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ERRATUM TO HYPERBOLIC PREDATORS VS. PARABOLIC PREY*

RINALDO M. COLOMBO † AND ELENA ROSSI ‡

Abstract. We correct an error in the proof of the main result in [R.M. Colombo and E. Rossi, *Commun. Math. Sci.,* 13(2):369-400, 2015]. The theorem, with all the provided estimates, remains true. The *online* version is corrected.

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AMS subject classification: 35L65, 35M30, 92D25.

(Here, we refer to [1] as it appears on paper, the *online* version being correct.)

The proof of Theorem 2.2 in [1] relies on the construction of a Cauchy sequence of approximate solutions. However, the space $\mathbf{L}^1([0,T];\mathcal{X}^+)$, where $\mathcal{X}^+ = (\mathbf{L}^1 \cap \mathbf{L}^\infty \cap \mathbf{BV})(\mathbb{R}^n;\mathbb{R}^+) \times (\mathbf{L}^1 \cap \mathbf{L}^\infty)(\mathbb{R}^n;\mathbb{R}^+)$, see [1, Formula (2.1)], is clearly *not* complete. In fact, the sequence

$$f_n(x) = \begin{cases} 0 & x \in [0,1/n] \\ x\sin(1/x) & x \in \,]1/n,1] \end{cases}$$

is in $(\mathbf{L}^1 \cap \mathbf{L}^\infty \cap \mathbf{BV})([0,1];\mathbb{R})$ but its $\mathbf{L}^1$ limit has unbounded total variation.

To correct the proof, proceed as follows.

Choose an in initial datum $(u_o,w_o) \in \mathcal{X}_r$, as in [1, Formula (4.10)], with $w_o \in (\mathbf{C}^1 \cap \mathbf{W}^{1,1})(\mathbb{R}^n;\mathbb{R}^+)$. Set, for $t \in [0,T]$, $(u_0(t),w_0(t)) = (u_o,w_o)$. For $i \in \mathbb{N}$ and for $(t,x) \in [0,T] \times \mathbb{R}^n$,, define recursively $a_{i+1}(t,x) = \gamma - \delta\, u_i(t,x)$, $b_{i+1}(t,x) = \alpha\, w_i(t,x) - \beta$ and $c_{i+1}(t,x) = v(w_i(t))(x)$. Exactly as in [1, § 4.3], let (u_{i+1},w_{i+1}) be such that

$$\begin{cases} \partial_t u_{i+1} + \nabla \cdot (c_{i+1} u_{i+1}) = b_{i+1}\, u_{i+1} \\ \partial_t w_{i+1} - \mu \Delta w_{i+1} = a_{i+1}\, w_{i+1} \\ u_{i+1}(0) = u_o \\ w_{i+1}(0) = w_o. \end{cases}$$

Apply [1, Point 7. in Proposition 2.8] in the explicit form [1, Formula (4.8)] to obtain

$$\begin{aligned} \mathrm{TV}(u_i(t)) \le{}& \left[\mathrm{TV}(u_o) + I_n \int_0^t \|\nabla(b_i - \nabla \cdot c_i)(\tau)\|_{\mathbf{L}^1(\mathbb{R}^n;\mathbb{R}^n)} \|u_i(\tau)\|_{\mathbf{L}^\infty(\mathbb{R}^n;\mathbb{R})} \,\mathrm{d}\tau \right] e^{\kappa_o^* t} \\ \le{}& \mathrm{TV}(u_o) e^{\kappa_o^* t} \\ &+ I_n \left[\|\nabla b_i\|_{\mathbf{L}^1([0,t]\times\mathbb{R}^n;\mathbb{R}^n)} + \|\nabla \nabla \cdot c_i\|_{\mathbf{L}^1([0,t]\times\mathbb{R}^n;\mathbb{R}^n)} \right] \|u_i\|_{\mathbf{L}^\infty([0,t]\times\mathbb{R}^n;\mathbb{R})} e^{\kappa_o^* t}. \end{aligned}$$

All terms in the right hand side above are estimated in the proof of [1, Theorem 2.2] by means of quantities independent of i. More precisely, using [1, ***(v)***] and [1, **Claim 1**]:

$$\begin{aligned} \kappa_o^* &= (2n+1) \|\nabla c_i\|_{\mathbf{L}^\infty([0,t]\times\mathbb{R}^n;\mathbb{R}^{n\times n})} + \|b_i\|_{\mathbf{L}^\infty([0,t]\times\mathbb{R}^n;\mathbb{R})} \\ &\le ((2n+1)K + \alpha) \|w_o\|_{\mathbf{L}^\infty(\mathbb{R}^n;\mathbb{R})} e^{\gamma t} + \beta \end{aligned}$$

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†Unità INdAM, Università di Brescia, Via Branze 38, 25123 Brescia, Italy (rinaldo.colombo@unibs.it).

‡Dipartimento di Matematica ed Applicazioni, Università di Milano-Bicocca, Via Cozzi 55, 20125 Milano, Italy (e.rossi50@campus.unimib.it).

whereas I_n is a numerical constant, see [1, Formula (4.5)]; $\|\nabla b_i\|_{\mathbf{L}^1([0,t]\times\mathbb{R}^n;\mathbb{R}^n)}$ is bounded in [1, Formula (4.21)]; $\|\nabla\nabla\cdot c_i\|_{\mathbf{L}^1([0,t]\times\mathbb{R}^n;\mathbb{R}^n)}$ is bounded in [1, Formula (4.22)] and $\|u_i\|_{\mathbf{L}^\infty([0,t]\times\mathbb{R}^n;\mathbb{R})}$ is bounded by [1, **Claim 2**].

Therefore, $\mathrm{TV}(u_i(t)) \leq \mathcal{F}(t)$ where $\mathcal{F} \in \mathbf{C}^0([0,T];\mathbb{R}^+)$ depends only on the hypotheses and initial data, being in particular independent of i. The Cauchy sequence constructed in [1] thus belongs to the set

$$\left\{(u,w) \in \mathbf{L}^1([0,T];\mathcal{X}^+) \colon \mathrm{TV}(u(t)) \leq \mathcal{F}(t) \text{ for all } t \in [0,T]\right\}$$

which is a complete metric space with the distance

$$\begin{aligned} d((u_1,w_1),(u_2,w_2)) &= \|u_2-u_1\|_{\mathbf{L}^1([0,T];\mathbf{L}^1(\mathbb{R}^n;\mathbb{R}))} + \|w_2-w_1\|_{\mathbf{L}^1([0,T];\mathbf{L}^1(\mathbb{R}^n;\mathbb{R}))} \\ &= \int_0^T \int_{\mathbb{R}^n} \left(|u_2(t,x)-u_1(t,x)| + |w_2(t,x)-w_1(t,x)|\right) \mathrm{d}x \, \mathrm{d}t\,. \end{aligned}$$

used in [1].

REFERENCES

[1] R. M. Colombo and E. Rossi. Hyperbolic predators vs. parabolic prey. *Commun. Math. Sci.*, 13(2):369–400, 2015.