Szu-Hoa Min (1913–1973): A Pioneer of Analytic Number Theory in China

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an abridged translation by Xun Jiang*

In memory of those who shouldn't be forgotten,
and for the times that will be remembered forever.

On March 8, 1913, a baby was born in the south area of Peking (Beijing). This boy showed significant mathematical talent ever since childhood. Two years after graduation from the College, he was offered a position of Assistant from Tsinghua University. When Tsinghua University was incorporated into Western-Southern United University (西南联大), he became an assistant of Professor Shiing-Shen Chern (陈省身, 1911–2004) and Professor Loo-Keng Hua (华罗庚, 1910–1985). From 1945, he studied and worked in England and the United States where he became a shining star. He returned to China in 1948. About seven or eight years later, at his prime time, he stopped the research on pure mathematics which was so dear to him, and switched to applied mathematics. He fell ill because of overwork; however, he continued to work hard as before. In the early morning of October 10, 1973, he passed away at the age of 60. Just one day before his death, he still worked on a mathematical problem in the digital seismic exploration.

His name is Szu-Hoa Min (Sihe Min, 闵嗣鹤, 1913–1973). He made tremendous contributions to the development of analytical number theory in China.

1. Born into a Family of Scholars

Min's ancestor is from Fengxin, Jiangxi Province, a place with rich culture heritage. Many poets, writers and the elite intellectuals are produced in this area in the past one thousand years. So was Szu-Hoa Min's grandfather Hesheng Min (闵荷生, 1847–1936) who passed the imperial examination at age 24. This 24 years old young man started a brand-new career. He became a prefectural magistrate of a north China city, ended up his career as a Congressman during the late years of the Qing Dynasty. Unfortunately he lost most of his saving in an attempt to fight against the Japanese merge-acquisition of Nanxun Railway (南浔铁路, a railway in Jianxi Province). It made Hesheng Min almost penniless and he was forced to spend a secluded life at Peking city.

Szu-Hoa Min's father Chizheng Min (闵持正) was fragile and not in good health always. He was a clerk at Capital Police Department. His mother Jintang Zheng (郑锦棠) was born to an intellectual family. She well understood the importance of education to her children. Even though the family financial budget was tight and they depended on pawn and borrowing money from relatives and friends, they never de-emphasized children's education.

When Szu-Hoa Min was about four or five years old, his seventy-years-old grandfather began to teach this grandson. Szu-Hoa Min did live up to Grandpa's expectation. He gradually mastered the ancient Chinese classics and built up a solid foundation in literature. The boy had bright eyes and graceful eyebrows.

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He was smart, hardworking, humble, polite, and was benevolent in nature. He won the love so much of his grandfather that Grandpa would keep him at home for private tutoring. In this way Szu-Hoa Min did not attend the elementary school at all.

However, Szu-Hoa Min learned mathematics himself from the textbooks of his playing mates in the neighborhood. He even became the little teacher of them by solving difficult arithmetic problems for them. In 1925, as he became 12 years old, he passed the entrance examination of the Affiliated Secondary School of Peking Normal College (which was renamed Beijing Normal University later).

Here are some chronological milestones in Min's life

- Born 1913,
- 1925, the Junior Middle School,
- 1928, the Senior Middle School,
- 1929, the Pre-College Program of Peking Normal College (Science Section),
- 1931, Peking Normal College, Department of Mathematics (Undergraduate),
- 1933, teaching at some private middle schools while he was a junior college student,
- 1935, graduated from Peking Normal College, a teacher of the Affiliated Secondary School and some private middle school,
- 1937 summer, an offer of Assistant from Tsinghua University,
- 1937, July 7, the Sino-Japan War broke out; Min and his family (Mom and three younger sisters) moved to Kunmin, Yunan Province,
- 1938, Assistant in Western-Southern United University to which Tsinghua University belonged,
- 1944, promoted to a Lecturer,
- 1945, Oxford University (mentor: Titchmarsh),
- 1947, Ph.D.; Member, Institute of Advanced Study, Princeton,
- 1948, Associate Professor, Tsinghua University,
- 1949, promoted to a Professor,
- 1952, Professor of Peking University because the colleges of science and liberal art of Tsinghua University and Yanching University were incorporated into Peking University.

Although Szu-Hoa Min received no elementary school education, he adapted to the middle school life easily and happily. He liked drawing and volleyball. He was a model student. Because he had rich knowledge in ancient Chinese literature and he wore plain cloth shoes hand-made by his mother all the year-round, he was nicknamed as "Old(-fashioned) Master" (老夫子) by his classmates. He performed excellently in all courses, especially in Chinese Literature and English. During the junior middle school years, his interest turned to mathematics notwithstanding his grandfather wished that he would major in Chinese literature.

Even though he honored the wish of his grandfather, he was a persistent teenager in his heart. Finally he enrolled into the Pre-College Program at Peking Normal College (Science Section), which guaranteed that he would be admitted into some department in the College of Science two years later (if he performed very well in the two years of the pre-college program). After rather long time, his grandfather realized the choice of his beloved grandson. He exclaimed, "How dare you try to study mathematics? 'Chou Pei Suan Ching' (周髀算经, a classical Chinese mathematical book) is such a profound and recondite classic!"

2. Mentor and Close Friend

One day in 1929, the Pre-College student Szu-Hoa Min asked a combinatorial question to his teacher Zhongsun Fu (傅种孙, 1898–1962): "Given five couples sitting in a round table, it is required that they must sit in a way male-female-male-.... such that no couple should sit next to each other. How many ways they may sit?" Mr. Fu didn't answer him right away because it was out of the syllabus. However, it drew Mr. Fu's attention to this student.

Figure 1. Szu-Hoa Min in College.

Zhongsun Fu was born also in Jiangxi Province (1898). He was thin, short, and looked weak, but he had sparkling eyes and deep voice. As a teacher, he was an eloquent lecturer and appeared extraordinarily. His father was an intellectual teaching him the

Figure 2. Zhongsun Fu.

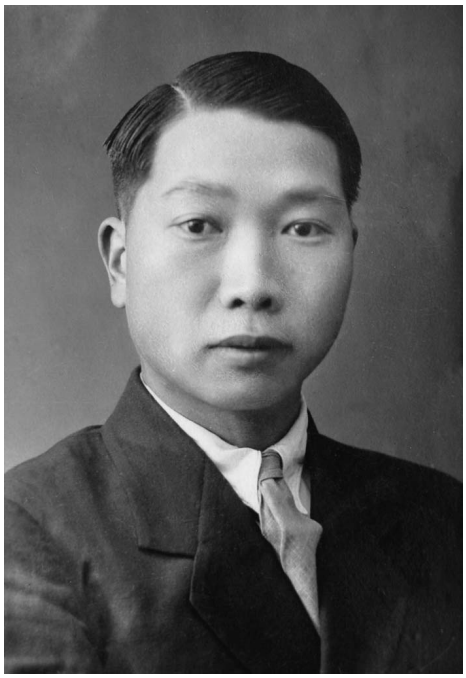

Figure 3. Cigeng Zhao (Min's best friend).

ancient Chinese literature. When his father died, he was merely 12 years old. His father had a wish that, at any circumstance, his son would never drop off from study. When Fu attended the middle school he liked the Euclidean plane geometry very much. When he graduated, his family could not afford the college tuition for him. It happened that he was recruited by Peking Normal College which was a tuition-free college. Fu was admitted into the mathematics-science section. He was an active student in the College. Being the president of the Mathematics-Science Society there, he published many articles in the official journal of the Society. When he graduated in 1920, he was offered the position of a teacher in the Affiliated Secondary School of Peking Normal College. One year later, he was appointed a lecturership in Peking Normal College. In 1923, Peking Normal College was renamed as Peking Normal University (Beijing Normal University). In the winter of 1928, Fu was offered a professorship in Peking Normal University. He was a teacher of Szu-Hoa Min for five years (from the Pre-College to the College). Fu believed that Min was an exceptional genius.

In 1930 just when Szu-Hoa Min was determined to dip himself in mathematical study, his grandmother and his father passed away. This is a heavy blow to the family. His mother managed single-handed to take care of the family which included Min's grandfather and three younger sisters. To save money, he had to commute for lunch everyday, rain or snow, by foot between his home and the College. From 1933, as a junior college student, he shared with his mother the duty of supporting the family by tutoring at some private middle schools.

Among his classmates in the College, Szu-hoa Min was the youngest one and the shortest one. His classmates treated him like a younger brother. His grades were excellent in all courses and he was a super-smart person. Polite and easy going, all the people liked him. He solved questions for his classmates, provided explanations to solve their puzzles. His notebooks were valued by his classmates. He preferred independent thinking; he didn't rely on the teachers too much. He respected his teachers and never criticized them. He was very active in the journal published by the Department of Mathematics. He contributed many papers, solicited other articles, edited the journal, helped in layout and proofreading, and even the cover design.

In 1935, Szu-Hoa Min graduated from Peking Normal University. Zhongsun Fu helped him find a job in the Affiliated Secondary School of Peking Normal University. Together with the dean of Peking Normal University and the chairman of the Mathematics Department, Fu went to see the principal of the Affiliated Secondary School. He argued that Szu-Hoa Min was the best student he had ever taught. He further told the principal that another teacher of the Secondary School was willing to curtail four hours of his own teaching time so as to create a half-position for Szu-Hoa Min. Fu's lobbying was approved. Besides, Min found some other part-time jobs to augment his income. He became the core person to support this family.

Figure 4. Graduates of Mathematics Department, Peking Normal University in 1935. Zhongsun Fu (second from left in front row), Cigeng Zhao (forth from left in back row), Szu-Hoa Min (forth from left in the third row).

Professor Wu-Chih Yang (Ko-chuen Yang, 杨武之, 1896–1973) of Tsinghua University was the first scholar in China, who investigated analytic number theory. He was a Ph.D. student of L. E. Dickson at the University of Chicago (1928); Yang was the father of the later Nobel Prize laureate in physics, Chen-Ning Yang. Wu-Chih Yang recognized and helped many outstanding members of Tsinghua University, e.g. Loo-Keng Hua and Shiing-Shen Chern. Yang taught at Peking Normal University as an adjunct professor. There he knew that Szu-Hoa Min was a bright student. Together with Zhongsun Fu, Yang recommended Min as an assistant of Tsinghua University. It succeeded in 1937. Szu-hoa Min happily got an offer from Tsinghua University. It was the starting point of the career of a professional mathematician for Min.

Unfortunately, just one month after getting the offer from Tsinghua University, the Sino-Japan War broke out at Marco Polo Bridge, a place not far away from Peking. The whole China, including Peking, was in total chaos. Another worse thing happened: Min's grandfather passed away in the meantime.

Many of Min's friends set off moving to the south part or the west-north part of China. Before leaving Peking, Cigeng Zhao (the best friend of Min) together with another classmate went to see Szu-Hoa Min. They left a checking account with a balance of 220 dollars as a gift. Zhao said, "Just use the money as much as you need."

With this money, Min's family were able to travel together with Fu's family to Jiangxi Province. Then they departed, because Fu joined the people of Peking Normal University going to Xian, a city in the west-north China where a new United University would be established. As to Min and his family, they toiled to Hunan Province; there they took the train to Guangdon Province, and then to Hong Kong, to Hanoi, and finally to Kunming (the capital city of Yunnan Province) where Western-Southern United University was situated and Tsinghua University was amalgamated into this United University.

3. Eight Years at Western-Southern United University

In 1938, Szu-Hoa Min started working at the United University. This was a special university consisting of Tsinghua University, Peking University and Nankai University during the Sino-Japan War. Most of the elite intellectuals in China, old and young, met each other in this University. It nurtured many first-rate scholars in various academic disciplines, a marvelous record in the education history of China.

Szu-Hoa Min supported his family, Mother and three sisters, by his meager salary as an assistant. Life was harsh, but his spirit was high. Many future great mathematicians worked in the same de-

partment. Loo-Keng Hua worked on number theory, Shiing-Shen Chern specialized in differential geometry, and Pao-Lu Hsu (许宝騄, 1910–1970) studied probability and statistics.

Szu-Hoa Min was the teaching assistant of Chern's course "Riemannian geometry". He was an active member of Hua's seminar on "Additive number theory". It was amusing to recall some episode of this seminar. In the first several meetings, so many students were attracted by this seminar that the classroom was fully packed. The audience size dwindled gradually and finally only four students remained. One week later, there were only two attendants besides Hua: Szu-Hoa Min and Kai-Lai Chung (鍾開萊, 1917–2009). Chung was then a student; later he became a distinguished mathematician working on probability theory and taught at Stanford University. Min and Chung lived near to Hua's home so that they might meet in Hua's home for the seminar, because only two students participated in the seminar.

In Kunming, Japanese fighters attacked almost every day. One day, the air raid alarm was proclaimed; people run into their air-raid dugouts as usual and expected the coming bombarding. However, for a long time no Japanese airplanes appeared. People thought that the enemy would not come this time. Thus Loo-Keng Hua decided to visit Min and discussed a mathematical problem with him. So he left the air-raid dugout. Suddenly the Japanese fighters appeared and threw a string of bombs. One bomb exploded near the entry of the air-raid dugout. Many people were thrown away and buried inside the ground. It took three hours to dig out all the victims; no one was killed. When Min and Hua were dug out, the robe of Szu-Hoa Min was torn into a short gown, and Hua's ears were bleeding. They experienced a live bury but survived luckily.

Szu-hoa Min showed extreme interests in analytic number theory and quickly strode into this research area. The war was going on and air attack was very often. Even though he was isolated from the outside world and no research resources were available, he still managed to publish seven papers. Except one paper in the field of combinations, all other six papers were in the field of number theory. Among them, there were four papers co-authored with Loo-Keng Hua between 1941 and 1944, and two papers were single-authored. He was promoted to a Lecturer in 1944.

The age difference of Loo-Keng Hua and Szu-Hoa Min was only three years. Their relationship was a kind of teacher-student, and they were also friends. They complemented to each other. During the years of cooperation, they had some conflicts inevitably. Hua was a genius, he was strong and dominating, regardless of feelings of other ones. Min was gentle, mild, reticent, but stubborn and unbending in his heart. Min worked conscientiously, providing the crucial proofs for the results in their joined papers via his adroit mathematical techniques. He couldn't tolerate Hua's fierce temper and quitted the joint programs several times. It was the senior professor Wu-Chih Yang who came to mediate, and then their cooperation continued.

Min's sisters grew up over the years. The eldest one studied chemistry, the second one studied physics, the youngest one studied biology, all in the United University. Min also volunteered to teach in high-schools in Kunmin. His lectures were very popular among the students, in particular, the lectures in Euclidean plane geometry and higher algebra. The style of his teaching was concise, accurate and logical. He would explain a new concept clearly and presented the deduction process step by step. His good friend Cigeng Zhao commented that Szu-Hoa Min enjoyed teaching and was a patient teacher. He didn't care about honorarium and praises; to him, the most important reward of teaching was the growth of young generation.

In early 1944, he passed the examination of the Program of the Boxer Indemnity Scholarship. In October 1945, Szu-Hoa Min went to England by an ocean liner. Although the War was over, travelling by sea was still rather risky. In the same ocean liner he met Hsien-chung Wang (王憲鍾, 1918–1978) and Szetsen Hu (胡世桢, 1914–1999). Wang was a student of S. S. Chern at the United University. Later he became an authority in Lie groups and differential geometry; he taught at Northwestern University and Cornell University. Hu graduated from the National Central University at Nanking in 1938. He was to become an expert of algebraic topology; he taught at UCLA. This group of students dreamed to revive their country by learning science and technology oversea. After a one-month journey, they arrived in England. Min went to Oxford University and studied analytic number theory under the guidance of E. C. Titchmarsh. Now he showed up in the academic arena. A challenging future was awaiting in front.

4. A Mathematical Genealogy

In the 1930s, Loo-Keng Hua visited Cambridge University. His research direction was influenced by G. H. Hardy and his school. Here is a genealogy pertaining to people working on analytic number theory:

- G. H. Hardy
- E. C. Titchmarsh
- Szu-Hoa Min

– Zongtao Chi (迟宗陶), Wenlin Yin (尹文霖), Pincong Shao (邵品琮), Chengdong Pan (潘承洞), Chengbiao Pan (潘承彪)
– Yitang Zhang (张益唐) etc. ……

Szu-Hoa Min was directly in the line handed down from Hardy and Titchmarsh. We will introduce these people briefly.

4.1 G. H. Hardy (1877–1947)

Hardy is certainly a prominent leader in the history of number theory. He and J. E. Littlewood (1885–1977) worked together and made pioneering contributions in many problems of number theory. Their cooperation set up a model of scientific cooperation. Here we present only two of Hardy's contributions which are related to our article.

The first contribution is related to the Riemann zeta function. As is well-known, the Riemann zeta function is the key to understand the prime numbers. The distribution of zeros of the zeta function plays a critical role in the distribution of prime numbers. B. Riemann (1826–1866) knew that the zeta function has infinitely many non-trivial zeros on the critical strip, namely the area consisting of those complex numbers whose real parts lie between 0 and 1. Riemann computed three such zeros by hand and found that their real parts are all equal to 1/2. Hardy first proved that the zeta function has infinitely many zeros on the critical line, i.e., the real parts are equal to 1/2. He then cooperated with Littlewood working on the density of those zeros on the critical line.

Figure 5. G. H. Hardy (left) and J. E. Littlewood in Cambridge.

Hardy's second contribution related to the present article was the Goldbach Conjecture. This was accomplished with Littlewood. In the preface to a sequence of papers, Hardy and Littlewood proudly claimed that this was the first serious study on the Goldbach Conjecture in history. They developed not only the famous Hardy-Littlewood circle method, but also proposed many well-known conjectures. For example, their k-tuple conjecture: If non-negative integers $a_1, a_2, \ldots, a_k$ satisfy some obvious necessary condition, then the polynomials

$$x+a_1+,\ x+a_2,\ \ldots,\ x+a_k$$

represent infinitely many k-tuples of simultaneous prime numbers. Note that when $k=2$ and $a_1=0$, $a_2=2$, it is just the twin prime conjecture.

4.2 E. C. Titchmarsh (1899–1963)

Titchmarsh was a Ph.D. student of Hardy at Oxford University (1923). He devoted his life to the study of analytic number theory. In 1931, Titchmarsh was offered the chair of Savilian Professor of Geometry of Oxford University, a great honor. Here is a procession of Savilian Professors beginning with Hardy: Hardy, Titchmarsh, Michael Atiyah, Ioan James, Richard Taylor, Nigel Hitchin, Frances Kirwan.

Titchmarsh was an undisputed authority of the Riemann zeta function. His monograph "*The theory of the Riemann zeta-function*" was first published in 1951. After several reprints, it is still ranked as an

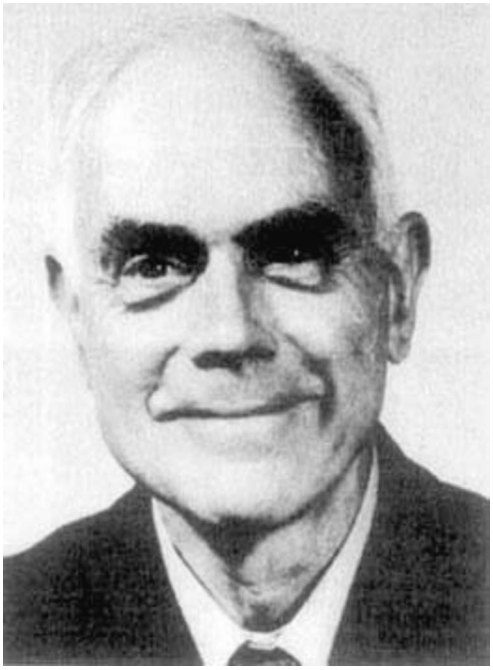

Figure 6. Edward Charles Titchmarsh.

Figure 7. Graduates of Oxford University in 1947, Szu-Hoa Min, the first from left in the third row.

important reference in this field. About this classic, a famous number-theorist once remarked: "If I were exiled to an isolated island, and were allowed to bring with me only one book about the zeta function, then no doubt I would choose the book of Titchmarsh."

4.3 Szu-Hoa Min

Szu-Hoa Min was admitted to Exeter College of Oxford University under the guidance of Titchmarsh. He arrived there at the end of 1945, received his Ph.D. degree in 1947 with a thesis of more than 200 pages. His dissertation focused on the Riemann zeta function, in particular, the rate of growth of the Riemann zeta function on the critical line, which is called the Lindelöf Hypothesis.

Min was recommended to the famous Carl Ludwig Siegel (1896–1981) who was then a professor of the Institute for Advanced Study (IAS) in Princeton, U.S.A. Min was a visitor of IAS in 1947 invited by Siegel. At the same time, another visitor hosted by Siegel was the Norwegian Alte Selberg (1917–2007) who was to win the Fields Medal in 1950.

IAS was established in 1930. The School of Mathematics had only a limited number of permanent members: besides Siegel, there were the renowned physicist A. Einstein, the distinguished mathematicians H. Weyl, K. Godel and J. von Neumann, among other members.

In Princeton, Szu-Hoa Min participated in the seminar of Weyl; there he obtained a lot of new results. Weyl proposed that Min might continue in IAS for another year, while Titchmarsh suggested Min returning to England. As to Min himself, the predominant factor was his motherland and his mother. Finally he accepted an offer of associate professorship of Tsinghua University in 1948. He was promoted to a professor one year later.

In 1952, the Chinese government reorganized all the universities in China by imitating the Russian model. Many universities were split, abolished and amalgamated together. In this way, the departments of mathematics of Peking University, Tsinghua University and Yanching University merged into the Mathematics-Mechanics Department of (the newly established) Peking University. Szu-Hoa Min became a professor of this new department.

4.3.1 The Lindelöf Hypothesis

The Lindelöf **Hypothesis** says, for any $\varepsilon > 0$, the estimate

$$\varsigma\left(\frac{1}{2}+it\right) = O\left(|t|^{\varepsilon}\right)$$

is valid for $|t| > 2$. It is an extremely difficult conjecture. In fact, it is a consequence of the Riemann Hypothesis. Using a general method in the theory of complex functions, one may show that for any $\varepsilon > 0$,

Figure 8. Taken at Fuld Hall, when Szu-Hoa Min worked at Institute for Advanced Study in Princeton in 1947–1948.

the estimate

$$\varsigma\left(\frac{1}{2}+it\right)=O\left(|t|^{\frac{1}{4}+\varepsilon}\right)$$

is valid for $|t|>2$. The constant 1/4 in the above formula is called the convex upper bound, or trivial upper bound. When we replace 1/4 by a smaller constant, the new constant is called a sub-convex upper bound. Sub-convex upper bounds have many important applications and hence is a celebrated open problem in analytic number theory. The first sub-convex upper bound of the Riemann zeta function was obtained by Weyl in 1921. Weyl used the traditional exponential sums to find his sub-convex upper bound. This implies that new results other than Weyl's constant would employ new method.

In Min's Ph.D. dissertation, he found a smaller bound

$$\frac{15}{92}.$$

Note that $\frac{1}{7}<\frac{15}{92}<\frac{1}{6}$. Min's result was published in the journal "*Tran. Amer. Math Soc.*" (1949). The main idea of Min's constant came from a new estimate of the 2-dimensional exponential sum

$$\sum_{m,n} e^{2\pi i f(m,n)}.$$

Such an idea is influential even in other fields. Min's sub-convex upper bound has been improved subsequently. The latest constant was found by Bourgain (2017): It is 13/84.

4.3.2 The Riemann Hypothesis

During the WWII, Atle Selberg stayed in a small town of Norway working lonely on his own problems. He found a world-class result: The zeros of Riemann zeta function on the critical line have positive density.

Let $N(T)$ be the number of zeros of the zeta function $\varsigma(\sigma+it)$ such that $0<\sigma<1$ and $0<t<T$, and let $N_0(T)$ be the number of zeros of $\varsigma(\frac{1}{2}+it)$ such that $0<t<T$. Then the Riemann **Hypothesis** simply says that, for all T, we have

$$N_0(T)=N(T).$$

With these notation, Selberg's theorem can be stated as follows: there exists a positive constant c, such that for all T, we have

$$N_0(T)>cN(T).$$

This result generalized the previous result of Hardy and Littlewood. After WWII, Selberg's Theorem was known to mathematicians all over the world. It was one of the major works of Selberg for receiving the Fields Medal. However, the constant c in Selberg's Theorem was not effectively computed.

Szu-Hoa Min was the first to find an effective bound for c in Selberg's theorem. Szu-Hoa Min proved that

$$c=\frac{1}{60000}$$

is an admissible bound. From this constant, we do know how far we are away from the Riemann Hypothesis. Min started to investigate this problem when he was in IAS and after he was back to China. The paper was published in the "*Journal of Peking University*".

In 1980, B. Conrey proved that $c>0.4$ and therefore, more than 40% of zeros of the zeta function fall on the critical line. A better bound was found by Shaoji Feng (冯绍继) in 2012 who showed that $c>0.4128$.

Since 1970, the Langlands program became an important mathematical project. Various generalized zeta functions (namely automorphic L-functions) were studied. For these automorphic L-functions, the corresponding generalized Riemann Hypothesis and generalized Lindelöf Hypothesis were proposed. Unfortunately, the corresponding Selberg's theorem is the only one which was generalized successfully (to the case of $GL(2)$ automorphic L-functions). For most automorphic L-functions, there are no positive answers to the corresponding Hardy's theorem (which says there are infinitely many zeros on the critical line). The results for Lindelöf Hypothesis for automorphic L-functions are rare either. So far as we

Figure 9. Snow landscape at Fuld Hall, Institute for Advanced Study in Princeton (by Jianya Liu).

Figure 10. In 1950s, Szu-Hoa Min (first from left) and Loo-Keng Hua (forth from left).

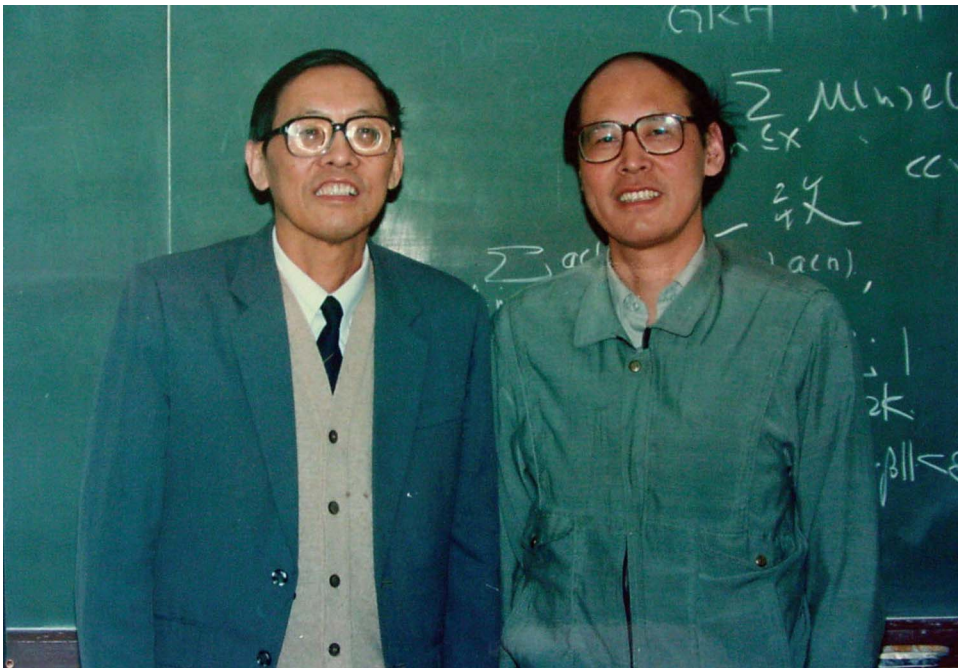

Figure 11. Chengdong Pan (left) and Chengbiao Pan in 1995 (by Tao Zhan).

know, Weyl type sub-convex upper bounds for automorphic L-functions are obtained only in the cases of $GL(2)$, $GL(3)$ and $GL(4)$. The effective bound of Szu-Hoa Min is completely unknown.

4.4 The Fourth and Fifth Generations: Min's Students and Hua's Students, etc.

At Tsinghua University, Szu-Hoa Min supervised Zongtao Chi to study analytic number theory. Using Szu-Hoa Min's idea on the sub-convex upper bound of the zeta function, Zongtao Chi improved the remainder term of the classical Dirichlet Divisor Problem. His new exponent is less than $1/3$. At Peking University, Szu-Hoa Min's graduate students were Wenlin Yin, Pincong Shao, Chengdong Pan. Chengdong Pan's younger brother (Chengbiao Pan) joined in a few years later.

After graduation from Peking University, Wenlin Yin and Pincong Shao were dispatched to Sichuan University and Qufu Normal University respectively. When Chendong Pan finished the master program at Peking University, he was ordered to accept a position at Shandong University, while Chengbiao Pan was assigned to a job at Beijing Agricultural and Mechanical College (now renamed as China Agriculture University). After the Culture Revolution ended, Chengbiao Pan was invited to teach at Peking University, but he chose to stay at the previous College and trained students in the master program of Peking University.

Under the guidance of Szu-hoa Min, Chengdong Pan studied a tough problem: The least prime numer in an arithmetic progression. The classical Dirichlet theorem says that, if $(a,q)=1$, then there are infinitely many prime numbers in the arithmetic progression

$$a+q, a+2q, a+3q, \ldots$$

A natural question is: At which position does the first prime number, denoted by$P(a,q)$, appear? The former Soviet Union mathematician Yu. V. Linnik showed that there exists a constant L, such that

$$P(a,q)=O(q^L).$$

This constant L is called the Linnik constant. Linnik didn't give an effective bound for L. The determination of the Linnik constant L depends on the distribution of zeros of a class of generalized zeta functions. Under Szu-hoa Min's advice, Chengdong Pan obtained the first effective bound: $L=5448$. After the subsequent work of several mathematicians, the British mathematician D. R. Heath-Brown was able to show that $L=5.5$. In 2011, Xylouris found that $L=5$, which is the latest result so far as we know.

The torch was passed from Szu-Hoa Min to his students. An illuminating example is Chengdong Pan's $(1+5)$ Theorem on Goldbach's conjecture.

Most of Szu-Hoa Min's academic descendants were from the branch of the Pan brothers. They trained over 30 graduate students with master and Ph.D. degrees. Most of them are active in the study of number theory and its applications.

In Peking University, Yitang Zhang wrote his master these under Chengbiao Pan. He is an outstand-

Figure 12. Yitang Zhang (by Yangbo Ye).

ing offspring of Szu-Hao Min. In 2013, he proved that there are infinitely many prime pairs with differences less than 70 million. This is a herald of the twin prime conjecture (= theorem?): Just imagine if we might replace the number 70 million by the number 2! After Zhang's method was known, the number 70 million has been reduced to smaller ones many times.

L. Kronecker said, "God creates the natural numbers; all the rest are the works of man." From this point of view, the twin prime conjecture is no doubt "God's conjecture". If Min and even Hardy knew this result in heaven, they would be pleased! As an educator, having a couple of such excellent descendants is the best gift that one may hope for.

Finally we will illustrate the students of Loo-Keng Hua after he returned to China from the United States in 1950. Hua visited Cambridge University during 1936–1938 at the invitation of Hardy. However, he didn't hasten to apply a Ph.D. there (as Hardy suggested to him), because he endeavored to devote all his time on research by finishing many papers in this period. After he returned to China in 1950, he had many students working on several complex variables, number theory, classical groups, etc. The most spectacular students in number theory were Yuan Wang (1930–) and Jingrun Chen (1933–1996).

Both of Wang and Chen were known for their works on Goldbach's conjecture. Wang proved that every sufficient large even integer is the sum of two positive integers, one of which is the product of at most two prime numbers, the other one is the product of at most three prime numbers (1958). This is the so-called the $(2+3)$ theorem. In 1966 Jingrun Chen announced an even more startling news: He proved the $(1+2)$ theorem. When Chen wrote the paper of this theorem and submitted it to the journal, Szu-Hao Min played a pivotal role. This is another story we will tell later in this article.

5. Coming Home

Szu-Hoa Min enjoyed a comfortable life when he was overseas. He was admitted to Exeter College of Oxford University, which was established in 1314. The campus was covered by green lawns everywhere. Beautiful churches and convenient libraries were invitations to scholars in the world. He visited the home of Professor Cyril Norman Hinshelwood and taught him Chinese. As a return, he learned the spoken English from Hinshelwood. Szu-Hoa Min's English improved greatly, which was a surprise to people who knew him. Hinshelwood also gave Min a gift "Shakespearean sonnet". Several years later Hinshelwood was awarded the Nobel Prize in Chemistry.

Figure 13. Szu-Hoa Min in 1950's.

The Institute of Advanced Studies at Princeton was another scholar paradise to MIn. Congenial environments allowing plenty of free time attracted top class scientists. Once Szu-Hoa Min met Albert Einstein, the scientific giant, in the hallway. He regretted later that he didn't greet Einstein because he was too shy.

In the summer of 1948 he accepted the offer of Mathematics Department of Tsinghua University. During the fall of 1948, the civil war of China was in progress. The army of KMT led by Chiang Kai-Shek fought with the army of the Communist Party first in the Northeast China, and then in Tianjin and Peking.

Tsinghua University was located in the northwestern suburban of Peking. In the campus of Tsinghua University, the voice of gunfire was audible clearly. Szu-Hoa Min settled down in the West Court of Tsinghua University (an area of faculty dormitories). In October 1948, he started the teaching duty; it was the required courses "Calculus" and "Mathematical Analysis" for sophomore students. He was an associate professor; one year later, he was promoted to a full professor.

5.1 That Pianist Girl

After returning to Peking, Szu-Hoa Min's marriage became an important issue to his mother. A lot of young girls were arranged to meet him via matchmakers. He himself got to know a short and cute girl who played piano at the church chorus. Her name was Jingyi Zhu (朱敬一), a student from Peking Normal University.

Figure 14. Mr. and Mrs. Min.

Zhu's father was a principal in some elementary school of Shandong Province. She was good at the Chinese calligraphy and also played piano excellently. When she was a high school student, she was invited to play "Maiden's Prayer" (by Tekla Badarzewska-Baranowska) at a broadcast station in the capitol of Shandong Province. At age 17, she was recommended and was admitted to Music Department of Peking Normal University. Unfortunately, her hands and feet were hurt by frostbite when she practiced in the piano room, because she didn't know how to adjust to the cold winter weather of Peking. She was forced to transfer to Department of Education. Still she played piano for the church chorus. Szu-Hoa Min and Jingyi Zhu got to know each other in the church. It was rather late when Jingyi Zhu realized that Szu-Hoa Min was a professor of Tsinghua University. Their wedding took place in 1950 at a church of Peking (then renamed as Beijing). This began a life-long marriage.

Szu-Hoa Min became a Christian in England due to an unsuccessful romance.

When he was about to finish his Ph.D., he met Shih-I Hsiung (熊式一, 1902–1991). Hsiung was from Jiangxi Province as Min's grand-father. He was a famous novelist and playwright. He translated a classic Chinese play "Lady Precious Stream" (1934), which was brought to the stage in England.

Min was invited by Hsiung to tutor Chinese classics for Hsiung's daughter and sons. Thus Min got acquaintance with Miss Hsiung who was ten years junior to Min (then Min was over 30 years old) and was studying British literature at Oxford University. Min was attracted by the young lady. But it didn't go anywhere. This romance turned out an unrequited love although Miss Hsiung treated Min like an elder brother. It was a heavy blow to the introverted Min. He couldn't get over it for a long time.

The landlord of Szu-Hoa Min, a senior lady and a devout Christian, helped him pass through the depressed period by inviting him to church. He calmed down gradually in the melodious music and the pastor's sermon. This was the second time that Min was saved by Christianity. In fact, once when he was in Western-Southern United University, he was infected with typhoid fever. He was so sick that he felled into shock. The hospital informed Min's family that the death was inevitable, while a doctor and a nurse would not give up. The nurse stayed in the bedside and prayed for him constantly. Min survived at last. From these two experiences, Szu-Hoa Min became a Christian. Christianity brought about unexpected disaster later in his life, it also equipped him with courage and strength.

5.2 Four Children and a Senior Colleague

In the summer of 1952, Chinese colleges and universities were re-organized. Many universities were abolished, incorporated into other ones, and many departments were split from the original ones. This was the so-called "institutional adjustment". The guiding line of the new organization was a principle of the "overall Sovietized" process.

For examples, Mathematics Department of Tsinghua University was split to three parts: Loo-Keng Hua and his students built up the Institute of Mathematics of Chinese Academy of Sciences. Hua became the first director of it. He and all of his graduate students moved to this new institute. Most other professors of Tsinghua University, say, Pao-Lu Hsu, Hsio-Fu Tuan (段学复, 1914–2005), Szu-Hoa Min were to join Peking University. The remaining people, just a few of them (such as Fangxiong Zhao (赵仿熊, 1908–1996)) stayed in Tsinghua University because engineering students of the new Tsinghua University would take mathematics courses and some mathematics teachers were in need. On the other hand, the

Figure 15. The family of Szu-Hoa Min in late 1950's.

newly established Mathematics Department (with the full name: Department of Mathematics and Mechanics) of Peking University consisted of people from three places: mathematicians belonging to the original Peking University, mathematicians immigrating from Tsinghua University, and mathematicians immigrating from Yanching University. In this way, Tsinghua University became an all-engineering university, Peking University became a university of science and humanity, and Yanching University vanished completely while the campus of the old Yanching University became the campus of the new Peking University.

Sometime between June and July in 1952, Szu-Hoa Min together with his family moved to an area of the staff dormitories of Peking University. This area was populated 600 years ago. Its north neighbor was the renowned Old Summer Palace (圆明园), and the south neighbor was another palace garden, the Zhong Guan Yuan (中观园). The street was narrow, about 100 meters in length and two meters in width. A creek flowed from the Langrun Garden (朗润园, situated in Peking University) and crossed the street. A small stone bridge was over the creek. During the 1950s and 1960s, the creek was very clear, full of fish and shrimp. In the summer time, frogs sang loudly everywhere.

Szu-Hoa Min's family consisted of three generations: The mother, Min and his wife, the children. Min's children were born in 1950s. Because his wife was from Jinan – the city with plenty of spring water and because of the bible-meaning of spring, he named his children with "quan" (the spring water): Lequan (乐泉), Huiquan (惠泉), Aiquan (爱泉), and Suquan (苏泉). They had a fifth child, a daughter, who was adopted by one of Min's sisters. The children brought in laugh and satisfaction, they also increased the family living expense.

The neighborhood of Min's family was Tong-Sun Zheng (郑桐荪, 1897–1963), a senior professor. Zheng was one of the first generation of modern mathematicians in China. He studied at Cornell University (1907–1910) and Harvard University (1910–1911). From 1920 he taught in Tsinghua School (reorganized as Tsinghua University in 1928). He was the founding chairman of Mathematics Department of Tsinghua University (1927), a professor of Tsinghua University and Western-Southern United University. Zheng retired in 1952 and became a neighborhood of Szu-Hoa Min for more than 10 years.

Zheng's wife passed away in 1940; he remained a single-person henceforth. In his senior years, Zheng kept a long beard with a pair of glasses. He often wore magua (马褂, a traditional Chinese dress) or the Zhongshan suit (a fashionable official dress), covered by a black vest, by which he looked rather dignified. One of Zheng's younger sister was married to the famous poet Yazi Liu (柳亚子, 1887–1958). The brothers-in-law met quite often (when Liu was still alive) and wrote poems to each other. Zheng was a person of broad knowledge: From mathematics to literature, history and poems.

Zheng's daughter Shining Zheng (郑士宁) was the wife of the celebrated Shiing-Shen Chern who was then a professor of the University of Chicago. Chern and his family left China in the late 1948. Zheng's eldest son Shizhuo (师拙) went to the United States one year before (1947). The other son Zhiqing (志清) also left China to study biochemistry in USA and Zheng saw this son off at the Main Train Station of Beijing in 1950. After the train disappeared in sight, he still stood on the platform for a long time. It seemed that Zheng understood that he would never see his children again. It was more than a decade later when they were permitted to travel back to China.

Zheng and Min had known each other, because they were colleagues at Western-Southern United University. Since they were neighborhood from 1952, they became close friends. Min would visit Zheng's home for chat. The lonely Zheng didn't cook himself. Sometimes, he would take dinner together with Min's family; sometimes, Min's mother made some South-China cuisine and had the children delivering the food for Zheng (Zheng came from South China). Min's children were Zheng's honored guests. They called him Grandpa Zheng. When he wrote Chinese calligraphy, the children would practice on ink grinding. When he took walks, the children cheered up to be Grandpa's life walking stick.

One morning in 1963, Zheng suffered from serious stomach pain and he found the eldest boy Lequan who was about to go to school. When Szu-Hoa Min learned the news, he arranged a car immediately to take Zheng to the hospital and had a surgery right away. During hospitalization, Min's family visited Zheng quite often by taking bus from Peking University (in the west part of Beijing) to the hospital (in the east Beijing). Unfortunately the seniority Zheng didn't survive. He stopped breathing in another operation. All the funeral matters including cremation and bury were taken care of by Min and Zheng's relatives. Szu-Hoa Min never mentioned to his colleagues what he had done for Zheng.

6. Christianity Interrogation

Szu-Hoa Min was a pure scholar. He carefully chose not to get involved in politics. However, he was a devout Christian, which was the main reason of political disasters to him.

By the end of 1954, Tsinghua University and Peking University offered the gathering space for Christians. Since 1955, the offerings were abolished. Christians should seek for the space of the family gatherings. Thus Min's home became one of the meeting places. On July 1, 1955, the Communist Party issued a decree about the clarification of the hidden counter-revolutionaries which signified the beginning of a new mass political movement. Beijing City Government announced that an anti-counter-revolutionaries group led by Mingdao Wang (王明道, 1900–1991) was caught up. Wang was a Christian insisting on religious freedom and he refused to join the "Three-Self Patriotic Sect" which collaborated with the Communist Party. He was arrested in 1955 and was sentenced to the life imprisonment (the sentence was altered and Wang was released in 1979). Wang once participated in the family gatherings at Min's home. As a result, Min became one object to be suspected and would be cleared off if convicted.

Two faculty members of Mathematics Department were commissioned to dissuade Min from keeping his Christian belief. Min's wife were also interviewed several times. As a simple and honest young woman, she gave in at last and she quit Christianity. On the other hand, Min kept silent without showing his opinion during the interrogation. It was obvious that Min had not had any anti-Communist-Party words or actions. They retreated because they understood the job was finished.

After this incident, Szu-Hoa Min became very cautious in speech, but the shadow of religious belief accompanied him throughout his life. Later Min confided to Zhong Li, a student of him, "I will never quit Christianity." It spoke for the stubborn personality hidden inside his mild appearance. Zhong Li followed Min rather closely in subsequent years; he was the Department chairman sometime in 1990's.

6.1 Education Reforms in 1960's

In 1960, a political movement was launched in the universities of China: The Great Educational Revolution. Its aim was to criticize "idealism" and "metaphysics" in the educational system. It was emphasized that theory and practice should be integrated together. As a result, every concept and every theorem in the mathematics syllabus ought to be tied up with the needs in practice.

The students were mobilized, especially freshman students, because they did not learned a lot and hence had open mind and dared to dream (according to the university leaders). They were assigned to write textbooks of senior students. They were encouraged to criticize Newton and Cauchy.

In the class of mathematical analysis, the teaching of the $\varepsilon - \delta$ language was denounced. It became the main target of the movement and Szu-Hoa Min happened to teach this course many times. The faculty members were asked to put off all business other than this movement and to listen to students' criticisms in the evening. The professors were enforced to tell their opinions. If the opinion was not the orthodox one, the professor would get into trouble. Such meetings usually lasted from p.m. 7 or 8 in the evening till a.m. 2 or 3 of the midnight. Without question Min had to show up at these meetings.

Szu-Hoa Min was not in good health. He suffered with high blood pressure when he was just 40 years old. At the same time, his wife was also sick: She was chronically depressed and was mentally disordered sometimes. Min had to deal with the activities of endless political movements and took care of the health of his wife and he himself. Even when he returned home so late in the midnight, he would rehearsed the lecture of the next morning. Szu-Hoa Min became the only one person who supported the family as he had done 30 years ago.

6.2 The Death of Zhongsun Fu (傅种孙)

Frankly speaking, the political situation of Szu-Hoa Min was not that bad comparing to many other intellectuals in the period 1950–1976. During that period, most intellectuals were classified as "old intellectuals". It was declared that their thoughts, living styles, and academic viewpoints were out of date and were reactionary; and therefore they should be converted altogether. In 1957, many Chinese intellectuals and their families were involved into a political disaster: The Anti-Rightist Movement, because

Min was cautious (especially after the 1955 Christianity interrogation), he was not affected by this dreadful movement. But many faculty members and many students of Peking University were convicted as Rightists, which were equivalent to claim that they were the outcasts of the society. There was even a class in Peking University, in which more than half of the students were Rightists. Szu-Hoa Min's teacher Zhongsun Fu didn't get away with this wave of purge.

Fu was a righteous man who would be in favor of the broken-down rather than the bending-over. During the Anti-Rightist Movement, he was the Vice-President of Beijing Normal University (renamed from Peking Normal University). He had many contacts with the party leaders in the university and well understood the university policies. Imaginably he disagreed with his party colleagues on many issues of the University. Nevertheless, the party boss, a senior communist, Xilin He (何錫麟), treated him with respect still.

Before the Anti-Rightist Movement started, a journalist interviewed Fu regarding the policy of the Communist Party toward intellectuals, because the Communist-Party leader, Mao Tse-tung, encouraged "the blooming of one-hundred flowers and the debating of different schools of thought". This interview was published in the campus magazine. Fu said, "Every time when a political movement is campaigned, many intellectuals are frightened very much, no matter whether the name of the movement is called a study, called an Ideological purification, or called a counter-revolutionary purge. They are beaten badly. This is unprecedented in the Chinese history."

All of a sudden, Mao's encouragement became an "Open Conspiracy" (阳谋) which was just a strategy to "lure the snake away from its hole" (引蛇出洞), i.e., to seek out for the insidious class enemies. So the Anti-Rightist Movement progressed vigorously (1957).

Fu was condemned as a criminal who attacked furiously the Communist Party. He was deprived of all the administrative duties as well as the rank-one professorship. He was banned from teaching and was coerced to accept students' criticisms. After one of these criticism meetings, a student asked about his feeling. Fu's answer was that "you should resume your study and prepare for the examinations." Fu was demoted to a librarian. He had been such a nice person that one door man of the University gate would make a cup of tea for him everyday and left it on his desk in secret.

The conviction of the Rightist for Fu was waived in December 1961. It was a good news to all the former students of Fu. On January 14 1952, Szu-Hoa Min and his best friend Cigeng Zhao visited Zhongsun Fu at Fu's home.

Fu complained the economic control of collectivism. Zhao was apprehensive that Fu would say something wrong again. Fu didn't pay attention to Zhao's worry. He asked, "What is the gain of the education reform?" Zhao was happy that Fu stopped talking about political issues. He turned to Min: "People at Peking University intended to eliminate Cauchy's definition of limits. How could it happen?" Min said, "It was initiated by some students. You may not know that they also tried to eliminate the definition of a function, which was defined as a correspondence of elements in two sets, because they couldn't find applications of a continuous function without derivatives. Thus they insisted such a function was phony. They declared that a legitimate function should possess continuous derivatives. As we knoe, in this situation the only possible functions are analytic functions. So I asked them what happens to the antiderivative of a continuous but nowhere differential function. Was it a "legal" function?"

Fu got excited. After a few minutes he pointed to his temples and said, "I am in pain." Then he fell down. Fu was hospitalized immediately. Four days later he passed away.

7. The Graduate Students

In Peking University Szu-Hoa Min trained a team of students specializing in analytic number theory. He taught in Beijing Normal University as an Adjunct Professor; Shijian Yan (严士健, 1929) was his assistant. Yan worked very hard and wrote up the lecture notes of Min's course "Elementary Number Theory". Two years later Min recommended that Yan as his successor to teach this course.

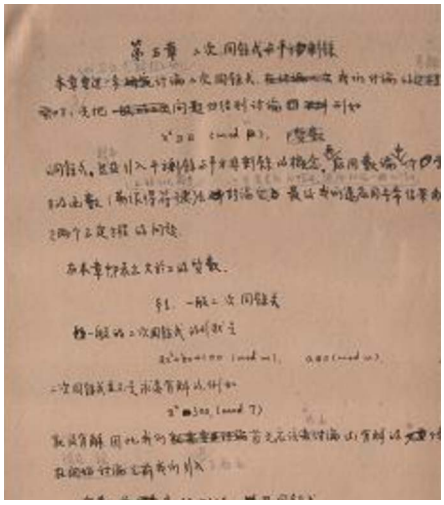

Figure 16. Manuscript of Elementary Number Theory by Szu-Hoa Min and Shijian Yan.

Figure 17. Left to right: Chengbiao Pan, Yuan Wang, Jingyi Zhu (Mrs. Min), Chengdong Pan during the 15th anniversary of Szu-Hoa Min's death in 1988.

Zhong Li was a student of Szu-Hoa Min in Peking University. At first he studied number theory. Since number theory was condemned as a non-applicable subject, Min moved to the area generalized analytic functions. He advised the students for writing bachelor theses on this new subject. He conducted the seminar "Generalized analytic function and its application on the theory of thin shells". He moved rapidly into this research area. Zhong Li continued to work along this research direction.

Szu-Hoa Min planned to help his master program student Chengdong Pan to get a job in Peking University, because Pan had some significant work in analytic number theory. Pan was dispatched to Shandong Province however, because the political performance evaluation for him recorded that he was politically indifferent, and that his parents and ancestry were potentially politically unreliable.

The Institute of Mathematics of Chinese Academy was about to hire another student of Min, Pincong Shao. Alas, Shao was very soon convicted as a Rightist in the Anti-Rightist Movement. He was enforced to leave Beijing and suffered hardship for many years.

Another student, Wenlin Yin, was impossible to stay at Beijing also, because "his personal (political) history was ambiguous".

In 1960, Chengbiao Pan, the younger brother of Chengdong Pan, finished the bachelor degree from Peking University. He was so brilliant as a student that Mathematics Department decided that Pan was allowed to take the master program entrance examination. Again his case was rebuked because of the political performance evaluation like his brother. He was dispatched to a remote place in Guizhou Province. Fortunately Chengbiao Pan got ill and took a sick leave going back to his home city. After he returned to Beijing, he was reassigned to Beijing Agricultural Mechanization Institute (later renamed as China Agricultural University). Because he lived in Beijing, Chengbiao Pan had the chance to learn from Szu-Hoa Min. In the years of the Cultural Revolution (1966–1976) knowledge was denounced as junk, while Chengbiao Pan had the wisdom to study analytical number theory.

7.1 Jingrun Chen (陈景润, 1933–1996)

Szu-Hoa Min had a group of students in Peking University working on analytic number theory. Loo-Keng Hua organized a seminar on analytic number theory for his students in Chinese Academy. Hua and Min encouraged the students to participate in activities of both places. Thus Chengdong Pan showed up in Hua's seminar very often and Hua's students, e.g., Yuan Wang and Jingrun Chen, were frequent visitors of Min's home.

In 1962, Chengdong Pan had a breakthrough in the research of Goldbach Conjecture. He improved the theorem of $(1+c)$ by A. Renyi (where c is an undetermined "astronomical" number) to the theorem of $(1+5)$: Every large even number is the sum of a prime number and a product of at most 5 prime numbers. Shortly after, Yuan Wang and Chengdong Pan proved the $(1+4)$ theorem independently.

Figure 18. Jingrun Chen.

Szu-Hoa Min and Jingrun Chen communicated from late 1950s. Chen was a humble young man; he regarded Szu-Hoa Min as one of his teachers. In holidays, he would pay greeting visits to Min and played with Min's children.

Chen was inarticulate, but was very smart in fact. He grew up in Fuzhou of Fujian Province. His father was a clerk in post office and his mother passed away when he was a little kid. He finished the high school education at Fuzhou, then studied in Xiamen University (1949–1953). He became a high-school teacher in Beijing, but was fired because that he was regarded disqualified after one-year-teaching. He was offered a job as a clerk at Xiamen University in 1955. He immersed himself in the study of number theory and wrote a paper improving a result in the book of Loo-Keng Hua ("Additive prime number theory"). He became an assistant of Chinese Academy at the invitation of Loo-Keng Hua in 1957. Thus he went to Beijing and got acquaintance with Szu-Hoa Min. Chen often went to Min's house by wearing faded blue uniforms with an old bag containing his manuscripts. Speaking in the strong accent of the Fujian dialect, he would utter, "Professor Min, please take a look." These words contained all his respect and trust to Szu-Hoa Min.

Chen's $(1+2)$ theorem on Goldbach conjecture was announced on May 15, 1966: A large even integer is the sum of a prime number and a product of at most two prime numbers. The full proof was not published because the manuscript was too long, more than 200 pages. Thus only the announcement was published in Kexue Tongbao (科学通报), a Chinese journal of Chinese Academy. The referee of Chen's paper was Szu-Hoa Min. Min's conclusion was that the proof in the 200-page manuscript was correct, but it should be simplified. In Chen's manuscript we may still found a lot of comments written by Szu-Hoa Min. This manuscript was returned to Chen when Min passed away in 1973.

Chen sent a copy of the offprint of his 2-page announcement to Min. He wrote: "Dear Professor Min, Thank you so much for your guidance over the years, especially your thorough guidance on the proof of this paper. Your student Jingrun Chen, May 19, 1966." Min told Jingrun Chen, "People verified the $(1+3)$ theorem with the aid of a high-speed computer last year

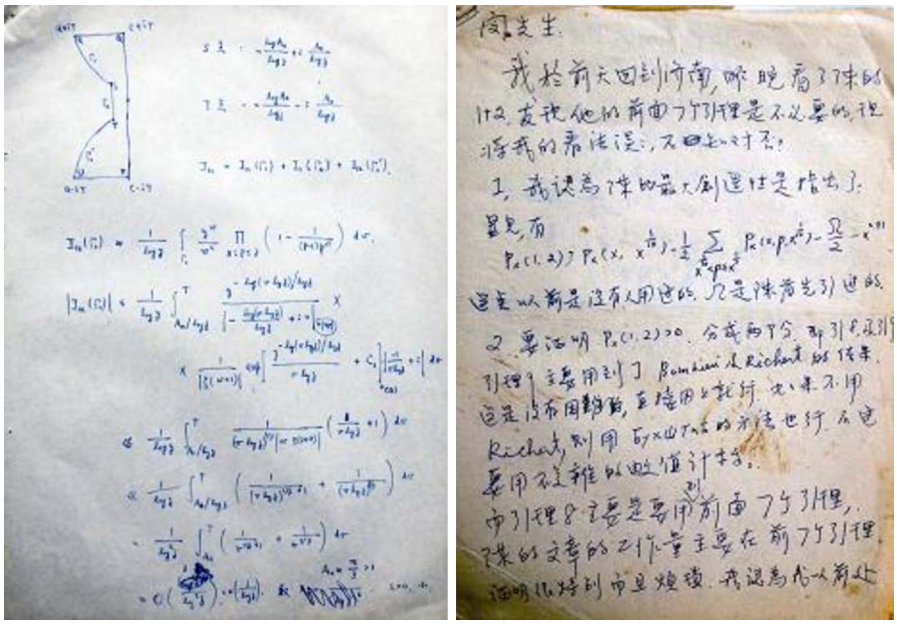

Figure 19. Manuscript by Szu-Hoa Min in 1973 The letter from Chengdong Pan on June 7, 1973.

(Buchstab, Bombieri, A. I. Vinogradov). Your proof of the $(1+2)$ theorem was finished by bare-hand. It is not surprising that your manuscript is so long." In fact, when Min read the manuscript of Chen, he checked and re-checked again until he was completely sure that the proof was correct and reliable. When Szu-Hoa Min conducted such a tremendous labor work of reviewing Chen's manuscript, he had suffered from heart attacks for years. Min would like to joke that it would lessen his life at least by three years to be the referee of this paper.

So Chen proceeded to simplify his proof. But the Cultural Revolution broke out in the summer of 1966. All the people were once again involved in the political turbulence. Chen was accused of the crime: A follower of Revisionists, a typical professional of political indifference, an idiot, a parasite, an exploiter." The "Revolution Masses" shouted, "Goldbach goes to the hell!", "$(1+2)$ worth nothing!", "Isn't $(1+2)$ equal to 3?"

Fortunately Chen was not one of the main objects of the "Revolutionary Masses", nor did he belong to the "Revolution Masses". He carefully shunned the exciting Revolution. He studied his own mathematical problem single-mindedly and desperately. To be unnoticed by the "Revolutionary Masses", he volunteered to live in an abandoned restroom, 3 square-meters only. Later he moved to a 6-square-meter panhandle-shaped room. No table was in this room, no light bulb either. He lighted kerosene lamp on bed board so that he might work. He committed suicide once at a moment when he was extremely depressed. Luckily, he was blocked by a tree when he jumped from the third floor.

Seven years later Jingrun Chen's paper was published in "Science Sinica 16 (1973), page 157–176". The proof was simplified considerably. The printed page was 20 pages only. It was submitted to the journal on March 13, 1973. The referees were Szu-Hoa Min and Yuan Wang.

Before the publication of Chen's paper, Szu-Hoa Min communicated with Chengdong Pan trying to generalize the recent method in Jingrun Chen's proof. Pan was worried about the health of his teacher. In every letter of their correspondences, Pan always wrote something like "Do take a rest". Chengdong Pan even wrote such lines: "I was sick for three days after I worked on number theory." Chengdong Pan's apprehension came true: Min died shortly.

When Szu-Hoa Min passed away, Jingrun Chen was also very sick. The memorial service of Szu-Hoa Min was held in mid-October of 1973 at Beijing Dongjiao Funeral Parlor. Chen appeared there and wore a thick cotton coat. He kept crying and crying. Since then, he visited Ms. Min during holidays although Szu-Hoa Min was no more in this world. When

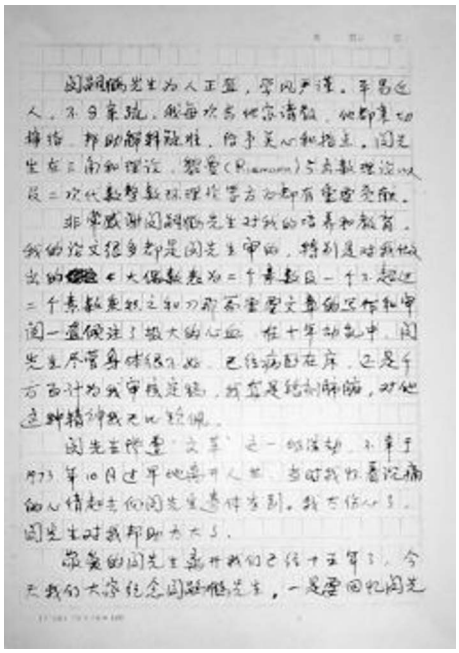

Figure 20. The congratulatory letter from Jingrun Chen to Shandong University for the 15th anniversary of Szu-Hoa Min's death on September 21, 1988.

Figure 21. Kun You (Jingrun Chen's wife, left) and Yingbo Zhang.

he got married in 1980 finally (!), the first place where the new couple visited after the wedding was Min's home.

8. The Cultural Revolution (1966–1976)

In the summer of 1966, the Cultural Revolution swept the whole China. No one was spared the blow of this holocaust. The "Maxist-Leninist Poster" (马列主义大字报) posted in Peking University was the herald of this political campaign. This poster was the first

one of similar posters in subsequent years throughout China. The top leaders of Peking University were revoked. The professors and the Communist-Party-leaders in most departments were denounced as "Monsters and Demons" (牛鬼蛇神).

On a wall of the building of Department of Mathematics a banner read: "Down with the bourgeois reactionary academic authority Szu-Hoa Min". Min was asked to stand on a stool listening to the accusations of the Red Guards (the Revolutionary students and some junior faculty members). After the humiliation was over, he went home with a gloomy face. He asked his children tentatively, "Do you decide to break up with me, too?" The children were just teenagers and they didn't know how to reply to Daddy. Following the motion of his second son who was the most mischievous one, they dug a pit in the backyard, threw in a copy of Bible (a Chinese translation) together with some other books which were political taboos (then called the "Four Olds" (四旧)), and burned down all of them. Min watched helplessly the act of the young rebels.

The Red Guards broke in his home several times and confiscated his belongings. The furniture was covered by seals. Once Zhong Li came to visit his teacher and he happened to witness such a confiscation. The Red Guards took away Min's books and manuscripts. Min stopped them and begged to take away a copy of the English Bible. He said, "Leave it for me. I read this book every day." Certainly those Red Guards didn't know it was the Bible.

In 1968, the "Mao Tse-tung Thought Propaganda Team" which comprised of soldiers and workers was ordered to take over Peking University. All the faculty members whom were accused of were ordered to live together in a common accommodation. The teachers of Department of Mathematics were imprisoned in a gray building: 6–8 persons shared a room, males and females were separated. Shengming Leng (冷生明) was a roommate of Szu-Hoa Min. He saw that every night Min would pray in secret under his blanket. Leng thought, without the religious belief, Min might not be able to get through this horrible period. After imprisonment of one month, he was released to home. Because no one was allowed to take a bath in prison, his body was covered and bit by lice. It took a long while to get away the lice with the help of his four children. Even after setting-free, the morning jogging was still necessary because it was believed to be a must to transform the bourgeois intellectuals into the proletarian ones. When he suffered from heart attack and asked for exemption of jogging, he was scolded severely.

Dozens of staff and faculty members of Peking University were cleared off or chose to clear themselves off. For instance, four faculty members of Mathematics Department committed suicide within one month. The most notable one was Professor Tiebao Dong (董铁宝, 1917–1968), a pioneer of computer scientist and engineering mechanics.

Dong was a bachelor of civil engineering in Jiaotung University (Shanghai, 1939). On finishing the degree, he rushed to Yunnan Province and worked for repairing the bombarded roads and bridges in the Yunnan-Burma Highway which was the only pathway of China to get in touch of western countries and supporting resources during the Sino-Japan War. After the War ended he studied at University of Illinois (Champagne-Urbana) and obtained a Ph.D. (1949). Dong was fortunate to participate in the design and the construction of one of the first computers in the world, ENIAC. In 1956 Dong returned to China together with his wife and three children, two sons and one daughter. He became a professor of Department of Mathematics of Peking University. Dong was an expert in computation mechanics and computation mathematics; he was instrumental in the development of computer science in China.

Szu-Hoa Min and Tiebao Dong became good friends, so did the children of their families. Dong was imprisoned as Szu-Hoa Min. In an October morning of 1968 people found that Dong was hanged in a tree: He chose a most extreme means to protest the violence and the insults to an intellectual. Twelve years before Dong and his family spent three months by detouring via Europe and returned to China, his beloved home country. After the Cultural Revolution was over, Dong's wife and her broken family left China forever.

During the chaos, the "Revolutionary Masses" came up with various tricks to torture the "Monsters and Demons". One of the trick was a new regulation of a food-shop which refused to sell "fine food" (e.g., flour and rice) to people belonging to the Five Classes (landlords, rich peasants, counter-revolutionaries, bandits, and rightists, 地富反坏右). Ms. Min was so naïve to ask the counter staff, "My husband is a reactionary academic authority. Does he belong to the Five Classes?" The answer was "Of course!". Thus all the people in Min's family would take the coarse grains for several months.

Min's mother did the shopping duty and other minor housework even in the old age. She died of rectal cancer in 1971 when the Cultural Revolution was still in progress. This was a heavy blow to Min. Two years later he passed away. Ms. Min lamented telling her children several times, "Your father came to this world to be an obedient son of his mother."

8.1 Applied Mathematics

During the Cultural Revolution, many pure mathematicians in China switched to applied math. Loo-

Keng Hua turned to optimization and numerical analysis. Szu-Hoa Min also tried to work on applied mathematics.

In 1969, Szu-Hoa Min was dispatched to Beijing Geological Instrument Factory to be re-educated by the working-class people. At that time, the factory was at a critical moment to device a marine gravimeter. Only a few companies in the western countries had the know-how, but it was an embargo supply to China. Min proposed a "Chebyshev weighted coefficient" to improve the digital filtering method. As a result, the self-devised gravimeter successfully obtained weak signals from a 50 times strong noises. This equipment was named as ZY-1 Marine Gravimeter. After exploring experiments in the sea over five years, it was patented by the National Standard Bureau. Its quality was much better than that produced in Japan.

In October 1971, Szu-Hoa Min was missioned to the Bureau of Petroleum Geophysical Prospecting of the Ministry of Fuel and Chemical Industry. He worked in Factory 646 which was engaging in petroleum seismic exploration. The task was to explore oil fields by analyzing the refracting waves of the artificial seismic waves on the surface of the earth. In the 1950s and 1960s, seismic waves were recorded using simulated tapes. When Min came to this research area in 1971, simulated tapes were being replaced by digital tapes in the west countries. Many countries tried hard to develop digital recording, multiple coverage technology, and seismic data process technology by using high-speed digital computers to improve the precision of the recordings.

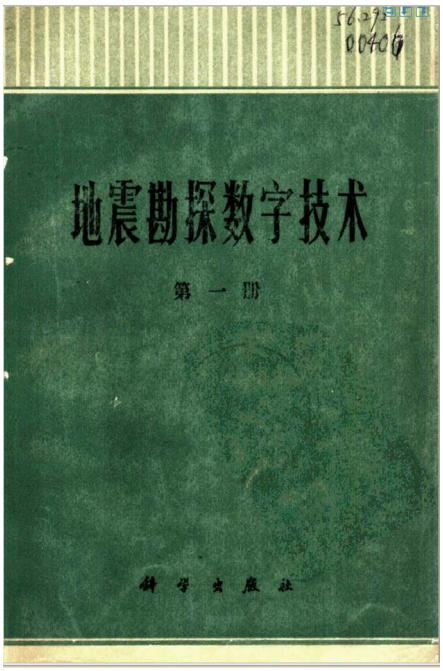

Figure 22. The cover of the first edition of Seismic Exploration Digital Technology in Seismic Exploration.

In the early 1970, Prime Minister Chou Enlai (周恩来) approved to embark the "150 Project". Within three years, the "150 Project" completed the assembly of "150 Computer" which included from integrated circuit to computer hardware system, from the software system to the application software. This was China's first MIPS computer which was a joint research output of Peking University, the Ministry of Fuel and Chemical Industry, and the Ministry of Electronics Industry. The 646 Factory where Szu-Hoa Min worked developed 3 software packages on "150 Computer", one of which was a C program, China's first seismic data process system with 18 seismic data processing methods available.

At that time, a problem that defeated people was: The digital seismic recorder recorded not only the effective waves from underground layers, but also other noises, i.e., various interference waves from the ground and underground. This made it difficult to identify correctly the effective waves; and even worse it would increase the chances of misinterpretation errors. Digital filtering technology was a handy method to suppress noises. It highlighted effective waves, and was one of the urgent key technologies in the oil seismic exploration at that time. Szu-Hoa Min studied this difficult subject in 1-D digital filtering, 2-D digital filtering, shifted superposition, and seismic holography. Together with colleagues from Peking University, he published their research works on "*Acta Mathematica Sinica*" (数学学报) under the pen-name Lihua Shu (舒立华). His joint research works with colleagues in the Bureau of Petroleum Geophysical Prospecting were also published on "Acta Mathematica Sinica" under another pen-name Youbing Hong (宏油兵). Min presided over the publication of the series books "*Seismic Exploration Digital Technology*" Volumes 1 and 2 (published by the Science Press). This series comprised of 4 volumes, but Min didn't live to see the publication of the last two volumes. It educated a whole generation of experts in 1970's, and it is still a classic of this field.

On April 2, 1974, China's first digital seismic profile generated by a C program was honored as a "show profile" (争气剖面). The "150 Project" helped the promotion of the digitalization of seismic exploration in China. Seismic data interpretation paved the way towards automation. The "150 Project" was an important contribution to the oil exploration in China. It was engraved on China Millennium Monument at Beijing as a milestone of Chinese civilization from the ancient time to the year 2000. It represents the efforts of Szu-Hoa Min and his friends, a whole generation of scientists.

Figure 23. Szu-Hoa Min (the third from right) receiving re-education by working class people in 1970's.

In this way he found the applications of mathematics in the front lines of practical production. On the other hand, he was still attached to number theory, his first love. He told Zhong Li many times, "Sooner or later I will turn back to analytic number theory". Compared to Min, the strong and decisive Loo-Keng Hua switched to operations research without returning to pure mathematics.

9. Ice-Thawing

The Communist China opened the doors to USA. The USA President Richard Nixon and his Secretary of States Henry Kissinger visited China in February 1972.

Subsequently many American scholars came to China. So did Shiing-Shen Chern in 1972. He returned to his motherland after 23 years. He visited Peking University where most of his friends at Western-Northern United University were here. He gave a small gift to each of them. The one for Min was an expensive Swiss watch.

When Chern was in Beijing, he had several conversations with Szu-Hoa Min. But Min didn't talk a lot because he was warned to obey faithfully the "discipline of conversations with foreigners". Once Chern asked him about his ongoing research program, he simply replied, "Geology".

All the professors Chern met were required to report to the Communist Party their gifts from Chern and their conversations with Chern. When Chern

Figure 24. Right to left: Hsien-chün Wang (Professor in Department of Philosophy, Peking University), Szu-Hoa Min, Mrs. Wang, S.-S. Chern, Mrs. Chern, Pu Chern (daughter of Chern).

was going to leave Beijing, the department organized a farewell meeting, but they forgot to notify Szu-Hoa Min this meeting. The next time when Chern visited Peking University, Szu-Hoa Min had passed away.

9.1 The End

In 1973, Min's eldest son, a worker, was admitted to Southwest Petroleum University. The second son, an educated-youth working in the farm, was recommended to study in Philosophy Department of Peking

University. It was a great relief to Szu-Hoa Min that, at least his sons had the chance for higher education.

Min was still busy with the mathematical problems of digital technology. He was exhausted, physically and mentally. He revised the book "*Seismic Exploration Digital Technology in Seismic Exploration*". Whenever his wife suggested him to take a rest, he would complain, "Don't interrupt me. I will be uneasy so long as the job hasn't be finished." In the meantime he found a migration problem of wave equations which was a hot topic then. He caught up with the recent progress, organized a crash training program, wrote the lecture notes. He worked so hard that he often went to sleep till midnight. Even three or four days before his death, he was busy discussing the problem with his team in the Bureau of Petroleum Geophysical Prospecting.

In the morning of October 9, 1973, Szu-Hoa Min suffered from heart attack again. His wife suggested him to see doctor. He refused and took several tablets of Nitroglycerin. The pain got worse that he agreed to be hospitalized at 10 o'clock late in the evening. As two of his sons lived away from Beijing, he was accompanied to the hospital by his daughters, the 19 years old Aiquan and the 17 years old Suquan. In the Clinic of Peking University, they happened to meet a couple of students of Min. Min asked the students to escort his daughters back to home.

Szu-Hoa Min died of heart attack the next morning. No relatives, no doctors nor nurses were in his bedside. The only thing left there was an empty oxygen bag. He was gone suddenly. He didn't close his eyes. He had so many works in ahead. He didn't have the chance getting back to analytical number theory. The Ministry of Fuel and Chemical Industry organized a memorial service in honor of his contribution to the petroleum industry.

Szu-Hoa Min was survived by his wife and their children. His daughters repented they left Min alone in the hospital that night. Many years later, his older son became a professor at Beijing University of Science and Technology, the younger son was a professor at China Communication University. The first daughter had got a job at the Instrument Factory of Peking University since 1970. The second daughter, Suquan, passed the university entrance examination after the Cultural Revolution was put to an end; she was a computer engineer. Min's wife lived together with Suquan and her family till she passed away at the age of 82 years old. Min and his wife had another daughter who was adopted by one of Min's sisters. This youngest daughter went far away the most; she and her family immigrated to USA in 1980's.