in these fields.

My first close encounter with Chinese number-theorists came in 1988, when Chunlai Zhao came to Cambridge University to work with me on the arithmetic of elliptic curves. He had had a very difficult time during the cultural revolution, but had finally returned to teach at Peking University. Despite his lost years of research, I soon saw Zhao's talents, and

he quickly did some interesting research proving that the complex L-series of certain explicit infinite families of quadratic twists of the congruent number elliptic curve $y^2 = x^3 - x$ did not vanish at $s = 1$. Zhao also invited me to come for my first visit to China at Peking University in the summer of 1990, and introduced me to a number of other Chinese number-theorists who, like him, were very keen to develop research in number theory in China in the new era after 1990. It was clear after this visit that there was a great reservoir of young talent for number theory in China if only one could train people in the modern developments of key parts of number theory, such as arithmetic geometry. I could see no practical way myself of how to achieve this.

However, during a visit to the newly founded Isaac Newton Institute in Cambridge in 1994, Yau first suggested to me that he was not only keen to see Chinese number theory flourish in the future, but, even more importantly, he was willing and able to give concrete practical help of a unique kind in achieving this. He also told me about his ambitious plans to hold the triennial International Congresses of Chinese Mathematicians (ICCM), and how he had persuaded Ronnie Chan to fund large meetings which would bring together Chinese mathematicians from all over the world to discuss the most recent developments in the major fields of mathematics, and to give prizes to recognize outstanding work by Chinese mathematicians in these fields. Having served on the Executive Committee of the International Mathematical Union which organised every four years the International Congress of Chinese Mathematicians, I was aware of the immense practical difficulties and cost of such a venture, and I must confess I wondered at the time whether Yau would be able to deliver his plans in practice. It is, of course, now history that not only did these International Congresses of Chinese mathematicians take place every three years as planned by Yau (with the first Congress in 1998 held in Great Hall of the People in Beijing), but also that they have completely transformed the whole atmosphere of mathematical research amongst Chinese mathematicians. To return to Chinese number theory, as the first ICCM approached, Yau made it clear to me that he would like to see the work of some very promising young Chinese number theorist recognised by one of the top prizes awarded at the Congress. Happily, there was an excellent candidate in Shouwu Zhang, and, although still very young, he was duly given one of the first Morningside Gold Medals of the ICCM thanks to Yau's insistence that research in number theory should be given priority. This strong recognition of Chinese number theory by top awards of the ICCM has continued ever since 1998, with Jun Li, Ye Tian and Wei Zhang being awarded Gold Medals in 2001, 2013, and 2016, respectively, and Daqing Wan, Ye Tian, and Zhiwei Yun being awarded Silver Medals in 2001, 2007, and 2016, respectively.

3. Later Years

It is remarkable that the recognition of number theory by the ICCM was only one part of Yau's very practical efforts to build up research in this field in China. As soon as the Morningside Center for Mathematics in the Chinese Academy of Sciences was created, Yau made it clear that he wanted a large part of its activities devoted to number theory. I was delighted that he arranged the appointment of Ye Tian as one of the first permanent members of the Center. Without Yau's active support, it would not have been possible to appoint highly promising, but very young, Chinese number-theorists like Tian to long term posts in the Academy of Sciences. Yau has continued to support such appointments until the present day, with Xin Wan being the latest one in number theory at the Morningside Center. It is now one of the leading centers in the world for research in arithmetic geometry.

Yau is unquestionably the most successful fund-raiser in the whole history of mathematics. Those of us who have struggled to find some private money to support infrastructure and research in mathematics in our own universities know how difficult it is to persuade businessmen and wealthy individuals to support a subject whose great charm can only be fully appreciated by its practitioners. But Yau has had a genius for convincing those outside mathematics to support it. Apart from the ICCM, the Morningside Center, and Centers at Hong Kong University, and Tsinghua University, Yau has raised money for Summer Schools, Competitions for gifted High School, and many other activities in mathematics. One of the best Summer Schools I have personally attended was one in arithmetic geometry held at Zhejiang University in Hangzhou in the early 2000s, and the generous funding for this School was found by Yau.

I hope these brief remarks convey a little of how much the present flourishing state of number theory in China owes to the generous practical and spiritual support given by Yau to the subject and its practitioners over the last 20 years. He has been a truly remarkable friend and benefactor of number theory. I would like to end by wishing him many of the happiest returns of his 70th birthday, reminding him of the ending two lines of Li Po's poem about his friend Meng Hao-jan:

> *You tower above me, a hill too high to climb;*
> *What more can I do than greet your fragrance from afar?*