Homology, Homotopy and Applications, vol. 16(2), 2014, pp.159–169

RAMIFIED COVERINGS OF SMALL CATEGORIES

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(*communicated by Claude Cibils*)

Abstract

We introduce ramified coverings of small categories, and we prove three properties of the notion: the Riemann-Hurwitz formula holds for a ramified covering of finite categories, the zeta function of B divides that of $\widetilde{E}$ for a ramified covering $\widetilde{P}\colon \widetilde{E} \to B$ of finite categories, and the nerve of a d-fold ramified covering of small categories is also a simplicial d-fold ramified covering.

1. Introduction

A covering is an interesting and important tool in algebraic topology. For example, a covering space is used for computing fundamental groups; furthermore, there exists an analogy between Galois theory and covering space theory (see, for example, [**Hat02**] and [**May99**]). A covering space should be called an "unramified covering." A ramified covering for topological spaces has been defined by Smith [**Smi83**] and Dold [**Dol86**], and a well-known example for ramified coverings is the one for Riemann surfaces. Ramified coverings for Riemann surfaces have important properties; for example, the Riemann-Hurwitz formula that states a relationship between the Euler characteristic of a total space and base space.

In this paper, we define a ramified covering of small categories. An unramified covering of small categories has already been defined, and several authors have studied it. Bridson and Haefliger presented many important properties of unramified coverings in [**BH99**]; for example, the monodromy theorem and the path lifting theorem. May studied unramified coverings of groupoids [**May99**]. Tanaka defined a model structure on the category of small categories, called the one-type model structure [**Tan13**]. An unramified covering is a fibration in the sense of the one-type model structure. Cibils and MacQuarrie studied Galois coverings of small categories [**CM**].

In this paper, we show that ramified coverings of small categories have several desirable properties.

The first result is an analogue of the Riemann-Hurwitz formula for Riemann surfaces (Theorem 2.5). For a ramified covering $p\colon \widetilde{X} \to X$ of Riemann surfaces under

Received March 29, 2013, revised July 25, 2013, April 29, 2014; published on September 1, 2014.
2010 Mathematics Subject Classification: 18G30, 55R05, 55U10.
Key words and phrases: ramified covering of small categories, zeta function of a finite category, Euler characteristic of categories.
Article available at http://dx.doi.org/10.4310/HHA.2014.v16.n2.a8

certain conditions, the following formula is popularly recognized as the Riemann-Hurwitz formula:

$$\chi(\widetilde{X}) = d \cdot \chi(X) - V,$$

where d is the degree of p, χ is the Euler characteristic of Riemann surfaces, and

$$V = \sum_{\tilde{x} \in \widetilde{X}} (e(\tilde{x}) - 1).$$

Here, $e(\tilde{x})$ is the ramification number of p at $\tilde{x}$.

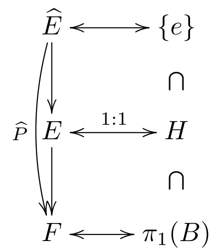
The Euler characteristic for categories has varying definitions. Leinster defined the *Euler characteristic of a finite category* in [**Lei08**], which is the first Euler characteristic for categories. Later, several authors defined the *Euler characteristic*: the *series Euler characteristic* by Berger-Leinster [**BL08**], the *L^2-Euler characteristic* by Fiore-Lück-Sauer [**FLS11**], the *extended L^2-Euler characteristic* [**NogB**], and the *Euler characteristic of $\mathbb{N}$-filtered acyclic categories* by the author [**Nog11**]. See [**NogB**] for relationships among them. In this paper, we only prove that the Riemann-Hurwitz formula holds for the series Euler characteristic. Accordingly, Theorems 5.30 and 5.37 of [**FLS11**] are analogues for unramified coverings of groupoids, isofibrations, and the L^2-Euler characteristic, and Proposition 2.8 of [**Lei08**] is an analogue for Grothendieck fibrations and Leinster's Euler characteristic of a finite category. Moreover, the graph-theoretic analogue of the Riemann-Hurwitz formula is considered in [**BN09**].

The second result indicates that for a ramified covering of finite categories $\widetilde{P} \colon \widetilde{E} \to B$, the zeta function of B divides that of $\widetilde{E}$ (Theorem 2.6). This is a generalization of Theorem 4.5 of [**NogA**]. The graph-theoretic analogue of this result is also considered in [**MM10**], [**ST96**], and [**Ter11**].

This result is a categorical analogue of the Dedekind conjecture that states that if K_1 and K_2 are number fields and $K_1 \subset K_2$, then the Dedekind zeta function of K_1 divides that of K_2. A covering of small categories is an analogy of Galois theory. The fundamental theorem of Galois theory states that if K/F is a finite Galois extension, the set of intermediate fields of K and F is naturally bijective to the set of subgroups of the Galois group $\mathrm{Gal}(K/F)$:

$$\begin{array}{ccc} K & \longleftrightarrow & \{e\} \\ \Big| & & \cap \\ L & \overset{1:1}{\longleftrightarrow} & \mathrm{Gal}(K/L) \\ \Big| & & \cap \\ F & \longleftrightarrow & \mathrm{Gal}(K/F). \end{array}$$

For a covering of small categories $\widehat{P} \colon \widehat{E} \to B$, where $\widehat{E}$ is the universal covering of B, the set of the isomorphism classes of intermediate coverings of $\widetilde{P}$ is bijective to the set of subgroups of the fundamental group $\pi_1(B)$:



(see Corollary 2.24 of [**Tan13**]). We have the correspondences

$$\begin{aligned} \text{coverings} &\leftrightarrow \text{extension of fields} \\ \pi_1 &\leftrightarrow \text{Galois groups} \\ \text{intermediate coverings} &\leftrightarrow \text{intermediate fields.} \\ \vdots \quad &\quad \vdots \end{aligned}$$

From the above diagrams, we can conclude that the relationship between zeta functions and coverings of small categories is an analogue of the Dedekind conjecture.

For an unramified covering $P\colon E \to B$, it is known that $N_*(P)$ is a simplicial covering [**GZ67**], where N_* is the nerve functor from the category of small categories to the category of simplicial sets. The third result indicates that for a d-fold ramified covering $\widetilde{P}$ of small categories, $N_*(\widetilde{P})$ is also a simplicial d-fold ramified covering [**AP08**, Theorem 2.8]. As a consequence, we show that the classifying space $B\widetilde{P}$ is a d-fold ramified covering in the sense of Dold [**Dol86**].

Acknowledgments

I thank the very helpful referee.

2. Ramified coverings of small categories

2.1. Notation and terminology

Before we introduce a ramified covering of small categories, let us recall unramified coverings of small categories (p. 579 of [**BH99**], Definition 3.1 of [**CM**], Definition 2.8 of [**Tan13**]).

Let C be a small category. For an object x of C, let $S(x)$ be the set of morphisms of C whose source is x:

$$S(x) = \{f\colon x \to * \in \mathrm{Mor}(C)\},$$

and $T(x)$ be the set of morphisms of C whose target is x:

$$T(x) = \{g\colon * \to x \in \mathrm{Mor}(C)\}.$$

A category C is *connected* if C is a nonempty category and there exists a zig-zag sequence of morphisms in C

$$x \xrightarrow{f_1} x_1 \xleftarrow{f_2} x_2 \xrightarrow{f_3} \cdots \xleftarrow{f_n} y$$

for any objects x and y of C. We do not need to consider the direction of the last morphism f_n since we can insert an identity morphism into the sequence.

A functor $P\colon E \to B$ is an *unramified covering* if B is connected and the following two restrictions of P are bijections for any object x of E:

$$P\colon S(x) \longrightarrow S(P(x))$$

$$P\colon T(x) \longrightarrow T(P(x)).$$

It is easy to show that our definition of unramified coverings is equivalent to the definition of coverings in [**Tan13**]. This condition is an analogue of the condition of an unramified covering of graphs (see [**ST96**]).

2.2. Definition

Definition 2.1. Let $P\colon E \to B$ be an unramified covering. An equivalence relation $\sim$ on the set of objects $\mathrm{Ob}(E)$ is called *for ramifications* of P if the following conditions are satisfied:

1. If $x \sim y$, then x and y belong to the same fiber
$$P^{-1}(b) = \{z \in \mathrm{Ob}(E) \mid P(z) = b\}$$
for some object b of B.
2. If $x \sim y$ and $x \neq y$, then $S(x) = \{1_x\}, S(y) = \{1_y\}$ or $T(x) = \{1_x\}, T(y) = \{1_y\}$.

Definition 2.2. Suppose that $P\colon E \to B$ is an umramified covering and $\sim$ is an equivalence relation for ramifications of P. Define a category $\widetilde{E}$ as follows: The set of objects of $\widetilde{E}$ is $\mathrm{Ob}(E)/\sim$. For objects $[x]$ and $[y]$ of $\widetilde{E}$, define

$$\mathrm{Hom}_{\widetilde{E}}([x],[y]) = \coprod_{x_0 \in [x],\, y_0 \in [y]} \mathrm{Hom}_E(x_0, y_0).$$

The composition of E naturally induces that of $\widetilde{E}$.

Define a functor $\widetilde{P}\colon \widetilde{E} \to B$ by $\widetilde{P}([x]) = P(x)$ for any object $[x]$ of $\widetilde{E}$ and $\widetilde{P}(f) = P(f)$ for any morphism f of $\widetilde{E}$. We call $\widetilde{P}$ the *ramified covering* of P by $\sim$.

For an object $[x]$ of $\widetilde{E}$, we define the *ramification number* of $[x]$ by its cardinality. It is clear that there exists the following bijection:

$$\bigcup_{[y] \in \widetilde{P}^{-1}(b)} [y] \cong P^{-1}(b)$$

for an object b of B. Define the *degree* of $\widetilde{P}$ by the cardinality of $P^{-1}(b)$. Since P is an unramified covering and B is connected, this definition does not depend on the choice of b.

For a natural number d, $\widetilde{P}$ is *d-fold* if $\deg \widetilde{P} = d$.

Example 2.3. We introduce two simple examples.

1. An unramified covering is a ramified covering of itself by the trivial equivalence relation.
2. Let E be the category

$$x_1 \xrightarrow{f_1} y_1$$

$$x_2 \xrightarrow{f_2} y_2$$

$$x_3 \xrightarrow{f_3} y_3.$$

Let $B = a \xrightarrow{f} b$. Define a functor $P\colon E \to B$ by $P(f_i) = f$ for any i. Then P is an unramified covering. Define an equivalence relation for ramifications of P by $x_1 \sim x_2$ and $y_1 \sim y_2 \sim y_3$. Then $\widetilde{E}$ is the category

$$\begin{array}{ccc} [x_1] & \overset{f_1}{\underset{f_2}{\rightrightarrows}} & [y_1]. \\ & \nearrow_{f_3} & \\ [x_3] & & \end{array}$$

The ramification numbers of $[x_1], [x_3]$, and $[y_1]$ are two, one, and three, respectively.

2.3. Proof of main theorem

In this subsection, we present the proof of our main theorem.

Throughout this section, $\widetilde{P}\colon \widetilde{E} \to B$ is a ramified covering of an unramified covering $P\colon E \to B$ by an equivalence relation for ramifications $\sim$ of P.

For a small category C, let $N_n(C)$ be the set of chains of morphisms in C of length n:

$$N_n(C) = \{\ (x_0 \xrightarrow{f_1} x_1 \xrightarrow{f_2} \cdots \xrightarrow{f_n} x_n)\ \text{ in } C\},$$

and let $\overline{N_n}(C)$ be the set of nondegenerate chains of morphisms in C of length n:

$$\overline{N_n}(C) = \{\ (x_0 \xrightarrow{f_1} x_1 \xrightarrow{f_2} \cdots \xrightarrow{f_n} x_n)\ \text{ in } C \mid f_i \neq 1\}.$$

The difference between them is that one of them uses identity morphisms whereas the other does not. For $n = 0$, we set $N_0(C) = \overline{N_0}(C) = \mathrm{Ob}(C)$.

Lemma 2.4. *Suppose that the categories E and B are finite. Then we obtain the following results.*

1. *For any $n \geqslant 1$,*

$$\#\overline{N_n}(\widetilde{E}) = \#\overline{N_n}(E).$$

2. For any $n \geqslant 0$,

$$\#N_n(\widetilde{E}) = \#N_n(E) - V,$$

where

$$V = \sum_{[x] \in \mathrm{Ob}(\widetilde{E})} (\#[x] - 1).$$

Proof. For

$$\mathbf{f} = [x_0] \xrightarrow{f_1} [x_1] \xrightarrow{f_2} \cdots \xrightarrow{f_n} [x_n]$$

of $\overline{N_n}(\widetilde{E})$, each $[x_i]$ is a one-point set for $1 \leqslant i \leqslant n-1$, and f_1 and f_n belong to $\mathrm{Hom}_E(x'_0, x_1)$ and $\mathrm{Hom}_E(x_{n-1}, x'_n)$ for some objects x'_0 and x'_n of E, respectively. Define $\varphi_n \colon \overline{N_n}(\widetilde{E}) \to \overline{N_n}(E)$ by

$$\varphi_n(\mathbf{f}) = \; x'_0 \xrightarrow{f_1} x_1 \xrightarrow{f_2} \cdots \xrightarrow{f_n} x'_n.$$

Then it is clear that φ_n is a bijection.

When $n = 0$, the second result is clear. If $n \geqslant 1$, the first result and Lemma 2.10 of [**NogA**] imply the second result. □

A finite category C has *series Euler characteristic* if we can substitute $t = -1$ in the rational function

$$\frac{\mathrm{sum}(\mathrm{adj}(I - (A_C - I)t))}{|I - (A_C - I)t|},$$

where I is the unit matrix, A_C is the adjacency matrix of C, $\mathrm{sum}(\cdot)$ denotes the summation of all the entries of the matrix, and $\mathrm{adj}(M)$ is the adjugate matrix of a matrix M [**BL08**]. In this case, the *series Euler characteristic* $\chi_{\sum}(C)$ of C is defined by the value of the rational function at $t = -1$. The rational function is the analytic continuation of the power series $\sum_{n=0}^{\infty} \#\overline{N_n}(C)t^n$ (Theorem 2.2 of [**BL08**]).

Theorem 2.5 (Riemann-Hurwitz for finite categories)**.** *Suppose that $\widetilde{P}$ is a d-fold ramified covering of finite categories. Then $\widetilde{E}$ has series Euler characteristic if and only if B has series Euler characteristic. In this case,*

$$\chi_{\sum}(\widetilde{E}) = d \cdot \chi_{\sum}(B) - V,$$

where

$$V = \sum_{[x] \in \mathrm{Ob}(\widetilde{E})} (\#[x] - 1).$$

Proof. By Lemma 2.4, we have

$$\sum_{n=0}^{\infty} \#\overline{N_n}(\widetilde{E})t^n = \left(\sum_{n=0}^{\infty} \#\overline{N_n}(E)t^n \right) - V.$$

Hence, $\widetilde{E}$ has series Euler characteristic if and only if E has series Euler characteristic.

In this case, we have

$$\chi_{\sum}(\widetilde{E}) = \chi_{\sum}(E) - V.$$

Proposition 4.10 of [**NogA**] completes this proof. □

Let C be a finite category. Then the *zeta function* $\zeta_C(z)$ of C is defined by

$$\zeta_C(z) = \exp\left(\sum_{n=1}^{\infty} \frac{\#N_n(C)}{n} z^n\right)$$

(see [**NogA**]). This function belongs to the power series ring $\mathbb{Q}[[z]]$. If one prefers, the zeta function can be considered as a function of a complex variable by choosing z to be a sufficiently small complex number.

Theorem 2.6. *Suppose that $\widetilde{P}$ is a d-fold ramified covering of finite categories. Then we have*

$$\zeta_{\widetilde{E}}(z) = \zeta_B(z)^d (1-z)^V.$$

Proof. By Lemma 2.4, we have

$$\begin{aligned}
\zeta_{\widetilde{E}}(z) &= \exp\left(\sum_{n=1}^{\infty} \frac{\#N_n(\widetilde{E})}{n} z^n\right) \\
&= \exp\left(\sum_{n=1}^{\infty} \frac{\#N_n(E)}{n} z^n - V \sum_{n=1}^{\infty} \frac{1}{n} z^n\right) \\
&= \zeta_E(z) \exp(V \log(1-z)) \\
&= \zeta_E(z)(1-z)^V.
\end{aligned}$$

Note that $\sum_{n=1}^{\infty} \frac{1}{n} z^n = -\log(1-z)$. Theorem 4.5 of [**NogA**] completes this proof. □

Let us recall the definition of simplicial ramified coverings [**AP08**]. A *simplicial set* X consists of a graded set $\{X_n\}_{n \geqslant 0}$, *face operators* $\partial_i \colon X_n \to X_{n-1}$, and *degeneracy operators* $s_i \colon X_n \to X_{n-1}$ for $0 \leqslant i \leqslant n$, which satisfy the simplicial relations (see [**May92**]). Let $p \colon X \to Y$ be a map of simplicial sets. We say that p is a *simplicial d-fold ramified covering map* if the following hold:

1. For each n, $p_n \colon X_n \to Y_n$ has finite fibers.
2. The restricted function $\partial_i|_{p_n^{-1}(y)} \colon p_n^{-1}(y) \to p_{n-1}^{-1}(\partial_i(y))$ is surjective for all i.
3. There is a family of *multiplicity functions* $\mu_n \colon X_n \to \mathbb{N}$, such that
   (a) For all y of Y_n, one has $\sum_{x \in p_n^{-1}(y)} \mu_n(x) = d$.
   (b) $\mu_{n+1} \circ s_i = \mu_n$.
   (c) For all y of Y_n and x of $p_n^{-1}(y)$, one has $\mu_{n-1}(\partial_i(x)) = \sum_{\alpha=1}^{l} \mu_n(x_\alpha)$, where $\{x_1, \ldots, x_l\} = (\partial_i)^{-1}(\partial_i(x)) \cap p_n^{-1}(y)$.

Next, let us recall the nerve of a small category C and its classifying space BC. The *nerve* $N_*(C)$ of C is a simplicial set that consists of the graded set $\{N_n(C)\}_{n \geqslant 0}$, the face operators ∂_i

$$\partial_i \colon N_n(C) \to N_{n-1}(C) \quad (n \geqslant 1, 0 \leqslant i \leqslant n),$$

and the degeneracy operators s_i

$$s_i \colon N_n(C) \to N_{n+1}(C) \quad (n \geqslant 0, 0 \leqslant i \leqslant n).$$

The maps are defined by

$$\partial_i(f_1, \ldots, f_n) = \begin{cases} (f_2, \ldots, f_n) & \text{if } i = 0 \\ (f_1, \ldots, f_{i+1} \circ f_i, \ldots, f_n) & \text{if } 1 \leqslant i \leqslant n-1 \\ (f_1, \ldots, f_{n-1}) & \text{if } i = n \end{cases}$$

and

$$s_i(f_1, \ldots, f_n) = \begin{cases} (1_{s(f_1)}, f_1, \ldots, f_n) & \text{if } i = 0 \\ (f_1, \ldots, f_i, 1_{t(f_i)}, f_{i+1}, \ldots, f_n) & \text{if } 1 \leqslant i \leqslant n. \end{cases}$$

The *classifying space* BC of C is the geometric realization of the simplicial set $N_*(C)$ (see p. 55 of [**May92**]).

For a functor $F \colon C \to D$ between small categories, define a simplicial map

$$N_*(F) \colon N_*(C) \to N_*(D)$$

by

$$N_n(F)((f_1, \ldots, f_n)) = (F(f_1), \ldots, F(f_n))$$

for any $n \geqslant 0$ and $(f_1, \ldots, f_n)$ of $N_n(C)$. The nerve functor and the geometric realization functor induce a continuous map $BF \colon BC \to BD$.

Let X be a simplicial set. For x of X_n, the *geometric dimension* of x is defined as follows: If x do not belong to the images of the degeneracy operators $s_i \colon X_{n-1} \to X_n$ for $0 \leqslant i \leqslant n-1$, the geometric dimension of x is n; otherwise it is defined by the smallest k such that there exist y of X_k and $i_1, i_2, \ldots, i_{n-k}$ such that

$$s_{i_1} \circ s_{i_2} \circ \cdots \circ s_{i_{n-k}}(y) = x.$$

The geometric dimension of x is denoted by $\mathrm{g.dim}\, x$.

Theorem 2.7. *Suppose that $\widetilde{P} \colon \widetilde{E} \to B$ is a d-fold ramified covering. Then the nerve $N_*(\widetilde{P}) \colon N_*(\widetilde{E}) \to N_*(B)$ is a simplicial d-fold ramified covering.*

Proof. For any $n \geqslant 0$ and $\mathbf{g}$ of $N_n(B)$, we have

$$\# N_n(\widetilde{P})^{-1}(\mathbf{g}) = \begin{cases} d & \text{if } \mathrm{g.dim}(\mathbf{g}) \neq 0 \\ \# P^{-1}(b)/\sim & \text{if } \mathrm{g.dim}(\mathbf{g}) = 0 \text{ and } (s_0)^n(b) = \mathbf{g}. \end{cases} \tag{1}$$

Hence, condition 1 is satisfied.

Next, consider the following diagram:

$$\begin{array}{ccc} N_n(P)^{-1}(\mathbf{g}) & \xrightarrow{\partial_i|_{N_n(P)^{-1}(\mathbf{g})}} & N_{n-1}(P)^{-1}(\partial_i(\mathbf{g})) \\ \downarrow Q_n & & \downarrow Q_{n-1} \\ N_n(\widetilde{P})^{-1}(\mathbf{g}) & \xrightarrow{\partial_i|_{N_n(\widetilde{P})^{-1}(\mathbf{g})}} & N_{n-1}(\widetilde{P})^{-1}(\partial_i(\mathbf{g})), \end{array}$$

where P is the unramified covering and Q_n and Q_{n-1} are the natural projections. If $\partial_i|_{N_n(P)^{-1}(\mathbf{g})}$ is surjective, then $\partial_i|_{N_n(\widetilde{P})^{-1}(\mathbf{g})}$ is also surjective; therefore, condition 2 is satisfied. Given an element $\mathbf{f}$ of $N_{n-1}(P)^{-1}(\partial_i(\mathbf{g}))$, we have the following commutative diagram:

$$\begin{array}{ccc} \Delta[n-1] & \xrightarrow{\mathbf{f}} & N_*(E) \\ \downarrow \delta_i & & \downarrow N_*(P) \\ \Delta[n] & \xrightarrow[\mathbf{g}]{} & N_*(B), \end{array}$$

where δ_i is the ith coface inclusion. Since $N_*(P)$ is a fibration and δ_i is a trivial cofibration in the Quillen model structure of the category of simplicial sets, there exists a lifting $\tilde{\mathbf{f}}\colon \Delta[n] \to N_*(E)$. Namely, there exists $\tilde{\mathbf{f}}$ of $N_n(P)^{-1}(\mathbf{g})$ such that $\partial_i(\tilde{\mathbf{f}}) = \mathbf{f}$. Hence, $\partial_i|_{N_n(P)^{-1}(\mathbf{g})}$ is a surjection.

Define $\mu_n\colon N_n(\widetilde{E}) \to \mathbb{N}$ by

$$\mu_n(\mathbf{f}) = \begin{cases} 1 & \text{if g.dim}(\mathbf{f}) \neq 0 \\ \#[x] & \text{if g.dim}(\mathbf{f}) = 0 \text{ and } (s_0)^n([x]) = \mathbf{f} \end{cases}$$

for any $\mathbf{f}$ of $N_n(\widetilde{E})$ and $n \geqslant 0$. By equality (1), condition 3(a) is satisfied. Moreover, it is clear that condition 3(b) is also satisfied. For $\mathbf{f}$ of $N_n(\widetilde{P})^{-1}(\mathbf{g})$, if

$$\mathbf{f} = [x] \xrightarrow{1} \cdots \xrightarrow{1} [x] \xrightarrow{f} [y] \quad \text{and } i = n$$

or

$$\mathbf{f} = [y] \xrightarrow{f} [x] \xrightarrow{1} \cdots \xrightarrow{1} [x] \quad \text{and } i = 0,$$

where f is a nonidentity morphism, then we have

$$\#(\partial_i)^{-1}(\partial_i(\mathbf{f})) \cap N_n(\widetilde{P})^{-1}(\mathbf{g}) = \#[x].$$

Hence, the equality in condition 3(c) holds. In other cases, the map $\partial_i|_{N_n(\widetilde{P})^{-1}(\mathbf{g})}$ is a bijection; therefore, it is easy to show that the equality holds. Hence, $N_*(\widetilde{P})$ is a simplicial d-fold ramified covering.

□

Corollary 2.8. *Suppose that the functor $\widetilde{P}\colon \widetilde{E} \to B$ is a d-fold ramified covering. Then the map $B\widetilde{P}$ is a d-fold ramified covering.*

Proof. Theorems 2.7 and 4.2 of [**AP08**] imply this result. □

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