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Erratum to "Soliton interaction with small Toeplitz potentials", O. Pocovnicu, Dynamics of PDE, vol.9, no.1, 2012, 1-27

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The last line in the proof of Lemma 5.1 in [**1**] is not correct and the purpose of this erratum is to fix this mistake. In the following, we state and prove the correct version of the lemma. Notice that the only change in the statement is on the constant: $\frac{1}{4}$ is replaced by $\frac{1}{20}$. Namely, we have $\langle \mathcal{L}(f), f\rangle \geq \frac{1}{20}\|f\|^2_{H^{\frac{1}{2}}_+}$ instead of $\langle \mathcal{L}(f), f\rangle \geq \frac{1}{4}\|f\|^2_{H^{\frac{1}{2}}_+}$. We emphasize that this correction does not affect in any way the proof of the main theorem (apart from the obvious modifications on constants as a result of this correction).

Lemma 1 (Lemma 5.1 in [**1**]). *For all* $f \in \mathrm{Ker}(H_{\eta^2}) \cap H^{\frac{1}{2}}_+$, *we have*

$$\langle \mathcal{L}(f), f\rangle \geq \frac{1}{20}\|f\|^2_{H^{\frac{1}{2}}_+}.$$

Proof. Since $\eta(x) = \frac{1}{x+i}$, we have $\mathrm{Ker}(H_{\eta^2}) = \left(\frac{x-i}{x+i}\right)^2 L^2_+$. Thus, for $f \in \mathrm{Ker}(H_{\eta^2}) \cap H^{\frac{1}{2}}_+$, we have $f = \left(\frac{x-i}{x+i}\right)^2 h$ for some $h \in H^{\frac{1}{2}}_+$. Then, we have

$$T_{|\eta|^2} f = \Pi\Big(\frac{1}{(x+i)(x-i)}\Big(\frac{x-i}{x+i}\Big)^2 h\Big) = \Pi\Big(\frac{x-i}{(x+i)^3}h\Big) = \frac{x-i}{(x+i)^3}h = \frac{1}{x^2+1}f,$$

and

$$-i\partial_x f = -i\Big(\frac{x-i}{x+i}\Big)^2\partial_x h + 4\frac{x-i}{(x+i)^3}h = -i\Big(\frac{x-i}{x+i}\Big)^2\partial_x h + \frac{4}{x^2+1}f.$$

Since $|\frac{x-i}{x+i}| = 1$, we thus have

$$\|f\|^2_{\dot{H}^{\frac{1}{2}}_+} = \langle -i\partial_x f, f\rangle = \langle -i\Big(\frac{x-i}{x+i}\Big)^2\partial_x h, \Big(\frac{x-i}{x+i}\Big)^2 h\rangle + \langle \frac{4}{x^2+1}f, f\rangle$$

$$= \|h\|^2_{\dot{H}^{\frac{1}{2}}_+} + 4\int \frac{1}{x^2+1}|f(x)|^2 dx. \qquad (1)$$

As a consequence, we obtain

$$\begin{aligned}\langle \mathcal{L}(f), f\rangle &= \langle -\frac{i}{2}\partial_x f - 2T_{|\eta|^2}f - H_{\eta^2}f + \frac{1}{4}f, f\rangle \\ &= \frac{1}{2}\|f\|^2_{\dot{H}^{\frac{1}{2}}_+} - 2\int \frac{1}{x^2+1}|f(x)|^2dx + \frac{1}{4}\|f\|^2_{L^2} \\ &= \frac{1}{2}\|h\|^2_{\dot{H}^{\frac{1}{2}}_+} + \frac{1}{4}\|f\|^2_{L^2} \geq \frac{1}{4}\|f\|^2_{L^2}. \qquad (2)\end{aligned}$$

Therefore, $\langle \mathcal{L}(f), f\rangle \geq \frac{1}{4}\|f\|^2_{L^2}$. On the other hand, it follows from (1) that

(3)

$$\langle \mathcal{L}(f), f\rangle = \frac{1}{2}\|h\|^2_{\dot{H}^{\frac{1}{2}}_+} + \frac{1}{4}\|f\|^2_{L^2} \geq \frac{1}{16}\|h\|^2_{\dot{H}^{\frac{1}{2}}_+} + \frac{1}{4}\int \frac{1}{x^2+1}|f(x)|^2dx = \frac{1}{16}\|f\|^2_{\dot{H}^{\frac{1}{2}}_+}.$$

From (2) and (3), we obtain $\langle \mathcal{L}(f), f\rangle \geq \frac{1}{20}\|f\|^2_{H^{\frac{1}{2}}_+}$. □

As a result of this correction on Lemma 5.1, the following two modifications need to be made. The statement of Proposition 5.3 should be "$\langle \mathcal{L}(w), w\rangle \geq \frac{1}{20}\|w\|^2_{H^{\frac{1}{2}}_+}$" and the constants in the proof of Proposition 6.2 (last 8 lines of the proof on p. 23) should be adjusted correspondingly.

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References

[1] Oana Pocovnicu, *Soliton interaction with small Toeplitz potentials for the Szego equation on* $\mathbb{R}$, Dyn. Partial Differ. Equ. 9 (2012), no. 1, 1–28.

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