New World Mathematics Awards

by Pei Lu

Mathematics and mathematics education are of great significance in human pursuit of knowledge and truth. They are the foundation of science and technology. In 2007, Dr Henry Cheng, Managing Director of New World Development Company Limited and Professor Yau Shing-Tung, an outstanding mathematician, co-initiated the "New World Mathematics Awards" (NWMA). The goal of this program is to encourage outstanding Chinese mathematics students worldwide in their pursuit of mathematical truth.

2017 NWMA is sponsored by the New World Development Company Limited and China Young Leaders Foundation, organized by Tsinghua University, co-organized by the International Congress of Chinese Mathematicians. Chinese undergraduate, master and PhD mathematics students from around the world who have completed or defended their thesis in 2016 or 2017 are qualified to participate. The topics of theses can be Pure or Applied Mathematics, Probability and Statistics, Biomathematics etc.

Application Process

Candidate's submission should have three parts: the thesis, an application form, one letter of recommendation. Student should fill in the application form online and upload the thesis on the webpage of NWMA. The application deadline is September 30, 2017.

Selection Process

All submissions are subjected to two rounds of review: A scientific committee chaired by Professor Shiu-Yuen Cheng, and then an international committee chaired by Professor Shing-Tung Yau. Members of these committees are all world-renowned mathematicians. The results will be announced around December 2017.

Qualification

This program is open to all undergraduate, master and PhD mathematics Chinese students.

The submitted thesis must be successfully defended and accepted by the students' college/university in 2016 or 2017. Students have to prepare to provide proof of defended of thesis.

The topics of thesis can be pure mathematics, applied mathematics, probability and statistics, biomathematics, etc.

The award winners will be announced around December 2017. Award ceremony is tentatively scheduled in 2019, at Beijing. To claim the award, awardees have to be present at the NWMA award ceremony.

Visit http://ymsc.tsinghua.edu.cn/nwma/.

International Committee

Shing-Tung Yau

Professor Shing-Tung Yau is the William Casper Graustein Professor of Mathematics at Harvard University. He is the inaugural Director of the Mathematical Sciences Center of Tsinghua University. Professor Yau has made fundamental contribution to differential geometry, differential equations and mathematical physics. Professor Yau is honored by numerous prestigious prizes and awards, including Oswald Veblen Prize (1981), John J. Carty Award for the Advancement of Science (1981), Fields Medal (1982), Humboldt Research Award, Alexander von Humboldt Foundation (1991), Crafoord Prize (1994), United States

National Medal of Science (1997), China International Scientific and Technological Cooperation Award (2003), and Wolf Prize in Mathematics (2010). He is a member of the United States National Academy of Sciences, a member of Russian Academy of Sciences, a foreign member of the Chinese Academy of Sciences, and a member of Academia Sinica.

Ben Andrews

Professor Ben Andrews is a senior fellow of Mathematical Sciences Institute, Australian National University. Professor Andrews works on differential geometry and related partial differential equation and is well known for his work in geometric evolutions. He is a leading international geometric analyst, who specializes in Riemannian geometry, submanifold geometry, heat flows, image processing, interface model and reaction-diffusion system. In 2011, Professor Andrews solved one of the most celebrated open problems in mathematics, the fundamental gap conjecture for the eigenvalues of the Laplacian, and he proved Firey's conjecture on the shape of rolling stone in 1999. He was an invited speaker at the International Congress of Mathematicians in Beijing, 2002. He was awarded the medal of Australian Mathematical Society in 2003. In 2012, Professor Andrews became a fellow of the American Mathematical Society, and he was elected a fellow of Australian Academy of Science in March 2013.

John H. Coates

Professor John H. Coates is the Sadleirian Professor of Pure Mathematics at the University of Cambridge. His research work concerns number theory, arithmetical algebraic geometry and Iwasawa theory. He became Head of the Department of Pure Mathematics and Mathematical Statistics at Cambridge in 1991, served as president of the London Mathematical Society during 1988–1990 and as vice-president of the International Mathematical Union from 1991 to 1995, as a member of Council of the Royal Society during 1992–1994. Professor Coates was elected a fellow of Emmanuel College in Cambridge twice, a fellow of the Royal Society of London in 1985 and awarded Senior Whitehead Prize by the London Mathematical Society in 1997. Professor Coates is the first receipt of the ICCM International Cooperation Award (2004).

Björn Engquist

Professor Björn Engquist is Director of the Parallel and Scientific Computing Institute. Engquist currently holds the Computational and Applied Mathematics Chair at the Institute for Computational Engineering and Sciences at the University of Texas at Austin. He has been a leading contributor in the areas of multiscale modeling and scientific computing, and a productive educator of applied mathematicians. His research field is computational and applied mathematics and numerical methods for differential equations with applications to multi-scale modeling, electromagnetism, and fluid mechanics. Engquist has authored more than 100 scientific publications and advised 31 PhD students. He is a recipient of numerous distinctions and awards: a member of the Royal Swedish Academy of Sciences and the Royal Swedish Academy of Engineering Sciences, and an invited speaker at the International Congress of Mathematicians (1982 and 1998), European Congress of Mathematics (1992), and European Congress of Fluid Mechanics (1991). He was selected to the Norwegian Academy of Science and Letters in 2011.

Akito Futaki

Akito Futaki is a Professor of Graduate School of Mathematical Sciences at University of Tokyo. His main research interests are differential geometry, complex algebraic geometry. He is interested in the existence problem of extremal Kähler metrics such as Kähler-Einstein metrics, and related problems in geometry. He received the Geometry Prize of Mathematical Society of Japan in 1990, Autumn Prize of Mathematical Society of Japan in 2011. He was elected professor emeritus of Tokyo Institute of Technology in 2012.

Dorian Goldfeld

Dorian Goldfeld is a professor at Columbia University. He is a member of the editorial board of Acta Arithmetica and of The Ramanujan Journal. He is a co-founder and board member of SecureRF, a corporation that has developed the world's first linear-based security solutions. Professor Goldfeld's research interests include various topics in number theory. In his thesis, he proved a version of Artin's conjecture on primitive roots on the average without the use of the Riemann Hypothesis. In 1987 he received the Frank Nelson Cole Prize in Number Theory, for his solution of Gauss' class number problem for imaginary quadratic fields. He has also held the Sloan Fellowship (1977–1979) and in 1985 he received the Vaughan prize. In April 2009 he was elected a Fellow of the American Academy of Arts and Sciences. In 2012 he became a fellow of the American Mathematical Society.

Thomas Yizhao Hou

Thomas Yizhao Hou is Charles Lee Powell Professor of Applied and Computational Mathematics in the Department of Computing and Mathematical Sciences at the California Institute of Technology. Hou is known for research on multiscale analysis. He is an author of the monograph Multiscale finite element methods. He has worked extensively on numerical analysis and applied analysis of the Navier-Stokes equations. His recent work focuses on adaptive data analysis. He received an Alfred P. Sloan Research Fellowship in 1990. He was awarded the Feng Kang Prize in Scientific Computing in 1997. He received the James H. Wilkinson Prize in Numerical Analysis and Scientific Computing from the Society for Industrial and Applied Mathematics (SIAM) in 2001, Morningside Gold Medal in Applied Mathematics in 2004, and Computational and Applied Sciences Award in 2005. Hou has also been inducted into several scholarly societies. He was elected Fellow of the Society for Industrial and Applied Mathematics in 2009, Fellow of the American Academy of Arts and Sciences (AAAS) in 2011, and Fellow of the American Mathematical Society (AMS) in 2012.

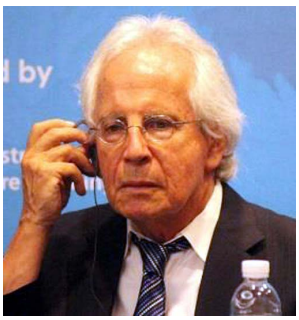
Jun Li

Dr. Jun Li is a professor at the Department of Mathematics, Stanford University. Since receiving his PhD from Harvard University in 1989, he was on the faculty of UCLA from 1992–1996, before joining Stanford University. His research interest is in algebraic geometry; he has made significant contribution to the research on moduli of vector bundles, stable morphisms and Gromov-Witten invariants. He was the recipient of Sloan fellowship, Terman fellowship; he was awarded the morningside medal in 2001.

Eduard Looijenga

Professor Eduard Looijenga is a Professor of the Yau Mathematical Sciences Center of Tsinghua University. His research started in singularity theory, but migrated via Torelli problems to locally symmetric varieties, then to mapping class groups and moduli spaces of curves. Professor Looijenga was an invited speaker at the ICM in 1978 and at the ECM in 1992. He is a member of the Royal Netherlands Academy of Arts and Sciences.

Stanley Osher



Stanley Osher is currently Director of Special Projects at the Institute for Pure and Applied Mathematics (IPAM) at the University of California Los Angeles, and Director of Applied Mathematics. Stanley Osher has made fundamental

contributions to applied mathematics, computational science, and scientific computing, and has cofounded three companies based on his research. He has applied level set methods for partial differential equations to the field of image processing, to video image enhancement, and movie animation. He has been featured in international media such as Science News, Die Zeit, and Los Angeles Times. Stanley Osher is a recipient of the 2007 USACM Computational and Applied Sciences Award, he was awarded Docteur Honoris Causa in 2006, and elected to the National Academy of Sciences in 2005. Stanley Osher has received the SIAM Kleinman Prize in 2005, the SIAM ICIAM Pioneer Prize in 2003, the NASA Public Service Group Achievement Award, and the Japan Society of Mechanical Engineers Computational Mechanics Award.

Lo Yang

Professor Lo Yang was the Director of Institute of Mathematics (1987–1995) and the President of Academy of Mathematics and System Science (1998–2002), CAS. Now he is the professor and Chairman of Scientific Committee of AMSS. He was elected as the academician of the Chinese Academy of Sciences in 1980. Besides, he was the President (1992–1995) of Chinese Mathematical Society.

He was mainly engaged in the research on complex analysis. He has made a through study of deficient values and deficient functions. He, cooperated with Guang-hou Zhang, established for the first time a close relation between the numbers of deficient values and Borel directions of entire and meromorphic functions. Among his research on normal families, he built the relationship between normal families and fix-points, as well as that between normal families and differential polynomials. He also made the systematic research on the angular distribution: finding a new kind of singular direction and establishing a necessary and sufficient condition for the distribution of singular directions. Distribution Theory was published by the Springer-Verlag. He was invited as the main or invited speaker for over 20 international conferences and delivering lectures in 60 famous universities in the world.

Horng-Tzer Yau

Horng-Tzer Yau is a professor of Harvard University. Professor Yau is a leader in the fields of mathematical physics, analysis and probability. He is a powerful analyst who has introduced important tools and concepts to study probability, stochastic processes, nonequilibrium statistical physics and quantum dynamics. His insight and skilled teaching are invaluable to students. He has also been a member of the Institute for Advanced Study in Princeton, in 1987–1988, 1991–1992, and 2003, and a member of the American Academy of Arts and Sciences. He received Henri Poincaré Prize, MacArthur Fellowship and Morningside Gold Medal of Mathematics in 2000 and 2001. He is elected a fellow of the US National Academy of Sciences in 2013.

Scientific Committee

Yuefei Wang
AMSS, CAS

Zhiying Wen
Tsinghua University

Nanhua Xi
AMSS, CAS

Zhouping Xin
Chinese University of Hong Kong

Jing Yu
National Taiwan University

Pin Yu
Tsinghua University

Shouwu Zhang
Princeton University

Xiping Zhu
Sun Yat-sen University

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Shiu-Yuen Cheng
Tsinghua University

Xiaoxia Huang
Tsinghua University

Bangming Deng
Tsinghua University

Huihui Zeng
Tsinghua University

Zuoqiang Shi
Tsinghua University

Recipients of the 2014 New World Mathematics Awards

Doctor Thesis Awards, Gold Prize

Hao Wu, Universite Paris-Sud 11

Thesis title: On the relations between SLE, CLE, GFF, and the consequences

Abstract: This thesis focuses on various relations between SLE, CLE and GFF. In Chapter 2, we give a construction of $SLE_k(\rho)$ processes from CLE_k loop configuration and chordal restriction samples. Sheffield and Werner has proved that CLE_k can be constructed from symmetric $SLE_k(K-6)$ exploration processes. We prove in Chapter 3 that the loop configuration constructed from the asymmetric $SLE_k(K-6)$ exploration processes also give the same law CLE_k. SLE_4 can be viewed as level lines of GFF and CLE_4 can be viewed as the collection of level lines of GFF. We define a conformally invariant time parameter for each loop in CLE_4 in the second part of Chapter 3 and then give a coupling between GFF and CLE_4 with time parameter in Chapter 4. SLE_k can be viewed as flow lines of GFF. We derive the Hausdorff dimension of the intersection of two flow lines in GFF. Then, from there, we obtain the dimension of the cut and double point set of SLE curve in Chapter 5. In Chapter 6, we define the radial restriction measure, prove the characterization of these measures, and show the if and only if condition for the existence of radial restriction measure.

Advisor: Wendelin Werner
Current affiliation: C.L.E. Moore Instructor, Department of Mathematics, MIT

Lin Wang, Nanjing University

Thesis title: Converse KAM theory for positive definite Hamiltonian systems

Abstract: In this thesis, we consider converse (KAM) theory for positive definite Hamiltonian systems. Precisely speaking, what is the maximum of r if the Lagrangian torus with a given rotation vector or all Lagrangian tori can be destroyed by an arbitrarily small perturbation in the C^r topology for an integrable positive definite Hamiltonian with d ($d \geq 2$) degrees of freedom?

A similar problem was raised by Herman in ICM1998. On one hand, it is shown that all of the Lagrangian tori of an integrable system with d ($d \geq 2$) degrees of freedom can be destroyed by an arbitrarily small C^ω (resp. C^∞) perturbation in the $C^{d-\delta}$ (resp. $C^{d+1-\delta}$) topology. On the other hand, Some results are proved based on the arithmetic properties of given rotation vectors. An vector $\omega \in \mathbb{R}^d$ is called τ-approximated if there exists a positive constant C as well infinitely many integer vectors $k \in \mathbb{Z}^d$ such that $|\langle \omega, k\rangle| < C|k|^{-d+1-\tau}$. For the Lagrangian torus with a given τ-approximated rotation vector of an integrable system with d ($d \geq 2$) degrees of freedom, we obtain:

(1) it can be destroyed by an arbitrarily small C^∞ perturbation in the C^r topology, where $r < 2d + 2\tau$;
(2) it can be destroyed by an arbitrarily small C^ω perturbation in the C^r topology, where $r < d + \tau$;
(3) it can be destroyed by an arbitrarily small Gevrey-α ($\alpha > 1$) perturbation in the Cr topology, where $r < 2d + 2\tau - 2\alpha(d + \tau)$.

Advisor: Chong-Qing Cheng
Current affiliation: Assistant professor, Yau Mathematical Sciences Center

Junyi Xie, Ecole Polytechnique

Thesis title: Dynamique algebrique des applications rationnelles de surfaces

Abstract: This thesis contains three parts. The first one is devoted to the study of the set of periodic points for birational surface maps. We prove that any birational transformation of a smooth projective surface whose degree growth is exponential admits a Zariski-dense set of periodic orbits. In the second part, we prove the dynamical Mordell-Lang conjecture for all polynomial birational transformations of the affine plane defined over a field of characteristic zero. Our approach gives a new proof of this conjecture for polynomial automorphisms of the affine plane. The last part is concerned with a problem in affine geometry that was inspired by the generalization to any polynomial map of the dynamical Mordell-Lang conjecture. Given any finite set S of valuations that are defined on the polynomial ring $k[x,y]$ over an algebraically closed field k, trivial on k, we give a necessary and sufficient condition so that the field of fractions of the intersection of the valuation rings of S with $k[x,y]$ has transcendence degree 2 over k.

Advisor: Charles Favre
Current affiliation: Postdoctoral, Institut de Mathématiques de Toulouse

Jie Zhou, Harvard University

Thesis title: Arithmetic properties of Moduli spaces and topological string partition functions of some Calabi Yau threefold families

Abstract: This thesis studies certain aspects of the global properties, including geometric and arithmetic, of the moduli spaces of complex structures of some special Calabi-Yau threefolds (B-model), and of the corresponding topological string partition functions defined from them which are closely related to the generating functions of Gromov-Witten invariants of their mirror Calabi-Yau threefolds (A-model) by the mirror symmetry conjecture.

For the mirror families (B-model) of the families (A-model) of $K_{\mathbb{P}^2}$, K_{dp_n}, $n = 5,6,7,8$ with varying Kähler structures, the bases are the moduli spaces of complex structures of the corresponding mirror Calabi-Yaus. We identify them with certain modular curves by studying the Picard-Fuchs systems and periods of the corresponding mirror families. In particular, the singular points on the moduli spaces correspond to the cusps and elliptic points on the modular curves.

We take the BCOV holomorphic anomaly equations with boundary conditions as the defining equations for the topological string partition functions. Using polynomial recursion and the above identification, we interpret the boundary conditions as regularity conditions for modular forms and express the equations purely in terms of the language of modular form theory. This turns the problem of solving the equations into a combinatorial problem. We also solve for the first few topological string partition functions genus by genus recursively in terms of almost-holomorphic modular forms. Assuming the validity of mirror symmetry conjecture, we prove a version of integrality for the Gromov-Witten invariants of the original non-compact Calabi-Yau threefolds (A-model) as a consequence of the modularity of the partition functions.

Motivated by the results for the aforementioned non-compact Calabi-Yaus, we construct triples of differential rings on the moduli spaces of complex structures for some oneparameter families of compact Calabi-Yau threefolds (B-model), e.g., the quintic mirror family, in a systematic way. These rings are defined from the Picard-Fuchs equations and special Kähler geometry on the moduli spaces. They share structures similar to the triples of rings of modular forms, quasi-modular forms and almost-holomorphic modular forms defined on modular curves. Moreover, the topological string partition functions are Laurent polynomials in the generators of the differential rings.

Advisor: Shing-Tung Yau
Current affiliation: Postdoctoral Researcher, Perimeter Institute

Doctor Thesis Awards, Silver Prize

Xinxin Chen, University Paris VI

Thesis title: Branching random walks with selection

Abstract: We consider branching Brownian motion which is a mathematical object modeling the evolution of a population. In this system, particles diffuse on the real line according to Brownian motions and branch independently into two particles at rate 1. We are interested in the rightmost (resp. leftmost) position at time t, which is defined as the maximum (resp. minimum) among the positions occupied by the particles alive at this time. According to Lalley and Sellke [101], every particle born in this system will have a descendant reaching the rightmost position at some future time. We study this phenomenon quantitatively, by estimating the first time when every particle alive at time s has had such a descendant. We then study an analogous model the branching random walk in discrete time, in which random walks are indexed by a Galton-Watson tree. Similarly, we define the rightmost and the leftmost positions at the n-th generation. We consider the walk starting from the root which ends at the leftmost position. We show that this work, after being properly rescaled, converges in law to a normalized Brownian excursion. The last part of the thesis concerns the evolution of a population with selection. Given a regular tree in which

each individual has N children, we attach to each individual a random variable. All these variables are i.i.d., uniformly distributed in [0,1]. Selection applies as follows. An individual is kept if along the shortest path from the root to the individual, the attached random variables are increasing. All other individuals are killed. We study the asymptotic behaviors of the evolution of the population when N goes to infinity.

Advisor: Zhan Shi
Current affiliation: Assistant professor, University Lyon 1

Gang Liu, University of Minnesota

Thesis title: On manifolds with Ricci curvature lower bound and Kahler manifolds with nonpositive bisectional curvature

Abstract: In the thesis, first, we classify complete noncompact three dimensional manifold with nonnegative Ricci curvature. As a corollary, we confirms a conjecture of Milnor in dimension three. The proof is based on the minimal surface theory developed by Schoen and Yau, Schoen and Fischer Colbrie. Next we study compact Kähler manifolds with nonpositive bisectional curvature. In particular, we confirm a conjecture of Yau which states that for there is a canonical fibration structure for these manifolds. We also prove some comparison theorems for Kähler manifolds with Ricci curvature lower bound.

Advisor: Jiaping Wang
Current affiliation: Morrey Assistant Professor, UC Berkeley

Junbin Li, Sun Yat-sen University

Thesis title: Branching random walks with selection

Abstract: The vacuum Einstein equations describe the behavior of gravitation fields in General Relativity. Mathematically, it means that on a 4-dimensional Lorentzian manifold (M, g) which satisfies $\mathrm{Ric}(g) = 0$. As a hyperbolic equations, vacuum Einstein equations can be considered as initial data problems in a natural way. It is one of the important topics in studying vacuum Einstein equations to study the dynamical evolution of the equations.

One important prediction of Einstein equations is the existence of black holes. From dynamical point of view, it is natural to ask whether a black hole can form in the evolution of Einstein equations. Christodoulou firstly gave a dynamical mechanism of the formation of black hole based on his short pulse method in 2008. The initial data were given on a null hypersurface, i.e. he considered the characteristic initial data problem. It is interesting to seed a Cauchy data problem description in the formation of black holes. In this thesis, we discover some special structure in the space-times Christodoulou constructed, and firstly construct asymptotically flat Cauchy initial data sets which evolve to black holes, combining with the gluing techniques developed by Corvino-Schoen. This work [77] is done in collaboration with Pin Yu, and is to appear in Annals of Mathematics. The weak cosmic censorship conjecture, which states that for generic asymptotically flat initial data, the future null infinity is always complete, is one of the most important conjectures in General Relativity. We consider two intersected null cones, the outgoing one of which intersects with future null infinity. The data on the outgoing null $\underline{\mathrm{OBJ}}$cone decay suitably. We show that the solution of Einstein equations always exists in a uniform neighbourhood of the outgoing null cone. In particular, we prove that the future null infinity can always extend in the retarded time locally, which should be considered as the first step towards the weak cosmic censorship. In addition, we can apply the result to the black holes formation space-time which Christodoulou constructed to extend the solution. This work [78] is done in collaboration with Xi-Ping Zhu.

Advisor: Xi-Ping Zhu
Current affiliation: Associate professor, Sun Yat-sen University

Xue Luo, University of Illinois at Chicago

Thesis title: Branching random walks with selection

Abstract: This dissertation provides an affirmative answer to the well-known half century old engineering question raised by Office of Naval Research: "How can one solve nonlinear filtering (NLF) problems in real time without memory, if enough computational resources are provided?" Instead of the prestigious Kalman filter (KF) and its derivatives to estimate the mean and the covariance matrix of the states, we resort to solving the Duncan-Mortensen-Zakai (DMZ) equation, which is satisfied by the un-normalized probability density function of the states. In this dissertation, we develop a novel algorithm, which is applicable to the most general settings of the NLF problems and keeps two of the most important properties of KF: real-time and memory-less. Briefly speaking, in our algorithm, we split the approximation of the conditional density function into two parts: one part could be pre-computed before any on-line experiments ran (so-called off-line computation); the other part has to be synchronized the real-time data with the pre-computed data (so-called on-line computation). More precisely, the off-line computation solves a forward Kolmogorov equation (FKE) with the initial conditions, which are chosen to be a complete base

functions in square-integrable function space, while the on-line part computes the projection of the conditional density function at each time step onto the basis, and then synchronize them with the off-line data to obtain the conditional density function at the next time step. First, we validate our algorithm theoretically, by estimating the convergence rate with respect to the sampling frequency. Second, we tackle some difficulties in the implementation of our algorithm and apply it to some 1-D benchmark NLF problems. Compared with the two most widely used methods nowadays, extended Kalman filter and particle filters, our algorithm surpasses both of them in the real-time manner with comparable accuracy. Last, when we investigate the application of our algorithm to the high-dimensional state NLF problems, we combine the sparse grid algorithm with the Hermite spectral method to serve as the off-line solver of FKE. The convergence rate is investigated both theoretically and numerically.

Advisor: Stephen S.-T. Yau
Current affiliation: Lecturer, Beihang University

Shuang Miao, University of Chinese Academy of Sciences

Thesis title: Branching random walks with selection

Abstract: We consider the classical compressible Euler's Equations in three space dimensions with an arbitrary equation of state, and whose initial data corresponds to a constant state outside a sphere. Under suitable restriction on the size of the initial departure from the constant state, we establish theorems which give a complete description of the maximal development. In particular, the boundary of the domain of the maximal solution contains a singular part where the inverse density of the wave fronts vanishes and the shocks form. We obtain a detailed description of the geometry of this singular boundary and a detailed analysis of the behavior of the solution there.

Advisor: Demetrios Christodoulou
Current affiliation: Post-Doc Assistant Professor, University of Michigan

Tian Yang, Rutgers University – New Brunswick

Thesis title: Branching random walks with selection

Abstract: This dissertation is based on a joint work with Dr. Julien Roger. We define an associative $\mathbb{C}[[h]]$–algebra $\mathcal{AS}_h(\Sigma)$ generated by framed arcs and links over a punctured surface Σ which is a quantization of the Poisson algebra $C(\Sigma)$ of arcs and curves on Σ. We also construct a Poisson algebra homomorphism from $C(\Sigma)$ to the space of smooth functions on the decorated Teichmüller space endowed with the Weil-Petersson Poisson structure. The construction relies on a collection of geodesic lengths identities in hyperbolic geometry which generalizes Penner's Ptolemy relation, the trace identity and Wolpert's cosine formula.

Advisor: Feng Luo
Current affiliation: Assistant Professor, Stanford University

Recipients of the 2015 New World Mathematics Awards

Doctor Thesis Awards, Gold Prize

Shen Lin, Université Paris-Sud

Thesis title: Tree-indexed random walk and random walk on trees

Abstract: The aim of this Ph.D. thesis is to study several probabilistic models linking the random walks and the random trees arising from critical branching processes. In the first part, we consider the model of random walk taking values in a Euclidean lattice and indexed by a critical Galton–Watson tree conditioned by the total progeny. Under some assumptions on the critical offspring distribution and the centered jump distribution, we obtain, in all dimensions, the asymptotic growth rate of the range of this random walk, when the size of the tree tends to infinity. These results also allow us to describe the asymptotic behavior of the range of a branching random walk, when the size of the initial population goes to infinity. In parallel, we treat likewise some cases where the random walk has a non-zero constant drift. In the second part, we focus on the fractal properties of the harmonic measure on large critical Galton–Watson trees. By harmonic measure, we mean the exit distribution from a ball centered at the root of the tree by simple random walk on this tree. If the critical off spring distribution is in the domain of attraction of a stable distribution, we prove that the mass of the harmonic measure is asymptotically concentrated on a boundary subset of negligible size with respect to that of the boundary. Assuming that the critical offspring distribution has a finite variance, we are able to calculate the mass of the harmonic measure carried by a random vertex uniformly chosen from the boundary.

Advisor: Jean-François LE GALL
Current affiliation: Assistant professor, Ecole Normale Supérieure

Ziming Nikolas Ma, The Chinese University of Hong Kong

Thesis title: From Witten-Morse theory to Mirror symmetry

Abstract: Witten-Morse view the stable submanifold from a critical point of a Morse function f as semi-classical limit of an eigenform with respect to Witten Laplacian Δ_f. We prove an enhancement of this correspondence in Chapter 1 by showing operations on eigenforms, involving taking wedge product and Witten's twisted Green's operator G_f successively, has semi-classical limit counting gradient trees between critical points. The main tool is the semi-classical analysis for the Witten's twisted Green's operator.

Mirror symmetry is a duality relating A-model on a Calabi-Yau manifold X to B-model on its mirror $\check{X}$. Limiting to their large structure limits we obtain (non-compact) semi-flat mirror pairs which are torus bundle X_0 and $\check{X}_0$ over a base B_0. Strominger-Yau-Zaslow's approach suggests that fiberwise Fourier transform over the base, given in Chapter 2, is responsible for the mirror correspondence. In particular, the deformation theory from the large structure limits which is responsible for capturing X and should be identified via semi-flat transform.

We propose a differential graded Lie algebra (dgLa) $L^*_{X_0}$ on fiberwise loop space of X_0, which is transformed to Kodaira-Spencer dgLa on $\check{X}_0$, to capture the quantum deformation of X_0 that is necessary to recover X which is believed to have informations from holomorphic disk instantons. In Chapter 3, we interpret holomorphic disk instantons as semi-classical limit of 1-form on loop space, motivated from Witten-Morse theory. We prove that solving Maurer-Cartan equation in $L^*_{X_0}$ has semi-classical limit as the scattering process introduced by Kontsevich-Soibelman in [24], which is known to govern the deformation from X_0 to $\check{X}$.

Advisor: Naichung Conan Leung
Current affiliation: Assistant professor, National Taiwan University

Doctor Thesis Awards, Silver Prize

Cheng-Chiang Tsai, Harvard University

Thesis title: A Formula for Some Shalika Germs

Abstract: In this thesis, for nilpotent orbits of ramified quasi-split unitary groups with two Jordan blocks, we give the values of their Shalika germs at certain equi-valued elements with half-integral depth previously studied by Hales. These elements are parametrized by hyperelliptic curves defined over the residue field, and the values we obtain can be expressed in terms of Frobenius eigenvalues on the ℓ-*adic* H^1 of these curves, generalizing previous result of Hales on subregular Shalika germs. Using the Shalika germ formulas, we obtain some new results on stability and endoscopic transfer of nilpotent orbital integrals.

Advisor: Benedict Hyman Gross
Current affiliation: CLE Moore instructor at MIT

Jingyue Chen, Brandeis University

Thesis title: Existence and Rigidity of Calabi-Yau Bundles

Abstract: A Calabi-Yau (CY) bundle on a compact complex manifold X was a crucial ingredient in constructing differential systems for period integrals in [LY], by lifting line bundles from the base X to the total space. A question was therefore raised as to whether there exists such a bundle that supports the liftings of all line bundles from X, simultaneously. This was a key step for giving a uniform construction of differential systems for arbitrary complete intersections in X. In this dissertation, the existence question is answered in the affirmative if X is assumed to be Kähler, and also in general if the Picard group of X is assumed to be free. Furthermore, a rigidity property of CY bundles is proved if the principal group is an algebraic torus, showing that such a CY bundle is essentially determined by its character map.

Advisor: Bong H. Lian
Current affiliation: Post-doctoral, Yau Mathematical Sciences Center

Zhijian He, Tsinghua University

Thesis title: High Dimensionality and Discontinuity: New Challenges in QMC for Quantitative Finance

Abstract: Many finance problems cannot be solved analytically due to the complexity of financial model and the diversity of financial products. This thesis develops quasi-Monte Carlo (QMC) methods for solving these problems approximately. However, the efficiency of QMC methods depends highly on the dimension and smoothness of the target functions. High dimensionality and discontinuity in finance problems are new challenges for QMC methods. In order to improve the efficiency of QMC for finance problems, this thesis proposes some new methods for overcoming the two difficulties.

Path generations of underlying assets are crucial for the implementation of QMC methods. Different path generation methods could have an impact on the efficiency of QMC. With a special concern of discontinuity structures involved in finance problems (such as the pricing and hedging of financial derivatives), this thesis develops a new path generation method – QR method. The method transforms the discontinuity structures such that the discontinuity surfaces are parallel to as many as coordinate axes as possible. In doing so, the integrands become "QMC-friendly". Additionally, we propose a measurement for quantifying

the importance of different discontinuity structures. Numerical results show that the QR method can improve significantly the efficiency of QMC for pricing some exotic options.

Due to the impact of discontinuity structures on QMC methods, this thesis proposes a new smoothing method for removing the discontinuities. In order to make the smoothing method applicable for some common finance problems, we develop a modified QR method. The combination of the modified QR method and the smoothing method can weaken the adverse effects of high dimensionality and discontinuity simultaneously. Numerical results show that the combined method can reduce significantly the effective dimension, and improve markedly the efficiency of QMC.

This thesis also studies the convergence rate of randomized quasi-Monte Carlo (RQMC) methods for discontinuous functions. For certain discontinuous functions, we prove that for arbitrary >0, the root mean squared error (RMSE) of RQMC is $O(n^{-1/2-1/(4d-2)+\epsilon})$, where d is the dimension, and n is the sample size. It was previously known that the rate is only $O(n^{-1/2})$. If the discontinuity surface is parallel to some coordinate axes, we can get a faster rate $O(n^{-1/2-1/(4d_u-2)+\epsilon})$, where d_u is the "irregular dimension" (the number of axes to which discontinuity surface is parallel). Numerical results show better estimated rates than the theoretical rates, especially for low-dimensional discontinuous functions. Moreover, the estimated rates deteriorate greatly with increasing dimension d or irregular dimension d_u. These insights are in line with the theoretical rates.

Finally, this thesis makes use of Hilbert's space-filling curve to transform d-dimensional quadrature rules into one-dimensional quadrature rules. We prove that the randomized quadrature rules based on Hilbert's space-filling curve yield an RMSE of $O(n^{-1/2-1/d})$ for Lipschitz functions, when $d \geq 3$. For certain discontinuous functions, the RMSE turns out to be $O(n^{-1/2-1/(2d)})$, which is better than the aforementioned rate of RQMC.

Advisor: Xiaoqun Wang
Current affiliation: Associate Research Fellow, Sun Yat-sen University

Chen Jiang, University of Tokyo

Thesis title: On Boundedness of Volumes and Birationality in Birational Geometry

Abstract: This is a Ph.D. thesis submitted to Graduate School of Mathematical Sciences, the University of Tokyo. Throughout this thesis, we work over the field of complex numbers $\mathbb{C}$. We adopt the standard notations and definitions in [KaMaMa87] and [KoMo98], and will freely use them.

The aim of birational geometry is to classify all varieties up to birational equivalence. According to Minimal Model Program, minimal varieties and Fano varieties with mild singularities form fundamental classes in birational geometry. To understand these special classes of varieties, it is very natural and interesting to prove some boundedness results. The goal of this thesis is to collect my recent works in birational geometry centered around the theme of boundedness.

Chapter 1 contains a brief summary of the motivations, main problems, histories, and main results on boundedness of volumes and birationality.

Chapter 2 provides basic knowledge on volumes, Hirzebruch surfaces, nonklt centers, connectedness lemma, rational map defined by a Weil divisor, Reid's Riemann-Roch formula, and so on. Basic lemmas are also provided to support the following chapters.

Chapter 3 focuses on the boundedness of anti-canonical volumes. We prove Weak Borisov-Alexeev-Borisov Conjecture in dimension three which states that the anti-canonical volume of an ϵ-klt log Fano pair of dimension three is bounded from above. As a corollary, we give a different proof of boundedness of log Fano threefolds of fixed index.

Chapters 4 and 5 are devoted to the boundedness of birationality.

In Chapter 4, we investigate the pluri-anti-canonical linear systems of weak $\mathbb{Q}$-Fano 3-folds. We prove that, for a $\mathbb{Q}$-Fano 3-fold X, $|-mK_X|$ gives a birational map for $m \geq 39$, and for a weak $\mathbb{Q}$-Fano 3-fold X, $|-mK_X|$ gives a birational map for $m \geq 97$. We also consider the generic finiteness and prove that for a $\mathbb{Q}$-Fano 3-fold X, $|-mK_X|$ gives a generically finite map for $m \geq 28$. Plenty of examples are provided for discussing the optimality of these results.

In Chapter 5, we investigate minimal 3-fold X with numerically trivial canonical divisor and a nef and big Weil divisor L on X. We prove that $|mL|$ and $|K_X + mL|$ give birational maps for $m \geq 17$.

Chapters 3 and 5 are based on my preprints [Jiang14b, Jiang14a]. Chapter 4 is based on a joint work with Meng Chen [CJ14].

Advisor: Yujiro Kawamata
Current affiliation: Ostdoctoral Research Fellow, The University of Tokyo

Han Li, Yale University

Thesis title: Some effective results in homogeneous dynamics and number theory

Abstract: In the thesis we prove some effective results in homogeneous dynamics and number theory. The thesis consists of two major parts. In the first part, we study the distribution of the Frobenius numbers. The Frobenius number F(a) of a lattice point a in $\mathbb{R}^d$ with positive coprime coordinates, is the largest integer which can not be expressed as a non-negative integer linear combination of the coordinates of a. Marklof has recently proved the existence of the limit distribution of the Frobenius numbers, when a is taken to be random in an enlarging domain in $\mathbb{R}^d$. We will show that if the domain has piecewise smooth boundary, the error term for the convergence of the distribution function is at most a polynomial in the enlarging factor. We show that certain estimates in the classical reduction theory for integral quadratic forms can be improved, in an effective way, as long as the quadratic form is primitive and indefinite. The second part of the thesis is based on the joint work with my advisor Professor G. A. Margulis.

Advisor: Gregory A. Margulis
Current affiliation: Assistant Professor, Wesleyan University

Master Thesis Awards, Silver Prize

Qiuxia Liu, University of Science & Technology Beijing

Thesis title: Factor complexity and a permutation similar to the 3-fold Morse sequence

Abstract: Some combinatorial properties of the 3-fold Morse sequence are considered and the factor complexity formula is established. Furthermore, another sequence is constructed by iterating a morphism and some properties of it will be investigated. Finally we show that the above two sequences generate the same infinite permutation.

Advisor: Niu Min

Recipients of the 2016 New World Mathematics Awards

Doctor Thesis Awards, Gold Prize

Tony Yue Yu, Université Paris Diderot – Paris 7

Thesis Title: First steps of non-archimedean enumerative geometry

Abstract: In this thesis, we establish the first steps of non-archimedean enumerative geometry. We present several new results in tropical geometry and in Berkovich geometry. Our motivation comes from the study of mirror symmetry, especially from the non-archimedean approach suggested by Kontsevich-Soibelman. We start by studying tropicalization in a global setting, because the classical setting is not sufficient for our purposes. We prove the generalized balancing condition in terms of the intersection theory. Then, we pass to curves in families. We construct the moduli space of non-archimedean stable maps, introduce the Kähler structure, and establish the non-archimedean Gromov compactness theorem. Concerning the tropicalization of analytic curves in families, we prove the theorem of continuity and polyhedrality. We also include a foundation for higher analytic stacks, and prove analogs of Grauert's theorem and Serre's GAGA theorem. All these general theorems converge to a concrete application: the enumeration of holomorphic cylinders in log Calabi-Yau surfaces. This gives rise to new geometric invariants. An explicit computation is given for a del Pezzo surface in detail, which verifies the conjectured wall-crossing formula. Our tools include Berkovich spaces, formal models, étale cohomology, vanishing cycles, intersection theory, Artin's representability criterion, the geometry of stable curves, rigid subanalytic sets, Gromov-Witten theory and infinity categories.

Advisor: Maxim Kontsevich
Current affiliation: Research Fellow, Clay Mathematics Institute

Yunqing Tang, Harvard University

Thesis Title: Algebraicity criteria and their applications

Abstract: We use generalizations of the Borel–Dwork criterion to prove variants of the Grothedieck–Katz p-curvature conjecture and the conjecture of Ogus for some classes of abelian varieties over number fields. We prove a variant of this conjecture for the affine line minus two points, which asserts that if the equation satisfies a certain convergence condition for all primes, then its monodromy is trivial. We deduce from this a description of the differential Galois group of the equation in terms of p-curvatures and certain p-adic monodromy groups. We also prove similar variants of the p-curvature conjecture for a certain elliptic curve with j-invariant 1728 minus its identity. Ogus defined a class of cycles in the de Rham cohomology of smooth proper varieties over number fields. This notion is a crystalline analogue of l-adic Tate cycles. Ogus predicted that such cycles coincide with Hodge cycles for abelian varieties. We confirm Ogus' conjecture for some classes of abelian varieties, under the assumption that these cycles lie in the Betti cohomology with real coefficients. These classes include abelian varieties of prime dimension that have nontrivial endomorphism ring.

Advisor: Mark Kisin
Current affiliation: Member at IAS

Jian Xiao, Fudan University & Université Grenoble Alpes

Thesis Title: Positivity in Kähler geometry

Abstract: The goal of this thesis is to study various positivity in Kähler geometry. In particular, for a compact Kähler manifold of dimension n, we study the positivity of transcendental $(1,1)$ and $(n-1,n-1)$ classes. These classes include the divisor classes and curve classes over smooth complex projective varieties. In Chapter 3, we solve Teissier's proportionality theorem in Kähler geometry, and apply the main result to non-Kähler metrics. In Chapter 4, we give a partial solution to Demailly's conjecture on transcendental Morse inequality. We also give similar results for transcendental movable $(n-1,n-1)$ classes. We apply these results to study the numerical characterization problem on the convergence of the inverse σ_k-flow, giving some partial results towards the conjecture of Lejmi and Székelyhidi. In Chapter 5, by using the duality of cones, we give a new characterization of the volume of divisors. Inspired by this result, we study a new volume type function for $(n-1,n-1)$ classes on Kähler manifolds. In Chapter 6, we develop a theory of Zariski decomposition for curves on algebraic varieties by Legendre-Fenchel type transforms, and study the positivity of curve classes.

Advisor: Jean-Pierre Demailly & Jixiang Fu
Current affiliation: Northwestern University

Master Thesis Awards-Gold Prize

Jiaming Chen, Tsinghua University

Thesis Title: On the stable cohomology of the Satake compactification and its mixed Hodge structure

Abstract: In this paper we are concerned with the stable cohomology of the Satake compactification of the coarse moduli space of principally polarized abelian varieties and its mixed Hodge structure. In particular, we provide a more transparent proof of the Charney-Lee stability theorem using the categorical description of the rational homotopy type of $\mathcal{A}_g^{bb}$ and determine the Hodge type of the α-classes.

Advisor: Eduard Looijenga
Current affiliation: Ph.D., Université Pierre et Marie Curie

Doctor Thesis Awards, Silver Prize

Cheuk Yu Mak, University of Minnesota

Thesis Title: Rigidity of symplectic fillings, symplectic divisors and Dehn twist exact sequences

Abstract: We present three different aspects of symplectic geometry in connection to complex geometry. Convex symplectic manifolds, symplectic divisors and Lagrangians are central objects to study on the symplectic side. The focus of the thesis is to establish relations of these symplectic objects to the corresponding complex analytic objects, namely Stein fillings, divisors and coherent sheaves, respectively. Using pseudoholomorphic curve techniques and Gauge theoretic results, we systematically study obstructions to symplectic/Stein fillings of contact 3-manifolds arising from the rigidity of closed symplectic four-manifolds with non-positive Kodaira dimension. This perspective provides surprising consequences which, in particular, captures a new rigidity phenomenon for exact fillings of unit cotangent bundle of orientable surfaces and recovers many known results in a uniform way. The most important source of Stein fillings comes from smoothing of a complex isolated singularities. This motivates us to study when a symplectic divisor admits a convex/concave neighborhood and we obtain a complete and very computable answer to this local behaviour of symplectic divisors. Globally speaking, symplectic divisors in a closed symplectic manifold that represent its first Chern class are of particular importance in mirror symmetry. Such a symplectic divisor, together with the closed symplectic manifold together is called a symplectic log Calabi-Yau surface. We obtain a complete classification of symplectic log Calabi-Yau surface up to isotopy of symplectic divisors. Finally, we study algebraic properties of Fukaya category on the functor level and utilize Biran-Cornea's Lagrangian cobordism theory and Mau-Wehrheim-Woodward functor to provide a partial proof of Huybrechts-Thomas's conjecture.

Advisor: Tian-Jun Li
Current affiliation: Member at IAS

Teng Fei, MIT

Thesis Title: On the Geometry of the Strominger System

Abstract: The Strominger system is a system of partial differential equations describing the geometry of compactifications of heterotic superstrings with flux. Mathematically it can be viewed as a generalization of Ricci-flat metrics on non-Kähler Calabi-Yau 3-folds. In this thesis, I will present some explicit solutions to the Strominger system on a class of noncompact Calabi-Yau 3-folds. These spaces include the important local models like $\mathbb{C}^3$ as well as both deformed and resolved conifolds. Along the way, I also give a new construction of non-Kähler Calabi-Yau 3-folds and prove a few results in complex geometry.

Advisor: Shing-Tung Yau
Current affiliation: Ritt Assistant Professor, Columbia University

Chao Gao, Yale University

Thesis Title: Frequentist Justifications for Bayes Procedures

Abstract: Bayes procedures are widely used in statistics. This thesis studies their frequentist justifications. The goal is to establish posterior contraction when data is sampled from some true distribution. Problems that are related to adaptation, non-intrinsic loss functions and high-dimensional statistics are going to be addressed.

Advisor: Harrison Zhou
Current affiliation: Assistant professor, University of Chicago