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Some mirror partners with complex multiplication

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In this note we give examples of families of Calabi–Yau 3-manifolds over Shimura varieties, whose mirror families contain subfamilies over Shimura varieties. In the case of these families we obtain dense sets of complex multiplication fibers by using the fact that the base space is a Shimura variety. In view of the work of Gukov and Vafa [7] this is of special interest in theoretical physics.

0. Introduction

In theoretical physics rational conformal field theories are considered as particularly interesting class of conformal field theories. Let $(\mathscr{X}, \mathscr{Y})$ be a pair of families of Calabi–Yau 3-manifolds, which are mirror partners, X be a fiber of $\mathscr{X}$ and Y be a fiber of $\mathscr{Y}$. In [7] Gukov and Vafa explain that X and Y yield a rational conformal field theory, if and only if both fibers have complex multiplication (CM). A family of Calabi–Yau manifolds over a Shimura variety has a dense set of CM fibers, if the variation of Hodge structures (VHS) is related to the Shimura datum of the base space in a natural way as in [10]. At present several of such families of Calabi–Yau 3-manifolds over Shimura varieties are known [2, 6, 10, 11, 12]. In general, one does not know a Shimura subvariety of the base space on the mirror side.[1] Here we give new examples of pairs of families of Calabi–Yau 3-manifolds over Shimura varieties, which are subfamilies of mirror partners.

We start with a family $\mathscr{C}_3$ of degree 3 covers of $\mathbb{P}^1$ with six different ramification points over an open Shimura subvariety $\mathscr{M}_3 \subset (\mathbb{P}^1)^3$. By using the Fermat curve of degree 3 and $\mathscr{C}_3$, one can construct a family of $K3$ surfaces with a non-symplectic involution over $\mathscr{M}_3$ as described in [10, Section 8]. The Borcea–Voisin construction yields a family $\mathscr{W}$ of Calabi–Yau 3-manifolds, which has a dense set of CM fibers. Garbagnati and van Geemen [5] have given a more general method to construct $K3$ surfaces, which yields the same $K3$ surfaces for the fibers of $\mathscr{C}_3$. The latter method allows one to

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[1] Moreover, for some of these examples [6, 11] the existence of a mirror is not clear.

construct $K3$ surfaces with non-symplectic involutions over the boundary of $\mathscr{M}_3 \subset (\mathbb{P}^1)^3$, where branch points of the fibers of $\mathscr{C}_3$ collide. Here we show that the Borcea–Voisin construction yields a family of Calabi–Yau 3-manifolds over a Shimura subvariety contained in the boundary of the base space of $\mathscr{W}$, whose fibers are its own Borcea–Voisin mirrors. Moreover, here we find a Shimura surface on the boundary of the base space of $\mathscr{W}$ such that the fibers of a family of Calabi–Yau 3-manifolds over this surface are Borcea–Voisin mirrors of the fibers of $\mathscr{W}$. We will also see that these families contain dense sets of CM fibers.

1. Construction of $K3$ surfaces by automorphisms

In this section we recall the construction of $K3$ surfaces by the methods in [5]. For this construction we use the following families of curves:

1) The family $\mathscr{C}_1$ is the family of genus 2 curves given by

$$V(y^3 - x_1(x_1 - x_0)^2(x_1 - \lambda x_0)^2 x_0) \to \lambda \in \mathscr{M}_1 := \mathbb{P}^1 \setminus \{0, 1, \infty\}.$$

2) The family $\mathscr{C}_2$ is the family of genus 3 curves given by

$$V(y^3 - x_1(x_1 - x_0)(x_1 - \alpha x_0)(x_1 - \beta x_0)x_0^2) \to (\alpha, \beta) \in \mathscr{M}_2,$$

where

$$\mathscr{M}_2 := (\mathbb{P}^1 \setminus \{0, 1, \infty\})^2 \setminus \{\alpha = \beta\}.$$

3) The family $\mathscr{C}_3$ is the family of genus 4 curves given by

$$V(y^3 - x_1(x_1 - x_0)(x_1 - \alpha x_0)(x_1 - \beta x_0)(x_1 - \gamma x_0)x_0) \to (\alpha, \beta, \gamma) \in \mathscr{M}_3,$$

where

$$\mathscr{M}_3 := (\mathbb{P}^1 \setminus \{0, 1, \infty\})^3 \setminus (\{\alpha = \beta\} \cup \{\alpha = \gamma\} \cup \{\beta = \gamma\}).$$

Remark 1.1. The families $\mathscr{C}_1$ and $\mathscr{C}_2$ can be obtained by collision of the branch points of the fibers of $\mathscr{C}_3$ over the boundary divisor of $\mathscr{M}_3 \subset (\mathbb{P}^1)^3$. Let Γ denote the monodromy group of the VHS of $\mathscr{C}_3$. Note that for $j = 1, 2, 3$ one can apply the Deligne–Mostow theory [4] to the VHS of $\mathscr{C}_j$. For an overview of this topic see also [8]. Due to the Deligne–Mostow theory, the period domain of the family $\mathscr{C}_j$ is the complex ball $\mathbb{B}_j$ and $\mathscr{M}_j$ is a dense open subset of $\Gamma \backslash \mathbb{B}_j$. In this sense the base spaces $\mathscr{M}_j$ are modular. Moreover

$\mathscr{M}_2$ and $\mathscr{M}_1$ are contained in the complement of $\mathscr{M}_3$ in $\Gamma\backslash\mathbb{B}_3$ (follows from [8, Theorem 3.1] and the description of the period map in [8, Section 4]).

One can also see that $\mathscr{M}_j$ is an open dense subset of a Shimura variety, which is a ball quotient. This can be concluded from the type of VHS of the given families (compare [10, Subsection 6.3]) and the description of such a VHS in the proof of [10, Theorem 4.4.4] in combination with the description of the period map above.

For $j = 1, 2, 3$ and $p \in \mathscr{M}_j$ let $f_j(t) \in \mathbb{C}[t]$ be a degree 6 polynomial such that $(\mathscr{C}_j)_p$ is given by the equation $v^3 - f_j(t) = 0$. Moreover let $\xi = e^{2\pi i \frac{1}{3}}$. It is clear that $\mathscr{C}_j$ has the $\mathscr{M}_j$-automorphism fiberwise given by

$$\beta_j : (v, t) \to (\xi v, t).$$

Let

$$\mathbb{F}_3 = V(y^2 z - x^3 - z^3) \subset \mathbb{P}^2$$

be a genus 1 curve isomorphic to the Fermat curve of degree 3 and

$$\alpha_{\mathbb{F}_3} : \mathbb{F}^3 \to \mathbb{F}^3 \text{ be given by } (x : y : z) \to (\xi x : y : z).$$

We have chosen this explicit formula due to technical reasons. Moreover, let S_{f_j} be a minimal model of a surface given by the Weierstrass equation

$$Y^2 = X^3 + f_j^2(t).$$

For the following lemma we will use methods, which occur already in the proof of [5, Proposition 2.2].

Lemma 1.1. *The surface S_{f_j} is a $K3$ surface birationally equivalent to the quotient $\mathbb{F}_3 \times (\mathscr{C}_j)_p/(\alpha_{\mathbb{F}_3}, \beta_j)$.*

Proof. The rational map

$$m_j : \mathbb{F}_3 \times (\mathscr{C}_j)_p \to S_{f_j}$$

is given by

$$((t, v), (x, y)) \to (v^2 x, v^3 y, t).$$

The reader checks easily that m_j is $(\alpha_{\mathbb{F}_3}, \beta_j)$-invariant and of degree 3. Moreover, one computes

$$Y^2 = (v^3y)^2 = v^6y^2 = v^6(x^3+1) = (v^2x)^3 + f_j^2(t) = X^3 + f_j^2(t).$$

From [5] we know that the minimal model S_{f_j} is a $K3$ surface. □

2. Some automorphisms of our $K3$ surfaces

2.1. The surface S_{f_j} has an elliptic fibration given by

$$S_{f_j} \to \mathbb{P}^1 \text{ via } (X,Y,t) \to t$$

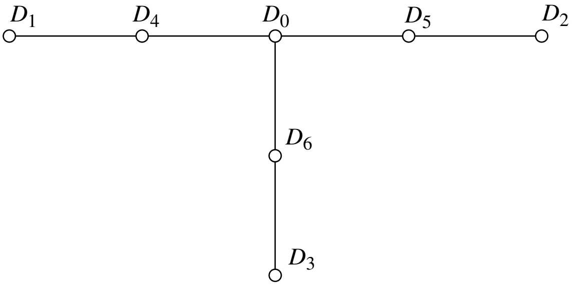
in the following way (see also [5]):

If $f_j(t_0) \neq 0$, the fiber of t_0 is given by the elliptic curve

$$V(Y^2Z = X^3 + uZ^3) \subset \mathbb{P}^2, \quad \text{where } u = f_j^2(t_0).$$

Now let $t_0 \in \mathbb{P}^1$ be a zero of $f_j(t)$. By using the Tate algorithm, one can compute the singular fibers. If $f_j(t)$ has a simple zero in t_0, the singular fiber $(S_{f_j})_{t_0}$ is of type **IV**. Thus it consists of three rational curves intersecting transversally in one point.

Now assume that $f_j(t)$ has a double zero in t_0. Then the fiber $(S_{f_j})_{t_0}$ is of type **IV***. Thus it is given by seven rational curves with the following intersection graph of type $\tilde{E}_6$:



Let $\iota_{f_j} : S_{f_j} \to S_{f_j}$ denote the involution given by

$$(X,Y,t) \to (X,-Y,t)$$

and $\alpha_{f_j} : S_{f_j} \to S_{f_j}$ denote the automorphism of degree 3 given by

$$(X,Y,t) \to (\xi X, Y, t).$$

2.2. The fixed locus of ι_{f_j} contains clearly the section s_∞ of the elliptic fibration fiberwise given by $(0:1:0) \in \mathbb{P}^2$ for a general $t \in \mathbb{P}^1$ and the curve $C_{j,p}$ given by

$$X^3 + f_j^2(t) = 0,$$

which is isomorphic to $(\mathscr{C}_j)_p$ (see [10, Remark 2.1.8]). One can also verify by explicit computation that one obtains a fiber F of type **IV** over a simple zero of $f_j(t)$ by blowing up once. This computation shows that ι_{f_j} interchanges two irreducible components of F and $C_{j,p}$ intersects F in the intersection point of its irreducible components. The involution ι_{f_j} acts non-trivially on the third irreducible component of F with isolated fixed point given by the intersection point with s_∞.

Moreover, the fixed locus of the automorphism α_{f_j} contains the sections

$$s_\pm(t) = (0 : \pm f_j(t) : 1) \in \mathbb{P}^2.$$

One can assume that s_∞ is the zero-section and under this assumption one can easily check that $s_\pm$ are sections of 3-torsion points by considering the generic fiber. The algebraic group on the special fiber F^* of type **IV*** is given by

$$(D_1 \setminus D_4) \ \cup \ (D_2 \setminus D_5) \ \cup \ (D_3 \setminus D_6) \ \cong \mathbb{G}_{a,\mathbb{C}} \times \mathbb{Z}_3.$$

Thus one has three different 3-torsion points such that each connected component contains precisely one 3-torsion point. Due to the Neron property, the sections $s_\pm$ and s_∞ intersect F^* in the 3-torsion points. Thus we can assume without loss of generality that s_∞ intersects $D_3 \setminus D_6$, the section s_+ intersects $D_1 \setminus D_4$ and s_- intersects $D_2 \setminus D_5$.

Since the fixed locus of ι_{f_j} contains curves, ι_{f_j} is non-symplectic. Recall that a non-symplectic involution of a $K3$ surface has a fixed locus, which is either empty or consists of smooth disjoint curves (see [3]). For a general description of automorphisms of K3 surfaces see [1]. Thus from the graph of a fiber of type **IV*** one concludes:

Proposition 2.1. *The curve D_6 is contained in the fixed locus with respect to ι_{f_j} and ι_{f_j} interchanges the handle consisting of D_1 and D_4 with the handle consisting of D_2 and D_5. Moreover, the intersection point $F^* \cap C_{j,p}$ is the additional isolated fixed point of $\iota_{f_j}|_{D_0}$.*

3. Construction of mirror pairs with complex multiplication

Recall 3.1. Let S be a $K3$ surface with non-symplectic involution ι_S, which has a fixed locus consisting of the curves $C_1, \ldots, C_N$, and E be an elliptic curve with involution ι_E fixing four points. Moreover, let

$$N' = \sum_{i=1}^{N} g(C_i),$$

where $g(C_i)$ denotes the genus of C_i. Then the Calabi–Yau 3-manifold X obtained from the Borcea–Voisin construction given by blowing up the singularities of $S \times E/(\iota_S, \iota_E)$ once has the Hodge numbers

$$h^{1,1}(X) = 11 + 5N - N' \quad \text{and} \quad h^{2,1}(X) = 11 + 5N' - N \tag{1}$$

(see [13]).

In many cases, the action of ι_S on the integral cohomology lattice $H^2(S, \mathbb{Z})$ of S can be used to construct a second involution ι' on the same lattice. The involution ι' can be realized as an involution of a family of $K3$ surfaces $\mathscr{S}' \to \mathscr{B}$ over $\mathscr{B}$, whose restrictions to each fiber of $\mathscr{S}'$ are non-symplectic involutions. By a relative version of the construction above for $\mathscr{S}'$, one obtains the Borcea–Voisin mirror family of X (for details see [3, 13]).

Remark 3.2. By using $\mathscr{C}_3$, one has already constructed families of Calabi–Yau manifolds over Shimura varieties (see [10, Section 8 and Section 9]). For this construction in [10] one has used a family of $K3$ surfaces obtained from a tower of cyclic coverings of weighted projective spaces $\mathbb{P}(2, 2, 1, \ldots, 1)$. The family of $K3$ surfaces is precisely the family over $\mathscr{M}_3$, which occurs in [5, Remark 1.3] and also here. This follows from the fact that both constructions yield a $K3$ surface, which is a minimal model of $\mathbb{F}_3 \times (\mathscr{C}_j)_p/(\alpha_{\mathbb{F}_3}, \beta_j)$ (see [10, Section 8.2]).

In [10] the fibers of the family of $K3$ surface come with an embedding into a desingularization of the weighted projective space $\mathbb{P}(2, 2, 1, 1)$ given by

$$\begin{aligned}\tilde{\mathbb{P}}(2, 2, 1, 1) \ni \tilde{V}(y_2^3 + y_1^3 + x_1(x_1 - x_0)(x_1 - ax_0)(x_1 - bx_0)(x_1 - cx_0)x_0)\\ \to (a, b, c).\end{aligned}$$

The non-symplectic involution γ, which is used in [10] for a Borcea–Voisin construction, is obtained from the involution of $\tilde{\mathbb{P}}(2, 2, 1, 1)$ given by $y_2 \leftrightarrow y_1$. Thus one checks easily that γ generates the Galois group of a Galois cover of degree 2 onto the Hirzebruch surface

$$H_2 \cong \tilde{V}(y_2 - y_1) \subset \tilde{\mathbb{P}}(2,2,1,1)$$

with ramification divisor isomorphic to a disjoint union of $\mathbb{P}^1$ and $C_{j,p}$. Thus from [9], the fact that the ramification divisor of the quotient by ι_{f_3} is isomorphic to the ramification divisor of the quotient by γ tells us that both coverings are coverings onto H_2. Since the branch divisors in H_2 coincide up to isomorphism, γ and ι_{f_3} coincide also and yield isomorphic families of Calabi–Yau 3-manifolds by the Borcea–Voisin construction.

3.3. By 2.2, the involution ι_{f_j} on S_{f_j} has a fixed locus containing a rational curve s_∞ and the curve $C_{j,p}$ of genus $j+1$. Moreover, the elliptic fibration of S_{f_j} contains $3-j$ singular fibers of type $\mathbf{IV}^*$ and each of these fibers has one rational curve contained in the fixed locus of ι_{f_j} (see Proposition 2.1). Thus by using (1) and the family of elliptic curves

$$\mathscr{E} \to \mathscr{M}_1, \quad V(y^2z - x(x-z)(x-\lambda z)) \to \lambda,$$

the Borcea–Voisin construction yields families $\mathscr{X}_j \to \mathscr{M}_j \times \mathscr{M}_1$ of Calabi–Yau 3-manifolds with the following Hodge numbers:

j	$h^{1,1}$	$h^{2,1}$
3	17	29
2	23	23
1	29	17

Remark 3.4. By [3, Section 3 and 4], one can easily check that $\mathscr{X}_2$ is contained in a family, which is its own Borcea–Voisin mirror family. Moreover, the families $\mathscr{X}_1$ and $\mathscr{X}_3$ can be embedded in families, which are Borcea–Voisin mirrors of each other.

Remark 3.5. By the construction above, the families $\mathscr{X}_1$ and $\mathscr{X}_2$ are contained in the boundary of $\mathscr{X}_3$. Moreover, by using Remark 1.1, one can show that the period map of $\mathscr{X}_j$ is a multivalued map to a dense open subset of $\mathbb{B}_j \times \mathbb{B}_1$. From these results one can conclude that the base space of $\mathscr{X}_j$ is an open subset of a Shimura variety with associated Hermitian symmetric domain $\mathbb{B}_j \times \mathbb{B}_1$.

By analogous arguments, one can also see that $\mathscr{X}_1$ is defined over the boundary of $\mathscr{X}_2$.

By [13, 2.21], we have a precise description of the mirror map for the families $\mathscr{X}_j$ and $\mathscr{X}_{4-j}$. Due to [7] one can assume that each pair of complex multiplication fibers of $\mathscr{X}_j$ and $\mathscr{X}_{4-j}$ yields a rational conformal field theory. Now we are going to show that each $\mathscr{X}_j$ has a dense set of CM fibers for $j = 1, 2, 3$. First we introduce our definition of CM:

3.6. Let X be a compact Kähler manifold of complex dimension n and S^1 be the $\mathbb{R}$-algebraic group

$$S^1 = \operatorname{Spec}(\mathbb{R}[x, y]/x^2 + y^2 - 1),$$

where

$$S^1(\mathbb{R}) = \left\{ M = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in \mathrm{SL}_2(\mathbb{R}) \right\} \cong \{z \in \mathbb{C} : |z| = 1\}.$$

The rational Hodge structure on $H^n(X, \mathbb{Q})$ of weight n corresponds to the representation

$$\begin{aligned} &h_X : S^1 \to \mathrm{GL}(H^n(X, \mathbb{R})), \quad h_X(z)v = z^p \bar{z}^q v \quad (\forall v \in H^{p,q}(X) \\ &\quad \text{with } \ p + q = n). \end{aligned}$$

Since $H^{p,q}(X) = \overline{H^{q,p}(X)}$, one can see that $h_X(S^1) \subset \mathrm{GL}(H^n(X, \mathbb{R}))$. The Hodge group $\mathrm{Hg}(X)$ is the smallest $\mathbb{Q}$-algebraic subgroup G of $\mathrm{GL}(H^n(X, \mathbb{Q}))$ such that $h_X(S^1) \subset G_{\mathbb{R}}$. We say that X has CM, if $\mathrm{Hg}(X)$ is a torus.

For more details in the case of Calabi–Yau 3-manifolds see also [2].

Remark 3.7. Now let X be a Calabi–Yau 3-manifold. In another definition, which is also used in [7], the Calabi–Yau manifold X has complex multiplication, if the torus

$$J_G(X) = H^3(X, \mathbb{Z}) \backslash H^n(X, \mathbb{C}) / F^2(H^3(X, \mathbb{C}))$$

is of CM type. The Griffiths intermediate Jacobian $J_{\mathrm{G}}(X)$ is of CM type, if and only if the endomorphism algebra of $J_{\mathrm{G}}(X)$ has dimension $2(H^{2,1}(X) + 1)$. By [2, Theorem 2.3], a Calabi–Yau 3-manifold with CM in the sense of 3.6, which is also the same definition of CM introduced by Borcea in [2], satisfies that $J_{\mathrm{G}}(X)$ is of CM type.[2]

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[2] It is a common mistake to claim that the both definitions of complex multiplication would be equivalent. The assumption that $J_{\mathrm{G}}(X)$ is of CM type does not imply that X has CM in the sense of 3.6. For a counter example see [10, Example 1.6.9].

Proposition 3.1. *For $j = 1, 2, 3$, the family $\mathscr{X}_j$ has a dense set of CM fibers.*

Proof. By [10, Subsection 6.3], each family $\mathscr{C}_j$ has a dense set of CM fibers. Note that the family of elliptic curves

$$\mathscr{E} \to \mathscr{M}_1, \quad V(y^2z - x(x-z)(x-\lambda z)) \to \lambda$$

has also a dense set of CM fibers. Since the ramification locus of the involution ι_{f_j} on S_{f_j} consists of $C_{j,p} \cong (\mathscr{C}_j)_p$ and some rational curves, it remains to show that S_{f_j} has CM, if $(\mathscr{C}_j)_p$ has CM. Using this result one can then conclude as in [10, Subsection 7.2] that $(\mathscr{X}_j)_{(p,q)}$ has CM, if $(\mathscr{C}_j)_p$ and $\mathscr{E}_q$ have CM.

The singularities of the fibers of $\mathbb{F}_3 \times \mathscr{C}_j/(\alpha_{\mathbb{F}_3}, \beta_j)$ are given by the singular sections. Let m_j denote the quotient map by $(\alpha_{\mathbb{F}_3}, \beta_j)$. Near the sections of fixed points corresponding to the singular sections of $\mathbb{F}_3 \times \mathscr{C}_j/(\alpha_{\mathbb{F}_3}, \beta_j)$ the action of $(\alpha_{\mathbb{F}_3}, \beta_j)$ is given by (ξ, ξ) or $(\xi, \bar{\xi})$.

First consider the case $(\xi, \bar{\xi})$. In this case one blows up the corresponding sections on $\mathbb{F}_3 \times \mathscr{C}_j$ with exceptional divisor E_1. The automorphism $(\alpha_{\mathbb{F}_3}, \beta_j)$ does not act trivially on E_1. Thus we blow up the two fixed sections on each connected component of E_1 with smooth exceptional divisor E_2. This divisor is contained in ramification locus of m_j. Now the quotient by $(\alpha_{\mathbb{F}_3}, \beta_j)$ is smooth in a neighborhood of $m_j(E_1 \cup E_2)$.

In the case (ξ, ξ) we blow up the section of fixed points and obtain a smooth exceptional divisor contained in the ramification locus.

Let $\widetilde{\mathbb{F}_3 \times \mathscr{C}_j}$ denote the manifold obtained from the previous blowing up operations on $\mathscr{C}_j \times \mathbb{F}_3$ and

$$\mathscr{F}_j = \widetilde{\mathbb{F}_3 \times \mathscr{C}_j}/(\alpha_{\mathbb{F}_3}, \beta_j).$$

Thus we obtain a model $\mathscr{F}_j$ of the quotient $\mathbb{F}_3 \times (\mathscr{C}_j)_p/(\alpha_{\mathbb{F}_3}, \beta_j)$ consisting of smooth fibers over $\mathscr{M}_j$. The surface $\mathbb{F}_3 \times \mathscr{C}_j$ has CM, if $\mathbb{F}_3$ and $(\mathscr{C}_j)_p$ have CM. Note that monoidal transformations of surfaces do not have any effect to the property of CM (compare [10, Corollary 7.1.6]). Due to the fact that the Hodge structure on $H^2((\mathscr{F}_j)_p, \mathbb{Q})$ is a sub-Hodge structure of the one on $H^2((\widetilde{\mathbb{F}_3 \times \mathscr{C}_j})_p, \mathbb{Q})$, one concludes that $(\mathscr{F}_j)_p$ has CM, if $(\mathscr{C}_j)_p$ has CM. Moreover, we can use monoidal transformations to obtain S_{f_j} from $(\mathscr{F}_j)_p$. Thus S_{f_j} has CM, if $(\mathscr{F}_j)_p$ has CM. $\square$

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