COMMUN. MATH. SCI. Vol. 12, No. 8, pp. 1565–1578

H^1-RANDOM ATTRACTORS OF STOCHASTIC MONOPOLAR NON-NEWTONIAN FLUIDS WITH MULTIPLICATIVE NOISE*

CHUNXIAO GUO† AND BOLING GUO‡

Abstract. In this paper, the authors study the asymptotic dynamical behavior for stochastic monopolar non-Newtonian fluids with multiplicative noise defined on a two-dimensional bounded domain, and prove the existence of an H^1-random attractor for the corresponding random dynamical system. A random attractor is a random compact set absorbing any bounded subset of the phase space V.

Key words. Non-Newtonian fluids, random attractor, multiplicative noise.

AMS subject classifications. 76A05, 35Q30.

1. Introduction

In this paper, suppose that $D\subset R^2$ is a two-dimensional bounded smooth open domain. We consider the following stochastic monopolar incompressible non-Newtonian fluids with multiplicative noise:

$$du+\big(u\cdot\nabla u-\nabla\cdot\tau(e(u))\big)dt=g(x)dt+\sum_{j=1}^{m}b_j u\circ d\omega_j(t),\ x\in D,\ t>0, \tag{1.1}$$

$$u(x,0)=u_0(x),\ x\in D, \tag{1.2}$$

$$\nabla\cdot u(x,t)=0, \tag{1.3}$$

subject to the boundary conditions

$$u(x,t)=0,\ x\in\partial D, \tag{1.4}$$

where $\circ$ denotes the Stratonovich sense in the stochastic term, and $\omega_j(t),1\le j\le m$ are mutually independent two-sided Wiener processes on a probability space which will be specified later, $b_j\in R,1\le j\le m$ are given. In the equation (1.1), the unknown vector function u denotes the velocity of the fluid, $g(x)$ is the external body force, and $\tau=(\tau_{ij})$ is the constitutive relation of the fluid, whose components are

$$\tau_{ij}=\pi\delta_{ij}-\tau_{ij}^{\vartheta}.$$

Here π is the pressure and τ_{ij}^{ϑ} is the viscous part of the stress tensor, which has the following constitutive relation:

$$\tau_{ij}^{\vartheta}=(\nu+2\mu_0|e|^{p-2})e_{ij},\quad i,j=1,2,$$

$$e_{ij}(u)=\frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\Big),\quad |e(\cdot)|^2=\sum_{i,j=1}^{2}|e_{ij}(\cdot)|^2,$$

*Received: April 1, 2013; accepted (in revised form): December 4, 2013. Communicated by Tiejun Li.

†Department of Mathematics, China University of Mining and Technology Beijing, 100083 Beijing, P.R. China (guochunxiao1983@sina.com).

‡Institute of Applied Physics and Computational Mathematics, P.O. Box 8009, 100088 Beijing, P.R. China (gbl@iapcm.ac.cn).

where $\nu > 0$ is a constant, $\mu_0 > 0$ is the viscosity, and only the first derivative of the velocity field is involved in the stress tensor.

If $\tau_{ij}^{\vartheta}(u) = \kappa e_{ij}(u)$, κ is a constant, then the fluids conform to the Stokes law, and are called Newtonian fluids, such as water and alcohol, satisfying the linear constitutive relation. If the random perturbations are not involved in equation (1.1), then the equation turns out to be the well-known Navier-Stokes system [22]. Furthermore, if $\tau^{\vartheta} = 0$, the system is an Euler system. For some fluid materials, their flow behavior cannot be characterized by a Newtonian relationship in real life. The nonlinearity in the constitutive relationship must be considered, and such fluids are called non-Newtonian fluids; see [6, 18], e.g. molten plastics, dyes, adhesives, paints and greases. A typical model is the monopolar incompressible viscous non-Newtonian fluid. The fluids are shear thinning when $1 < p < 2$, and shear thickening when $p > 2$. We only consider the shear thickening case $2 < p < \frac{7}{3}$ throughout this paper.

We recall some results about the deterministic non-Newtonian fluids. There is an extensive literature on the existence and uniqueness of solutions, the existence of attractors for non-Newtonian fluids, etc.; see [1, 2, 3, 4, 5, 13, 14, 18, 19, 20, 21, 23, 24] for further details. Ladyzhenskaya [19] established the existence of a weak solution of the monopolar model for $p > 1 + \frac{2n}{n+2}$ $(n = 2,3)$ in a bounded domain and the existence of unique regular weak solution for $p \geq 2$, dimension $n = 2$, and for $p \geq \frac{5}{2}$, dimension $n = 3$. Du and Gunzburger [13] proved the existence of unique weak solution with the condition $n = 3$, and $p \geq \frac{11}{5}$. For the space-periodic version, Bellout et al. [1] have established the following results in the periodic domain: for $n = 2,3$, $p \geq 1 + \frac{2n}{n+2}$, there exists a unique regular weak solution. Nečasová and Penel [20] studied the decay of weak solutions to equations of monopolar non-Newtonian incompressible fluids in the whole space.

In fact, the deterministic system model usually neglects the impact of many small random perturbations, and stochastic equations can conform to physical phenomena better. These random perturbations are intrinsic effects in a variety of settings and spatial scales. It could be most obviously influential at the microscopic and smaller scales but indirectly it plays a vital role in microscopic phenomena. Thus many authors contributed their efforts to this stochastic field of research, and displayed interesting structures and phenomena in physics.

For important equations, such as the stochastic KdV equation, Navier-Stokes equation, Burgers equation, Schrödinger equation, etc., there have been much work and interesting results related to their existence, uniqueness, and attractors; for these topics and the progress in these fields, see [7, 8, 9, 11, 12]. There is also a series of papers which investigate stochastic non-Newtonian fluids. Some important results have been obtained, such as [15, 16, 17, 25, 26], and so on. Especially, Zhao et al. [25] proved the existence of a random attractor for two-dimensional stochastic bipolar non-Newtonian fluids with multiplicative noise in the case of $1 < p < 2$. Guo and Guo [16] expanded this result to the case of $2 < p < 3$. Along this line, we want to know whether a similar result is also true for stochastic monopolar non-Newtonian fluids. This is the main subject that we will develop in this work. Compared with the work on stochastic Navier-Stokes equations, we here need to deal with the nonlinear term $\nabla \cdot (|e(u)|^{p-2} e_{ij}(u))$, and compared with the work on stochastic bipolar non-Newtonian fluids, the lack of the highly regular four order term $\nabla \cdot (\Delta e(u))$ will make it more difficult to obtain estimates. In this paper, we prove that there exist global random attractors for two-dimensional stochastic monopolar non-Newtonian fluids with multiplicative noise in the case of $2 < p < \frac{7}{3}$.

Crauel, Debussche, and Flandoli [7, 8] present a general theory to study the random attractors by defining an attracting set as a set that attracts any orbit starting from $-\infty$. Given a probability space, the random attractors are compact invariant sets, which depend on chance and move with time. The main general result on random attractors relies heavily on the existence of a random compact attracting set. In this paper, we will apply this theory to prove the existence of random attractors for two-dimensional stochastic monopolar non-Newtonian fluids in the case of $2<p<\frac{7}{3}$. First, we make use of the Stratonovich transform to change the stochastic equation to a deterministic equation with random parameter; Second, we obtain the existence of bounded absorbing sets by some estimates of solutions in the spaces H and V; Third, we use the compact embedding of Sobolev space to obtain the existence of a compact random set.

The paper is organized as follows. In Section 2, we recall some definitions and already known results concerning random attractors. In Section 3, we develop all the results needed to prove the existence of random attractors in space V. In Section 4, we establish the existence of a compact random attractor in V by compactness of Sobolev embedding.

We introduce some functional spaces and some notation.

$L^q(D)$-the Lebesgue space with norm $\|\cdot\|_{L^q}$, and $\|\cdot\|_{L^2}=\|\cdot\|$. Particularly, $\|u\|_{L^\infty}=ess\sup_{x\in D}|u(x)|$, for $q=\infty$.

$H^\sigma(D)$-the Sobolev space $\{u\in L^2(D), D^k u\in L^2(D), k\le\sigma\}$ with norm $\|\cdot\|_{H^\sigma}=\|\cdot\|_\sigma$.

$\mathscr{C}(I,X)$-the space of continuous functions from the interval I to X.

Define a space of smooth functions

$$\mathscr{V}=\{u\in C_0^\infty(\overline{D}):\nabla\cdot u=0, x\in D, u=0, x\in\partial D\},$$

H=the closure of $\mathscr{V}$ in $L^2(D)$ with norm $\|\cdot\|$, and let $(\cdot,\cdot)$ denote the inner product in H. $V=$ the closure of $\mathscr{V}$ in $H^1(D)$ with norm $\|\cdot\|_1$, and V' is the dual space of V.

By simple computation, we can obtain the result $\nabla\cdot e(u)=\frac{1}{2}\Delta u$. For simplicity in writing, we put $\nu=2$.

For notational simplicity, C is a generic constant, and may assume various values from line to line throughout this paper. In addition, the summation convention of repeated indices is used in the whole paper.

2. Preliminaries

We introduce the linear operator A as follows: consider a bilinear form $a:V\times V\to R$ by

$$a(u,v)=\int_D \nabla u\nabla v dx, \qquad (u,v\in V).$$

As a consequence of the Lax-Milgram lemma, we obtain an isometry $A\in V\to V'$,

$$(Au,v)=a(u,v)=<f,v>,\ u\in V,\ f\in V', \tag{2.1}$$

where V' is the dual space of V, $A:V\to V'$ is a linear operator, and $D(A)=V\cap H^2(D)$. In fact $A=-P\Delta$, P is the projection from $L^2(D)$ to H.

According to the Rellich theorem, A^{-1} is compact in H, and

$$A\phi_n=\lambda_n\phi_n,\ \phi_n\in D(A), \tag{2.2}$$

where $\{\phi_n\}_{n=1}^{\infty}$ are the eigenfunctions and also are a basis of V. $\lambda_n > 0$ are eigenvalues, and $\lambda_n \to \infty$ when $n \to \infty$.

Moreover, $\forall u \in V$, we have $\|u\|_1^2 \geq \lambda_1 \|u\|^2$, where λ_1 is the first eigenvalue.

LEMMA 2.1 (Gagliardo-Nirenberg Inequality). *For $1 \leq p', q', r' \leq \infty$, and the dimension $n \geq 1$, for all integers $m > k \geq 0$, there exist two constants $0 \leq \alpha \leq 1$, $C > 1$, such that for all $u \in C^\infty(D)$,*

$$\|\nabla^k u\|_{p'} \leq C \|\nabla^m u\|_{r'}^{\alpha} \|u\|_{q'}^{1-\alpha},$$

for $\frac{1}{p'} - \frac{k}{n} = \alpha(\frac{1}{r'} - \frac{m}{n}) + \frac{1}{q'}(1-\alpha)$, and $\frac{1}{p'} \leq \frac{\alpha}{r'} + \frac{1-\alpha}{q'}$. The only exception is that $\alpha = 1$, if $m - \frac{n}{r'} = k$, $1 < r' < \infty$.

LEMMA 2.2. *If $u \in W^{s_1,p}(D)$, $0 \leq s_2 \leq s \leq s_1 < \infty$, then there exists a constant C, such that*

$$\|u\|_{s,p} \leq C \|u\|_{s_1,p}^{\alpha} \|u\|_{s_2,p}^{1-\alpha},$$

where $s = \alpha s_1 + (1-\alpha) s_2$.

Defining the trilinear form b on $V \times V \times V$ as

$$b(u,\upsilon,\psi) = \int_D u_i \frac{\partial \upsilon_j}{\partial x_i} \psi_j dx, \quad u,\upsilon,\psi \in V,$$

one can check that $b(u,\upsilon,\omega) = -b(u,\omega,\upsilon)$ and $b(u,\upsilon,\upsilon) = 0$.

Next, define a bilinear map B on $V \times V$ by

$$(B(u,u),\psi) = b(u,u,\psi), \quad u,\psi \in V.$$

Define the map $N(u)$ on V as follows:

$$(N(u),\psi) = 2\mu_0 \int_D |e(u)|^{p-2} e_{ij}(u) e_{ij}(\psi) dx, \quad u,\psi \in V.$$

Following these preparation, equations (1.1)-(1.4) can be translated into the following abstract problems in H:

$$du + [Au + N(u) + B(u,u)]dt = gdt + \sum_{j=1}^{m} b_j u \circ d\omega_j(t), \; t > s, \tag{2.3}$$

$$u(s) = u_s, s \in R, \tag{2.4}$$

where we assume that $u_s \in H, g \in H$.

We next recall some definitions and results concerning the random attractors, which can be found in [7, 8]. Let (X,d) be a separable metric space and $(\Omega, \mathscr{F}, \mathbb{P})$ be a complete probability space. We will consider a family of mappings $S(t,s;\omega): X \to X$, $-\infty < s \leq t < \infty$, parameterized by $\omega \in \Omega$ in the following.

DEFINITION 2.1. *Let $\{\theta_t : \Omega \to \Omega, t \in R\}$ be a family of measure preserving transformations of $(\Omega, \mathscr{F}, \mathbb{P})$ such that $\theta_0 = id_\Omega$ and $\theta_{t+s} = \theta_t \circ \theta_s$ for all $t,s \in R$. Here we assume θ_t is ergodic under $\mathbb{P}$. Especially, for all $s < t \in R$, and $x \in X$,*

$$S(t,s;\omega)x = S(t-s,0;\theta_s\omega)x, \qquad \mathbb{P}-a.e.$$

REMARK 2.1. For (2.3)-(2.4), we consider the probability space $(\Omega,\mathscr{F},\mathbb{P})$, where $\Omega=\{\omega\in C(R,R^m)|\omega(0)=0\}$, $\mathscr{F}$ is the Borel σ-algebra induced by the compact open topology of Ω, and $\mathbb{P}$ is the product measure of two Wiener measures on the negative and the positive time parts of Ω. Define the time shift θ_t by

$$(\theta_t\omega)(s)=\omega(t+s)-\omega(t),\ \omega\in\Omega\, s,t\in R.$$

Then $(\Omega,\mathscr{F},\mathbb{P},(\theta_t)_{t\in R})$ is a metric dynamical system (see [7] for more details).

DEFINITION 2.2. *Let $t\in R$ and $\omega\in\Omega$. A random dynamical system with time t on a separable metric space (X,d) with Borel σ-algebra $\mathscr{B}$ over $\{\theta_t\}$ on $(\Omega,\mathscr{F},\mathbb{P})$ is a measurable map*

$$S(t,s;\omega):X\to X,\ -\infty<s\le t<\infty,$$

such that $S(0,0,\omega)=id$ and $S(t,0;\omega)=S(t,s;\omega)S(s,0;\omega)$ for all $t,s\in R$ and $\omega\in\Omega$.

The random dynamical system $S(t,s;\omega)$ is called continuous if the mapping $x\mapsto S(t,s,\omega)x$ is continuous for all $t,s\in R$ and $\omega\in\Omega$.

DEFINITION 2.3. *Given $t\in R$ and $\omega\in\Omega$, $K(t,\omega)\subset X$ is an attracting set if for all bounded sets $B\subset X$*

$$d(S(t,s;\omega)B,K(t,\omega))\to 0,\ \ s\to-\infty,$$

where $d(A,B)$ is the semidistance defined by

$$d(A,B)=\sup_{x\in A}\inf_{y\in B}d(x,y).$$

DEFINITION 2.4. *A family $A(\omega)$, $\omega\in\Omega$ of closed subsets of X is measurable if for all $x\in X$, the mapping $\omega\mapsto d(x,A(\omega))$ is measurable.*

DEFINITION 2.5. *Define the random omega limit set of a bounded set $B\subset X$ at time t as*

$$A(B,t,\omega)=\bigcap_{T<t}\overline{\bigcup_{s<T}S(t,s;\omega)B}.$$

DEFINITION 2.6. *Let $S(t,s;\omega)_{t\ge s,\omega\in\Omega}$ be a random dynamical system, and suppose that $A(t,\omega)$ is a random set satisfying the following conditions:*

(1) *It is the minimal closed set such that for $t\in R$, $B\subset X$,*

$$d(S(t,s;\omega)B,A(t,\omega))\to 0,\ \ s\to-\infty,$$

which implies $A(t,\omega)$ attracts B (B is a deterministic set).

(2) *$A(t,\omega)$ is the largest compact measurable set which is invariant in sense that*

$$S(t,s;\omega)\,A(\theta_s\omega)=A(\theta_t\omega),\quad s\le t.$$

Then $A(t,\omega)$ is said to be the random attractor.

THEOREM 2.1 (see [7]). *Let $S(t,s;\omega)_{t\ge s,\omega\in\Omega}$ be a random dynamical system satisfying the following conditions:*

(1) $S(t,r;\omega)S(r,s;\omega)x = S(t,s;\omega)x$, *for all* $s \le r \le t$ *and* $x \in X$,

(2) $S(t,s;\omega)$ *is continuous in X, for all* $s \le t$,

(3) for all $s < t$ *and* $x \in X$, *the mapping* $\omega \mapsto S(t,s;\omega)x$ *is measurable from* $(\Omega,\mathscr{F})$ *to* $(X,\mathscr{B}(X))$,

(4) for all $t,x \in X$ *and* $\mathbb{P}$*-a.e.* ω, *the mapping* $s \mapsto S(t,s;\omega)x$ *is right continuous at any point.*

Assume that there exists a group θ_t, $t \in R$, *of measure preserving mappings such that*

$$S(t,s;\omega)x = S(t-s,0;\theta_s\omega)x, \quad \mathbb{P}-a.e.\ s<t, x \in X \tag{2.5}$$

holds and for $\mathbb{P}$*-a.e.* ω, *there exists a compact attracting set* $K(\omega)$ *at time 0. If for* $\mathbb{P}$*-a.e.* $\omega \in \Omega$, *we set* $\Lambda(\omega) = \overline{\bigcup_{B\subset X} A(B,\omega)}$, *where the union is taken over all the bounded subsets of X and* $A(B,\omega)$ *is given by*

$$A(B,\omega) = A(B,0,\omega) = \bigcap_{T<0} \overline{\bigcup_{s<T} S(0,s;\omega)B},$$

then $\Lambda(\omega)$ *is a random attractor.*

3. The existence of bounded absorbing set

Next, we show that there is a continuous random dynamical system generated by the stochastic monopolar non-Newtonian fluid with multiplicative noise. We introduce an auxiliary Stratonovich process which enables us to change the stochastic equation to an evolution equation depending on a random parameter. Considering the process $\eta(t) = e^{-\sum_{j=1}^m b_j\omega_j(t)}$ which satisfies the Stratonovich equation

$$d\eta(t) = -\sum_{j=1}^{m} b_j \eta(t) \circ d\omega_j(t). \tag{3.1}$$

We set $v(t) = \eta(t)u(t)$, so that it satisfies the equation

$$\frac{dv}{dt} + Av + \eta B(u,u) + \eta N(u) = \eta g, \tag{3.2}$$

$$v(x,s) = v_s = \eta(s)u_s(x),\ x \in D,\ s \in R. \tag{3.3}$$

Similar to [10, 19, 21], we can use the Galerkin method and some a priori estimates to prove that the following result holds for $\mathbb{P}$-a.e. $\omega \in \Omega$:

For $p \ge 2$, $v_s \in V$, $s < T \in R$, there exists a unique weak solution to (3.2)-(3.3) satisfying $v \in \mathscr{C}(s,T;V)\bigcap L^2(s,T;H_0^2(D))$ with $v(s) = v_s$.

The above result shows that $v(t,\omega;s,v_s)$ is unique and continuous with respect to the initial value v_s in V, where $v(t,\omega;s,v_s)$ is the solution of (3.2)-(3.3). We can define a random dynamical system $(S(t,s;\omega))_{t\ge s,\omega\in\Omega}$ by

$$S(t,s;\omega)u_s = u(t,\omega;s,u_s) = \eta^{-1}(t,\omega)v(t,\omega;s,\eta(s,\omega)u_s).$$

Obviously, $(S(t,s;\omega))_{t\ge s,\omega\in\Omega}$ satisfies the conditions in Definition 2.2. Therefore, it is a continuous random dynamical system on V. It can be easily checked that the assumptions (1)-(4) are satisfied in Theorem 2.1.

In the following, we will prove the existence of a compact attracting set $K(\omega)$ at time 0 in V. First, we would obtain the existence of a bounded absorbing set by some estimates in H and V; second, we use the compactness of the embedding to prove the existence of a compact random attractor.

LEMMA 3.1. *Letting $p>2, g\in H$, there exists a random radius $r_1(\omega)$, such that $\forall\rho>0$, there exists $\overline{s}(\omega)\le -1$, such that for all $s\le\overline{s}(\omega)$, and for all $u_s\in H$, with $\|u_s\|\le\rho$, the solution of equations (3.2)-(3.3) with $v_s=\eta(s)u_s$ satisfies the inequality*

$$\|v(-1,\omega;s,\eta(s,\omega)u_s)\|^2\le r_1^2(\omega),\quad \mathbb{P}-a.e.,$$

where $r_1^2(\omega)=e^{\lambda_1}(1+\frac{||g||^2}{\lambda_1}\int_{-\infty}^{-1}e^{\lambda_1\sigma}\eta^2(\sigma)d\sigma)$.

Proof. Taking the inner product of equation (3.2) with v in H, and noticing the fact that $b(u,u,v)=0$, we get

$$\frac{1}{2}\frac{d}{dt}\|v\|^2+\|\nabla v\|^2+2\mu_0\eta\int_D|e(u)|^{p-2}e(u)e(v)dx=(\eta g,v).\tag{3.4}$$

Noticing the condition $v=\eta u$, we obtain $e(v)=\eta e(u)$.

Letting $I=2\mu_0\eta\int_D|e(u)|^{p-2}e(u)e(v)dx$, we get

$$I=2\mu_0\int_D|e(u)|^{p-2}|e(v)|^2dx>0.\tag{3.5}$$

We drop the term I in the equation (3.4), and deduce that

$$\frac{1}{2}\frac{d}{dt}\|v\|^2+\|\nabla v\|^2\le\frac{\|\eta g\|^2}{2\lambda_1}+\frac{\lambda_1\|v\|^2}{2},\tag{3.6}$$

where λ_1 is the first eigenvalue of operator A.

$$\frac{1}{2}\frac{d}{dt}\|v\|^2+\lambda_1\|v\|^2\le\frac{\|\eta g\|^2}{2\lambda_1}+\frac{\lambda_1\|v\|^2}{2}.\tag{3.7}$$

By Gronwall's lemma on the interval $[s,-1]$ for above inequality, we can deduce

$$\begin{aligned}\|v(-1)\|^2&\le e^{-\lambda_1(-1-s)}\|\eta(s)u(s)\|^2+\int_s^{-1}e^{-\lambda_1(-1-\sigma)}\frac{\|g\|^2}{\lambda_1}\eta^2(\sigma)d\sigma\\&\le e^{\lambda_1}(e^{\lambda_1 s}\eta^2(s)\|u_s\|^2+\frac{\|g\|^2}{\lambda_1}\int_{-\infty}^{-1}e^{\lambda_1\sigma}\eta^2(\sigma)d\sigma).\end{aligned}\tag{3.8}$$

By a standard argument,

$$\lim_{t\to-\infty}\frac{1}{t}\sum_{j=1}^m b_j\omega_j(t)=0,\quad \mathbb{P}-a.s.$$

It follows that $s\mapsto e^{\lambda_1 s}\eta^2(s)$ is pathwise integrable over $(-\infty,0]$,

$$\lim_{s\to-\infty}e^{\lambda_1 s}\eta^2(s)=0\quad \mathbb{P}-a.s.$$

Let $r_1^2(\omega)=e^{\lambda_1}(1+\frac{\|g\|^2}{\lambda_1}\int_{-\infty}^{-1}e^{\lambda_1\sigma}\eta^2(\sigma)d\sigma)$. Given $\rho>0$, there exists $\overline{s}(\omega)$ such that $e^{\lambda_1 s}\eta^2(s)\rho^2\le 1$, for all $s\le\overline{s}(\omega)$, and it follows that $\|v(-1,\omega;s,\eta(s,\omega)u_s)\|^2\le r_1^2(\omega)$. □

LEMMA 3.2. *Letting $p>2, g\in H$, there exist random radii $r_2(\omega)$ and $r_3(\omega)$, such that $\forall\rho>0$, there exists $\overline{s}(\omega)\le -1$, such that for all $s\le\overline{s}(\omega)$, and for all $u_s\in H$, with $\|u_s\|\le\rho$, the solution of equations (3.2)-(3.3) with $v_s=\eta(s)u_s$ satisfies the following inequalities:*

$$\|v(t,\omega;s,\eta(s,\omega)u_s)\|^2\le r_2^2(\omega),\, t\in[-1,0],\quad \mathbb{P}-a.e.,$$

$$\int_{-1}^0\|\nabla v\|^2dt\le r_3^2(\omega),\, \mathbb{P}-a.e.,$$

where

$$r_2^2(\omega)=r_1^2(\omega)+\frac{\|g\|^2}{\lambda_1}\int_{-1}^0\eta^2(s)ds,$$
$$r_3^2(\omega)=\frac{r_1^2(\omega)}{2\mu_1}+\frac{r_2(\omega)\|g\|}{\mu_1}\int_{-1}^0\eta(t)dt.$$

Proof. From Lemma 3.1, using Gronwall's lemma for inequality (3.7) again with $t\in[-1,0]$, we get

$$\begin{aligned}\|v(t)\|^2&\le e^{-\lambda_1(t+1)}\|v(-1)\|^2+\frac{\|g\|^2}{\lambda_1}\int_{-1}^t e^{-\lambda_1(t-s)}\eta^2(s)ds\\&\le e^{-\lambda_1(t+1)}r_1^2(\omega)+\frac{\|g\|^2}{\lambda_1}\int_{-1}^t e^{-\lambda_1(t-s)}\eta^2(s)ds\\&\le r_1^2(\omega)+\frac{\|g\|^2}{\lambda_1}\int_{-1}^0\eta^2(s)ds\\&\doteq r_2^2(\omega).\end{aligned}\tag{3.9}$$

From (3.4) and (3.5), it follows that

$$\frac{d}{dt}\|v\|^2+2\|\nabla v\|^2\le 2\eta\|g\|\,\|v\|.\tag{3.10}$$

Integrating the above inequality with t from -1 to 0, then

$$\|v(0)\|^2+2\int_{-1}^0\|\nabla v\|^2dt\le\|v(-1)\|^2+2\int_{-1}^0\eta(t)\|g\|\,\|v\|dt.\tag{3.11}$$

We drop the first term in the left hand side of (3.11), and thus get

$$\begin{aligned}\int_{-1}^0\|\nabla v\|^2dt&\le\frac{r_1^2(\omega)}{2}+\int_{-1}^0\eta(t)\|g\|r_2(\omega)dt\\&=\frac{r_1^2(\omega)}{2}+r_2(\omega)\|g\|\int_{-1}^0\eta(t)dt\\&\doteq r_3^2(\omega).\end{aligned}\tag{3.12}$$

□

LEMMA 3.3. *Letting $p>2, g\in H$, there exists a random radius $r_4(\omega)$, such that $\forall\rho>0$, there exists $\overline{s}(\omega)\le -1$, such that for all $s\le\overline{s}(\omega)$, and for all $u_s\in H$, with $\|u_s\|\le\rho$, the solution of equations (3.2)-(3.3) with $v_s=\eta(s)u_s$ satisfies the inequalities*

$$\|v(t,\omega;s,\eta(s,\omega)u_s)\|_1^2\le r_4^2(\omega),\ t\in[-1,0], \mathbb{P}-a.e.,$$

$$\int_{-1}^{0}\|\Delta v\|^2 dt \le r_5^2(\omega), \mathbb{P}-a.e.$$

Proof. Taking the inner product of equation (3.2) with $-\Delta v$ in H, we can obtain

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|\nabla v\|^2+\|\Delta v\|^2-\eta b(u,u,\Delta v)&\\ +2\mu_0\eta\int_D |e(u)|^{p-2}e(u)e(-\Delta v)dx&=(\eta g,-\Delta v). \qquad (3.13)\end{aligned}$$

First, letting $J=2\mu_0\eta\int_D |e(u)|^{p-2}e(u)e(-\Delta v)dx$, we obtain

$$\begin{aligned}J&=2\mu_0\eta^2\int_D |e(u)|^{p-2}e(u)e(-\Delta u)dx\\ &=2\mu_0\eta^2\Big[\int_D |e(u)|^{p-2}\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(u)}{\partial x_k}dx+(p-2)\int_D |e(u)|^{p-2}\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(u)}{\partial x_k}dx\Big]\\ &=2\mu_0\eta^2(p-1)\int_D |e(u)|^{p-2}\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(u)}{\partial x_k}dx. \qquad (3.14)\end{aligned}$$

Obviously $J>0$, so we drop it in the following computation.

For the third term in the left hand side of equation (3.13), we deduce that

$$\begin{aligned}|\eta b(u,u,\Delta v)|&\le \eta\|u\|_{L^4}\|\|\nabla u\|_{L^4}\|\Delta v\|\\ &\le C\eta\|u\|^{\frac{1}{2}}\|\nabla u\|^{\frac{1}{2}}\|\nabla u\|^{\frac{1}{2}}\|\Delta u\|^{\frac{1}{2}}\|\Delta v\|\\ &=C\eta^{-1}\|v\|^{\frac{1}{2}}\|\nabla v\|\|\Delta v\|^{\frac{3}{2}}\\ &\le\frac{1}{4}\|\Delta v\|^2+C\eta^{-4}\|v\|^2\|\nabla v\|^4\\ &=\frac{1}{4}\|\Delta v\|^2+(C\eta^{-4}\|v\|^2\|\nabla v\|^2)\|\nabla v\|^2, \qquad (3.15)\end{aligned}$$

where the first inequality is due to the Hölder inequality, the second inequality is due to the Gagliardo-Nirenberg inequality

$$\|u\|_{L^4}\le C\|u\|^{\frac{1}{2}}\|\nabla u\|^{\frac{1}{2}}, \quad \text{for} \quad n=2,$$

and the third inequality is due to the ϵ-Young inequality.

Obviously, applying the ϵ-Young inequality, we get

$$|(\eta g,\Delta v)|\le \eta^2\|g\|^2+\frac{\|\Delta v\|^2}{4}.$$

Combining the above estimates, we can transfer (3.13) into the following inequality:

$$\frac{1}{2}\frac{d}{dt}\|\nabla v\|^2+\frac{1}{2}\|\Delta v\|^2\le \eta^2\|g\|^2+(C\eta^{-4}\|v\|^2\|\nabla v\|^2)\|\nabla v\|^2. \qquad (3.16)$$

Applying Gronwall's lemma for $-1\le\tau\le t\le 0$, then

$$\|\nabla v(t)\|^2\le\|\nabla v(\tau)\|^2 e^{\int_\tau^t C\eta^{-4}(\sigma)\|v\|^2\|\nabla v\|^2 d\sigma}+2\|g\|^2\int_\tau^t \eta^2(\sigma)e^{\int_\sigma^t C\eta^{-4}(\theta)\|v\|^2\|\nabla v\|^2 d\theta}d\sigma$$

$$\leq (\|\nabla v(\tau)\|^2 + 2\|g\|^2 \int_{-1}^{0} \eta^2(\sigma) d\sigma) e^{\int_{-1}^{0} C\eta^{-4}(\sigma)\|v\|^2\|\nabla v\|^2 d\sigma}. \tag{3.17}$$

Integrating with respect to τ over $[-1,0]$, we obtain

$$\begin{aligned}\|\nabla v(t)\|^2 &\leq \Big(\int_{-1}^{0} \|\nabla v(\tau)\|^2 d\tau + 2\|g\|^2 \int_{-1}^{0} \eta^2(\sigma) d\sigma\Big) e^{\int_{-1}^{0} C\eta^{-4}(\sigma)\|v\|^2\|\nabla v\|^2 d\sigma} \\ &\leq (r_3^2(\omega) + 2\|g\|^2 \int_{-1}^{0} \eta^2(\sigma) d\sigma e^{C\sup_{-1\leq\sigma\leq 0} \eta^{-4}(\sigma) r_2^2(\omega) r_3^2(\omega)} \\ &\doteq r_4^2(\omega).\end{aligned} \tag{3.18}$$

Thus,

$$\begin{aligned}\|u(t,\omega;s,u_s)\|_1^2 &= \|\eta^{-1}(t,\omega) v(t,\omega;s,\eta(s,\omega)u_s)\|_1^2 \\ &\leq \sup_{-1\leq t\leq 0} \frac{1}{\eta^2(t,\omega)} \|v(t,\omega;s,\eta(s,\omega)u_s)\|_1^2 \\ &\leq r_4^2(\omega) \sup_{-1\leq t\leq 0} \frac{1}{\eta^2(t,\omega)},\end{aligned} \tag{3.19}$$

and from above results, $r_4^2(\omega)\sup_{-1\leq t\leq 0} \frac{1}{\eta^2(t,\omega)}$ is bounded. This lemma implies the existence of a bounded absorbing set.

Integrating the inequality (3.16) with t from -1 to 0, then

$$\begin{aligned}\|\nabla v(0)\|^2 + \int_{-1}^{0} \|\Delta v\|^2 dt \leq & \|\nabla v(-1)\|^2 + 2\|g\|^2 \int_{-1}^{0} \eta^2(t) dt \\ &+ 2\int_{-1}^{0} (C\eta^{-4}\|v\|^2\|\nabla v\|^2)\|\nabla v\|^2 dt.\end{aligned} \tag{3.20}$$

We drop the first term in the left hand side of (3.20), and thus we get

$$\begin{aligned}\int_{-1}^{0} \|\Delta v(t)\|^2 dt &\leq r_4^2(\omega) + 2\|g\|^2 \int_{-1}^{0} \eta^2(t) dt + 2C r_2^2(\omega) r_4^4(\omega) \int_{-1}^{0} \eta^{-4}(t) dt \\ &\doteq r_5^2(\omega).\end{aligned} \tag{3.21}$$

□

4. The existence of random attractor

In this section, we will deduce some estimates in $H^2(D)$. Then we use these estimates and the compactness of the embedding to obtain the existence of a compact random attractor.

LEMMA 4.1. *Letting $2 < p < \frac{7}{3}, g \in H^1$, there exists a random radius $r_4(\omega)$, such that $\forall \rho > 0$, there exists $\overline{s}(\omega) \leq -1$, such that for all $s \leq \overline{s}(\omega)$, and for all $u_s \in H$, with $\|u_s\| \leq \rho$, the solution of equations (3.2)-(3.3) with $v_s = \eta(s)u_s$ satisfies the inequality*

$$\|v(0,\omega;s,\eta(s,\omega)u_s)\|_2^2 \leq r_6^2(\omega),\ \mathbb{P}-a.e.$$

Proof. Taking the inner product of equation (3.2) with $\Delta^2 v$ in H, we can obtain

$$\frac{1}{2}\frac{d}{dt}\|\Delta v\|^2 + \|v\|_3^2 + \eta b(u,u,\Delta^2 v) + 2\mu_0 \eta \int_D |e(u)|^{p-2} e(u) e(\Delta^2 v) dx = (\eta g, \Delta^2 v). \tag{4.1}$$

Next, we estimate these terms in equation (4.1) respectively, letting

$$
\begin{aligned}
Q =& 2\mu_0\eta\int_D |e(u)|^{p-2}e_{ij}(u)e_{ij}(\Delta^2 v)dx\\
=& 2\mu_0\eta^2\int_D |e(u)|^{p-2}e_{ij}(u)e_{ij}(\Delta^2 u)dx\\
=& -2\mu_0\eta^2\int_D |e(u)|^{p-2}\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(\Delta u)}{\partial x_k}dx\\
& -2\mu_0(p-2)\eta^2\int_D |e(u)|^{p-2}\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(\Delta u)}{\partial x_k}dx\\
=& -2\mu_0(p-1)\eta^2\int_D |e(u)|^{p-2}\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(\Delta u)}{\partial x_k}dx. \qquad (4.2)
\end{aligned}
$$

Furthermore, integrating by parts again,

$$
\begin{aligned}
Q =& -2\mu_0(p-1)\eta^2\int_D |e(u)|^{p-2}\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(\Delta u)}{\partial x_k}dx\\
=& 2\mu_0(p-1)(p-2)\eta^2\int_D |e(u)|^{p-4}e_{ij}(u)\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(u)}{\partial x_l}\frac{\partial^2 e_{ij}(u)}{\partial x_k\partial x_l}dx\\
& +2\mu_0(p-1)\eta^2\int_D |e(u)|^{p-2}\frac{\partial^2 e_{ij}(u)}{\partial x_k\partial x_l}\frac{\partial^2 e_{ij}(u)}{\partial x_k\partial x_l}dx\\
\doteq& Q_1+Q_2. \qquad (4.3)
\end{aligned}
$$

From the assumed condition $2<p<\frac{7}{3}$, we get $Q_2>0$, thus we drop it in the following computation. For the first term Q_1, we have

$$
\begin{aligned}
|Q_1| \le& 2\mu_0(p-1)(p-2)\eta^2\int_D |e(u)|^{p-3}\frac{\partial e_{ij}(u)}{\partial x_k}\frac{\partial e_{ij}(u)}{\partial x_l}\frac{\partial^2 e_{ij}(u)}{\partial x_k\partial x_l}dx\\
\le& 2\mu_0(p-1)(p-2)\eta^2\|e(u)\|_{L^\infty}^{p-3}\|\nabla^2 u\|_{L^4}^2\|u\|_3\\
\le& 2\mu_0 C(p-1)(p-2)\eta^2\|u\|_3^{p-3}\|u\|^{\frac{1}{3}}\|u\|_3^{\frac{5}{3}}\|u\|_3\\
=& 2\mu_0 C(p-1)(p-2)\eta^{2-p}\|v\|^{\frac{1}{3}}\|v\|_3^{p-\frac{1}{3}}, \qquad (4.4)
\end{aligned}
$$

where the second inequality is due to the Hölder inequality, the third inequality is due to the Sobolev embedding $H^2\hookrightarrow L^\infty$, for $n=2$, and the Gagliardo-Nirenberg inequality

$$\|\nabla^2 u\|_{L^4}\le C\|\nabla^3 u\|^{\frac{5}{6}}\|u\|^{\frac{1}{6}}.$$

Noticing the condition $2<p<\frac{7}{3}$, we can apply the ϵ-Young inequality $ab\le\epsilon a^{p'}+C(\epsilon)b^{q'}$ with $\epsilon=\frac{1}{4}$, $p'=\frac{6}{3p-1}$, $q'=\frac{6}{7-3p}$ for (4.4), and we obtain

$$
\begin{aligned}
|Q_1| \le& \frac{1}{4}\|v\|_3^2+\frac{7-3p}{6}\Big(\frac{6p-2}{3}\Big)^{\frac{3p-1}{7-3p}}\big(2\mu_0 C(p-1)(p-2)\eta^{2-p}\|v\|^{\frac{1}{3}}\big)^{\frac{6}{7-3p}}\\
=& \frac{1}{4}\|v\|_3^2+\frac{7-3p}{6}\Big(\frac{6p-2}{3}\Big)^{\frac{3p-1}{7-3p}}\big[2\mu_0 C(p-1)(p-2)\big]^{\frac{6}{7-3p}}\|v\|^{\frac{2}{7-3p}}\eta^{\frac{12-6p}{7-3p}}. \qquad (4.5)
\end{aligned}
$$

For the last term in the left hand side of (4.1), we have

$$\eta|b(u,u,\Delta^2 v)|=\eta^2|b(u,u,\Delta^2 u)|\le\eta^2\|u\|_{L^\infty}\|\Delta u\|\|D^3 u\|+\eta^2\|\nabla u\|_{L^4}^2\|D^3 u\|, \qquad (4.6)$$

and we estimate the right hand side of (4.6) respectively.

For the first term, applying the Agmon inequality

$$\|u\|_{L^\infty} \le C\|u\|^{\frac{1}{2}}\|\Delta u\|^{\frac{1}{2}},$$

then

$$\eta^2\|u\|_{L^\infty}\|\Delta u\|\|D^3u\| \le C\eta^2\|u\|^{\frac{1}{2}}\|\Delta u\|^{\frac{3}{2}}\|D^3u\|.$$

From Lemma 2.2, we know

$$\|u\|_2 \le C\|u\|_1^{\frac{1}{2}}\|u\|_3^{\frac{1}{2}},$$

$$\eta^2\|u\|_{L^\infty}\|\Delta u\|\|D^3u\| \le C\eta^2\|u\|^{\frac{1}{2}}\|u\|_1^{\frac{3}{4}}\|u\|_3^{\frac{7}{4}} = C\eta^{-1}\|v\|^{\frac{1}{2}}\|v\|_1^{\frac{3}{4}}\|v\|_3^{\frac{7}{4}}, \tag{4.7}$$

and applying the ϵ-Young inequality with $\epsilon = \frac{1}{8}$,

$$\eta^2\|u\|_{L^\infty}\|\Delta u\|\|D^3u\| \le \frac{1}{8}\|v\|_3^2 + \frac{7^7}{8}(C\|v\|^{\frac{1}{2}}\|v\|_1^{\frac{3}{4}})^8\eta^{-8}. \tag{4.8}$$

For the second term in the right hand side of (4.6), apply the Gagliardo-Nirenberg inequality

$$\|\nabla u\|_{L^4} \le C\|\Delta u\|^{\frac{3}{4}}\|u\|^{\frac{1}{4}}$$

to get

$$\eta^2\|\nabla u\|_{L^4}^2\|D^3u\| \le C\eta^2\|u\|^{\frac{1}{2}}\|\Delta u\|^{\frac{3}{2}}\|D^3u\|,$$

and similar to (4.7) and (4.8) we have

$$\eta^2\|\nabla u\|_{L^4}^2\|D^3u\| \le \frac{1}{8}\|v\|_3^2 + \frac{7^7}{8}(C\|v\|^{\frac{1}{2}}\|v\|_1^{\frac{3}{4}})^8\eta^{-8}. \tag{4.9}$$

From estimates (4.6)-(4.9), we can obtain

$$\eta|b(u,u,\Delta^2 v)| \le \frac{1}{4}\|v\|_3^2 + \frac{7^7}{4}(C\|v\|^{\frac{1}{2}}\|v\|_1^{\frac{3}{4}})^8\eta^{-8}. \tag{4.10}$$

Obviously, applying the ε-Young inequality, we get

$$|(\eta g, \Delta^2 v)| \le \eta^2\|g\|_1^2 + \frac{\|v\|_3^2}{4}.$$

Combining the above estimates, we can transfer (4.1) into the following inequality:

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|\Delta v\|^2 + \frac{1}{4}\|v\|_3^2 \le{}& \frac{7-3p}{6}\left(\frac{6p-2}{3}\right)^{\frac{3p-1}{7-3p}}\left[2\mu_0 C(p-1)(p-2)\right]^{\frac{6}{7-3p}}\|v\|^{\frac{2}{7-3p}}\eta^{\frac{12-6p}{7-3p}}\\ &+\eta^2\|g\|_1^2 + \frac{7^7}{4}C\|v\|^4\|v\|_1^6\eta^{-8}.\end{aligned}$$

It is clear that

$$\frac{1}{2}\frac{d}{dt}\|\Delta v\|^2 + \frac{1}{8}\|v\|_3^2 + \frac{\lambda_1}{8}\|\Delta v\|^2$$

$$\leq \frac{7-3p}{6}\left(\frac{6p-2}{3}\right)^{\frac{3p-1}{7-3p}}\left[2\mu_0 C(p-1)(p-2)\right]^{\frac{6}{7-3p}}\|v\|^{\frac{2}{7-3p}}\eta^{\frac{12-6p}{7-3p}}$$
$$+\eta^2\|g\|_1^2+\frac{7^7}{4}C\|v\|^4\|v\|_1^6\eta^{-8}, \tag{4.11}$$

where λ_1 is the first eigenvalue of operator A. ∎

Applying Gronwall's inequality for $-1\leq s\leq t\leq 0$, we can obtain

$$\begin{aligned}
&\|\Delta v(t)\|^2\\
\leq&\|\Delta v(s)\|^2 e^{-\frac{\lambda_1}{4}(t-s)}+2\|g\|_1^2\int_s^t e^{-\frac{\lambda_1}{4}(t-\tau)}\eta^2(\tau)d\tau\\
&+\frac{7^7}{2}Cr_2^4(\omega)r_4^6(\omega)\int_s^t e^{-\frac{\lambda_1}{4}(t-\tau)}\eta^{-8}(\tau)d\tau\\
&+\frac{7-3p}{3}\left(\frac{6p-2}{3}\right)^{\frac{3p-1}{7-3p}}\left[2\mu_0 C(p-1)(p-2)\right]^{\frac{6}{7-3p}}r_2^{\frac{2}{7-3p}}(\omega)\int_s^t e^{-\frac{\lambda_1}{4}(t-\tau)}\eta^{\frac{12-6p}{7-3p}}(\tau)d\tau\\
\leq&\|\Delta v(s)\|^2+2\|g\|_1^2\int_{-1}^0\eta^2(\tau)d\tau+\frac{7^7}{2}Cr_2^4(\omega)r_4^6(\omega)\int_{-1}^0\eta^{-8}(\tau)d\tau\\
&+\frac{7-3p}{3}\left(\frac{6p-2}{3}\right)^{\frac{3p-1}{7-3p}}\left[2\mu_0 C(p-1)(p-2)\right]^{\frac{6}{7-3p}}r_2^{\frac{2}{7-3p}}(\omega)\int_{-1}^0\eta^{\frac{12-6p}{7-3p}}(\tau)d\tau. \qquad (4.12)
\end{aligned}$$

Integrating with respect to s over $[-1,0]$, we obtain

$$\begin{aligned}
\|\Delta v(t)\|^2\leq&\int_{-1}^0\|\Delta v(s)\|^2ds+2\|g\|_1^2\int_{-1}^0\eta^2(\tau)d\tau+\frac{7^7}{2}Cr_2^4(\omega)r_4^6(\omega)\int_{-1}^0\eta^{-8}(\tau)d\tau\\
&+\frac{7-3p}{3}\left(\frac{6p-2}{3}\right)^{\frac{3p-1}{7-3p}}\left[2\mu_0 C(p-1)(p-2)\right]^{\frac{6}{7-3p}}r_2^{\frac{2}{7-3p}}(\omega)\int_{-1}^0\eta^{\frac{12-6p}{7-3p}}(\tau)d\tau\\
\leq&r_5^2(\omega)+2\|g\|_1^2\int_{-1}^0\eta^2(\tau)d\tau+\frac{7^7}{2}Cr_2^4(\omega)r_4^6(\omega)\int_{-1}^0\eta^{-8}(\tau)d\tau\\
&+\frac{7-3p}{3}\left(\frac{6p-2}{3}\right)^{\frac{3p-1}{7-3p}}\left[2\mu_0 C(p-1)(p-2)\right]^{\frac{6}{7-3p}}r_2^{\frac{2}{7-3p}}(\omega)\int_{-1}^0\eta^{\frac{12-6p}{7-3p}}(\tau)d\tau\\
\doteq&r_6^2(\omega). \qquad (4.13)
\end{aligned}$$

Especially, for $t=0$,

$$\|u(0)\|_2^2=\|v(0)\|_2^2\doteq r_6^2(\omega).$$

THEOREM 4.1. *Letting $2<p<\frac{7}{3}, g\in H^1$, there exists a random attractor for the stochastic monopolar non-Newtonian fluid with multiplicative noise* (2.3)-(2.4) *in V.*

Proof. Letting $K(\omega)$ be the ball in $H^2(D)$ of radius $r_6(\omega)$, we have proved that for any B bounded in V, there exists $\overline{s}(\omega)$ such that for $s\leq\overline{s}(\omega)$,

$$S(0,s;\omega)B\subset K(\omega)\,\mathbb{P}-a.e.$$

This clearly implies that $K(\omega)$ is an attracting set at time $t=0$. Because it is compact in V, Theorem 2.1 applies. ∎

Acknowledgment. This paper is supported by the National Natural Science Foundation of China (No. 11126160, 11201475, 11371183).

REFERENCES

[1] H. Bellout, F. Bloom, and J. Nečas, *Young measure-valued solutions for non-Newtonian incompressible viscous fluids*, Commun. Part. Diff. Eqs., 19, 1763–1803, 1994.

[2] H. Bellout, F. Bloom, and J. Nečas, *Existence, uniqueness, and stability of solutions to the initial boundary value problem for bipolar viscous fluids*, Diff. Int. Eqs., 8, 453–464, 1995.

[3] F. Bloom, *Attractors of non-Newtonian fluids*, J. Dyn. Diff. Eqs., 7, 109–140, 1995.

[4] F. Bloom and W. Hao, *Regularization of a non-Newtonian system in an unbounded channel: Existence and uniqueness of solutions*, Nonlin. Anal., 44, 281–309, 2001.

[5] F. Bloom and W. Hao, *Regularization of a non-Newtonian system in an unbounded channel: Existence of a maximal compact attractor*, Nonlin. Anal. TMA, 43, 743–766, 2001.

[6] W. Chen, *Non-Newtonian Fluids*, Science Press, 1984.

[7] H. Crauel, A. Debussche, and F. Flandoli, *Random attractors*, J. Dyn. Diff. Eqs., 9, 307–341, 1997.

[8] H. Crauel and F. Flandoli, *Attractors for random dynamical systems*, Prob. Th. Rel. Fields, 100, 365–393, 1994.

[9] G. Da Prato, A. Debussche, and R. Temam, *Stochastic Burgers' equation*, NoDEA, 1, 389–402, 1994.

[10] G. Da Prato and J. Zabczyk, *Stochastic Equations in Infinite Dimensions*, Cambridge University Press, 1992.

[11] A. de Bouard and A. Debussche, *On the stochastic Korteweg-de Vries equation*, J. Func. Anal., 154, 215–251, 1998.

[12] A. de Bouard and A. Debussche, *A stochastic nonlinear Schrödinger equation with multiplicative noise*, Commun. Math. Phys., 205, 161–181, 1999.

[13] Q. Du and M. Gunzburger, *Analysis of a Ladyzhenskaya model for incompressible viscous flow*, J. Math. Anal. Appl., 155, 21–45, 1991.

[14] B. Guo and C. Guo, *The convergence of non-Newtonian fluids to Navier-Stokes equations*, J. Math. Anal. Appl., 357, 468–478, 2009.

[15] B. Guo, C. Guo, and Y. Han, *Random attractors of stochastic non-Newtonian fluid*, Acta Math. Appl. Sinica (English Series), 28, 165–180, 2012.

[16] B. Guo and C. Guo, *Remark on random attractor for a two dimensional incompressible non-newtonian fluid with multiplicative noise*, Commun. Math. Sci., 10, 821–833, 2012.

[17] B. Guo, C. Guo, and J. Zhang, *Martingale and stationary solutions for stochastic non-Newtonian fluids*, Diff. Integral Eqs., 23, 303–326, 2010.

[18] B. Guo, G. Lin, and Y. Shang, *Non-Newtonian Fluids Dynamical Systems*, National Defense Industry Press, 2006.

[19] O. Ladyzhenskaya, *New equations for the description of the viscous incompressible fluids and solvability in the large of the boundary value problems for them*, in Boundary Value Problems of Mathematical Physics V, Amer. Math. Scoc., Providence, RI, 1970.

[20] Š. Nečasová and P. Penel, *L^2 decay for weak solution to equations of non-Newtonian incompressible fluids in the whole space*, Nonlin. Anal. TMA, 47, 4181–4192, 2001.

[21] M. Pokorný, *Cauchy problem for the non-Newtonian incompressible fluids*, Appl. Math., 41, 169–201, 1996.

[22] R. Temam, *Infinite Dimensional Dynamical Systems in Mechanics and Physics*, 2nd Edition, Springer, Berlin, 1997.

[23] C. Zhao and S. Zhou, *Pullback attractors for a non-autonomous incompressible non-Newtonian fluid*, J. Diff. Eqs., 238, 394–425, 2007.

[24] C. Zhao and Y. Li, *A note on the asymptotic smoothing effect of solutions to a non-Newtonian system in 2-D unbounded domains*, Nonlin. Anal., 60, 475–483, 2005.

[25] C. Zhao, Y. Li, and S. Zhou, *Random attractor for a two-dimensional incompressible non-Newtonian fluid with multiplicative noise*, Acta Math. Sci., 31, 567–575, 2011.

[26] C. Zhao and J. Duan, *Random attractor for the Ladyzhenskaya model with additive noise*, J. Math. Anal. Appl., 31, 241–251, 2010.