Preface

Dave Levermore

C. David (Dave) Levermore was born on November 19th, 1951 in Teaneck, New Jersey. After graduating from Clarkson College in 1974, he studied for his doctorate under the supervision of Peter Lax. He received his PhD from New York University in 1982. At the beginning of his career, Dave worked in the Lawrence Livermore National Laboratory from 1982 to 1988. He later joined the University of Arizona as an Associate Professor and became a Professor in 1992. In 2000, he moved to the University of Maryland as a Professor in the Department of Mathematics and in the Institute for Physical Science and Technology, his present position.

Scientific Contributions. Dave is at the origin of several fundamental ideas and results in the analysis of nonlinear PDEs, mostly in connection with mathematical physics. His lasting contributions to science already began during his undergraduate years, at a time when much of the fundamental early research in the field of soliton theory and integrable systems was being carried out by faculty at Clarkson College. Indeed, a careful reader of the famous 1974 paper of M. Ablowitz, D. Kaup, A. Newell, and H. Segur in *Studies in Applied Mathematics* will find a footnote therein referring to a calculation contributed by a certain precocious Clarkson undergraduate.

Dave's doctoral thesis involved what is now known as the Lax-Levermore theory of the small dispersion limit of the KdV equation, presented in a series of three articles published in *Communications on Pure and Applied Mathematics* in 1983. Starting from the KdV equation set on the real line, written as

$$\partial_t u_\epsilon + u_\epsilon \partial_x u_\epsilon = \epsilon^2 \partial_{xxx} u_\epsilon\,, \qquad u_\epsilon\big|_{t=0} = u^{in}$$

one seeks the limit of u_ϵ as $\epsilon \to 0$. The first major result in the Lax-Levermore theory is that the limit is given in terms of a variational problem, vaguely analogous to the case of the vanishing viscosity limit of the Burgers equation

$$\partial_t v_\epsilon + v_\epsilon \partial_x v_\epsilon = \epsilon \partial_{xx} v_\epsilon\,, \qquad v_\epsilon\big|_{t=0} = v^{in}$$

where the limit is also given by a much simpler variational problem (involving the inf-convolution of $x^2/2t$ with a primitive of v^{in}) known as the Hopf-Lax formula. If the initial data u^{in} is a simple well, rapidly decaying at infinity, the small ϵ limit of u_ϵ is governed by the Hopf equation

$$\partial_t u + u \partial_x u = 0\,, \qquad u\big|_{t=0} = u^{in}$$

as long as the solution of this equation remains smooth. After that, the dispersionless limit of the KdV equation, given by Lax-Levermore variational formula, differs from the usual extension of the solution of the Hopf equation involving shock waves — the "entropy solution" that is obtained as the vanishing viscosity limit of the solution of the Burgers equation. Instead, under some specific conditions, the Lax-Levermore variational problem leads to the solution of a hyperbolic system of PDEs previously obtained by G. Whitham by a formal averaging procedure that can be viewed as a nonlinear analogue of the classical WKB ansatz used in the semiclassical limit of the linear Schrödinger equation. In 1980, H. Flaschka, G. Forest and D. McLaughlin had obtained a vast generalization of Whitham's modulation ansatz, and the connection between these results and the dispersionless limit of the KdV equation was a striking success of the Lax-Levermore theory. Some time later, with Shan Jin and D. McLaughlin, Dave obtained an analogous theory for the one-dimensional cubic defocusing nonlinear Schrödinger equation. All these works are based on integrability techniques, but Dave was also very interested in dispersionless limits of nonintegrable problems, as seen for example in his work with J.-G. Liu on the behavior of dispersive numerical schemes for shock waves. It should also be pointed out that the variational approach for semiclassical limits of integrable PDE pioneered by Dave beginning with his thesis work has since found far-reaching and unanticipated applications in the areas of approximation theory and random matrix theory.

In connection with his work on plasmas at the Lawrence Livermore National Laboratory, Dave studied radiative transfer in various asymptotic regimes. In 1979, he came up with a systematic derivation of flux limited diffusion theory, based on an appropriately designed, formal asymptotic expansion. With G. Pomraning, D. Sanzo and J. Wong, he proposed a homogenization theory for radiative transfer in random media, leading to a great variety of nontrivial qualitative behaviors.

In the late 1980s, R. DiPerna and P.-L. Lions made considerable progress on the analysis of several kinetic models, by constructing global weak solutions of these equations for all physically admissible initial data. Most notably, they obtained global solutions of the Boltzmann equation in the three-dimensional Euclidean space for all initial data with finite total mass, energy and entropy. These solutions are somewhat weaker than what one would normally call weak solutions, and were called "renormalized solutions" by DiPerna and Lions. Being obtained by a compactness argument, these solutions are not known to be uniquely determined by their initial data. Moreover they satisfy a certain entropy inequality that is a weaker variant of Boltzmann's famous H-Theorem — and that would be equivalent to the H-Theorem in the case that the inequality becomes an equality.

The similarities between DiPerna-Lions solutions of the Boltzmann equation and Leray solutions of the incompressible Navier-Stokes equations were too obvious to remain unnoticed. Besides, the problem of deriving the equations of fluid mechanics from Boltzmann's kinetic theory had been famous in the mathematical community since Hilbert's plenary address to the 1900 International Congress of Mathematicians in Paris, where this question was formulated as an example in Hilbert's 6th problem on the axiomatization of physics. At the same time, functional analytic tools for deriving degenerate nonlinear diffusion approximations of various kinetic models had been developed in the Paris PDE school. While the Chapman-Enskog asymptotic expansion relating the Boltzmann equation to the compressible Navier-Stokes equations had been known for a long time in the mathematical community, the corresponding theory in the incompressible case remained unknown[1]. With C. Bardos and F. Golse, Dave formulated a general asymptotic theory leading to the incompressible Euler or Navier-Stokes equations, or to the time-dependent Stokes equations, depending on the relative sizes of the Knudsen and Mach numbers in the gas flow. Unlike earlier asymptotic theories (pioneered by Hilbert, Chapman and Enskog), this new asymptotic theory was based on taking appropriate closures in equations for moments (in the velocity variable) of the distribution function. Hopefully, these closures relations could be established by a careful study of the entropy production term in Boltzmann's H-Theorem — more exactly, in the DiPerna-Lions variant thereof. Together with C. Bardos and F. Golse, Dave proposed a program for deriving Leray solutions of the incompressible Navier-Stokes equations from DiPerna-Lions solutions of the Boltzmann equation, and laid out the main steps for a proof. Eventually, there remained three outstanding difficulties that were solved in the following decade. The first such difficulty was the control of fast oscillations due to acoustic waves, solved in 2000 by a beautiful argument due to P.-L. Lions and N. Masmoudi. A second difficulty was that the local conservation of momentum — a well-known property of *classical* solutions of the Boltzmann equation — is *not* a consequence of the DiPerna-Lions theory of renormalized solutions of the Boltzmann equation. That the renormalized solutions of the Boltzmann equation may fail to satisfy the local momentum conservation shed some doubts on their physical value — and on the soundness of the project of deriving fluid dynamical equations from such solutions. In 2000, Dave realized, with C. Bardos and F. Golse, that the local conservation laws of momentum and energy could be established *after* passing to the fluid dynamic limit, without necessarily being satisfied by solutions of the Boltzmann equation for positive Knudsen numbers, i.e. *before* passing to the limit. This was observed originally on the limit of the Boltzmann equation leading to the acoustic system; Dave subsequently extended this result to the Stokes-Fourier limit with F. Golse. The final difficulty to be handled was a nonlinear compactness of number density fluctuations, which, in the Navier-Stokes limit, controlled the defect of local momentum and energy conservation. Although the Leray energy inequality for Navier-Stokes solutions was obtained as the limiting form of the DiPerna-Lions variant of Boltzmann's H-Theorem, Boltzmann's entropy functional

$$H[F] := \iint F(t,x,v) \ln F(t,x,v)\, dx\, dv$$

fails to control functionals that are quadratic in the distribution function F, or fluctuations thereof about some uniform equilibrium distribution — i.e. Maxwell distribution. This difficulty was finally removed in 2004 by F. Golse and L. Saint-Raymond in the simplest possible case of particle interactions, by combining new ideas on the entropy production with a limiting L^1 case of velocity averaging involving dispersion arguments. The case of more general particle interactions, including elastic hard sphere collisions, and more generally all hard potentials satisfying Grad's angular cutoff assumption was subsequently obtained by the same authors. In collaboration with N. Masmoudi, Dave finally extended the incompressible limit to the most general case of cutoff potentials known at the date of this writing. This completed the Bardos-Golse-Levermore program.

In connection with his work on the fluid dynamic limits of the Boltzmann equations, Dave proposed a general framework for obtaining moment closures from dissi-

[1]Except in the steady case, obtained by Y. Sone as early as 1969.

pative kinetic models. Being based on entropy and entropy production, his approach is physically natural and intrinsic, and leads to a symmetrizable hyperbolic system in the sense of Godunov, and is therefore different from earlier, and sometime rather arbitrary closure strategies. (For instance, Dave's method leads to a 14-moment system instead of Grad's celebrated 13-moment system.) Various numerical methods in gas dynamics or radiative transfer are based on Dave's closure method.

With G.Q. Chen and T.-P. Liu, Dave studied hyperbolic systems of conservation laws pertrubed by stiff relaxation terms. Reduced systems, inviscid and viscous local conservation laws, and weakly nonlinear limits are derived through asymptotic expansions. There is an obvious analogy between this situation and the fluid dynamic limits of the Boltzmann equation. Here also, the role of entropy is extremely important: an entropy condition is introduced for $N \times N$ systems that entails the hyperbolicity of the reduced inviscid system, and the resulting characteristic speeds are shown to be interlaced with those of the original system. Besides, the first correction to the reduced system is shown to be dissipative. Interestingly, there is a partial converse of this result in the simplest case of 2×2 systems.

Dave has also strong interests in numerical analysis. His work with Shi Jin on numerical transport in diffusive regimes, and on hyperbolic conservation laws with stiff relaxation terms, were some of the earliest examples of the now-popular "asymptotic-preserving (AP) schemes" for multiscale kinetic equations that are efficient also in the hydrodynamic regimes. With F. Golse and Shi Jin, he provided the first theoretical framework to understand the uniform convergence of an AP scheme, using the example of numerical passage from (the boundary-value problem of) the linear transport equation to its diffusion limit.

Since Dave was trained as both a physicist and a mathematician, there is perhaps little surprise that core physical notions such as entropy and entropy production, or the transition from microscopic to the macroscopic models are so ubiquitous in his scientific work. In fact, kinetic models and integrable systems are two examples of mathematical theories rooted in very classical and distinguished branches of physics and which have gone through very exciting developments in the past 50 years, and Dave's contributions to both subjects are truly remarkable.

Dave as a Teacher, Mentor, Advisor, and Collaborator. To date, Dave Levermore has supervised 21 PhD dissertations written by his students at the University of California — Davis, at the University of Arizona, and most recently at the University of Maryland.

Two of us were students of Dave's at the University of Arizona in the late 1980s and early 1990's, and we are in a position to share some of our thoughts on his character as a teacher, mentor, and departmental citizen. As a teacher and thesis advisor, Dave has a well-deserved reputation for fostering self-confidence and independent thinking in his students. He has made repeated efforts to allow his students to thrive in the aftermath of graduation, at times even changing his research emphasis to avoid competing with his former students. Dave teaches his students to see the "big picture", but he also strongly emphasizes concern for details. Those who were lucky enough to have Dave as an advisor still remember after many years learning not only how to think of nature existing on a hierarchy of dynamically interacting space and time scales but also how (and why) one should always insert a "thin space" in TeX code between the integrand and the dx of an integral.

Not only does Dave teach his students to write well, and to write about important things, but he is equally concerned that his students learn to publicly present their work in an effective fashion. One of us recalls giving a talk as a student at a Los Alamos conference in which Dave stood in the back of the room listening carefully to the lecture. Immediately afterward Dave offered an honest postgame critique: "Good job," he said, "but starting out your talk by saying that the methodology you're going to use is essentially due to someone else is shooting yourself in the foot." The message of course was to be clear about what your unique contributions are; we are *all* "standing on the shoulders of giants" in Newton's famous words.

Another aspect of Dave's character as an advisor is that he is selfless about pointing out to his students and collaborators those specific problems or broader areas of applied mathematics that are "ripe for picking". One of us remembers several conversations in various cafes scattered around northern New Mexico in which Dave revealed what he thought were great opportunities for scientific advancement. This advice was not offered in a vague sense, but rather with words such as "If I were you, I would go and learn ... and start working on ..." Those who took this sort of advice and acted upon it can vouch for his uncanny ability to smell "the next big thing."

During his years in Tucson, Dave was actively involved in many projects with students, postdocs, and fellow faculty. Without giving too much away, we could mention that several of the faculty earned nicknames among the student population, and Dave's nickname reflected his dedication and extraordinary level of involvement; he was simply "Superdave." Not only did he work well with his colleagues and students, but he also socialized with them when work was over, for example, playing on the flag-football team with the graduate students, joining a weekend hike among the saguaro cacti in Sabino Canyon, or enjoying the annual "Derelict of the Year" party (even though he never actually won the award in spite of his vigorous campaigning, he remained a good sport about it all and was happy to raise his glass in honor of the winner).

Those days in the late 1980's and early 1990's in Tucson were filled with exciting intellectual activity centered around Dave. He had enough students simultaneously to form several "working groups" with whom he met weekly, either presenting material himself or listening as his students spoke on various topics. The density of his students was so high at one point that there were two consecutive days in 1994 during which at least three of Dave's students defended their dissertations in "tag-team" fashion. Even after the thesis defense, Dave remained a mentor and advisor, helping his students sort out the transition from school to work. One of us recalls that when Dave himself came up dry with advice on what to expect when going to work in Australia after graduation, he sought out a colleague from Los Alamos with the relevant experience and arranged a meeting to make sure the correct advice got delivered even if it couldn't come directly from him.

All three of us have benefited greatly from the experience of working with Dave as a collaborator. For one thing, Dave is a deeply thoughtful practitioner of scientific writing as a discipline of its own. He truly frets over exactly which phrase to coin to best describe a new phenomenon, or even which Greek letter should be used to best represent a certain quantity. He prefers not to name things after the symbols commonly used to write them (e.g. "theta-function") but rather wants the name to suggest to the reader what the thing named actually *is*. His grammatical sense is spot-on. But it is Dave's unique and insightful scientific skill and infectious enthusiasm that makes collaboration with him a real joy. We know that many others, including the numerous contributors to this volume, have also felt lucky to have worked with Dave on various scientific projects, and we know that there will be many more to come in the future. Dave, with great joy and sincerest respect, we wish you all the best on the auspicious occasion of your 60th birthday!

François Golse
Shi Jin
Peter Miller