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ON ROBUST H_∞ CONTROL FOR NONLINEAR UNCERTAIN SYSTEMS*

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Abstract. This paper addresses robust H_∞ control problems for nonlinear systems with parameter uncertainty. By using the technique of partition of unity, necessary and sufficient conditions for existence of strong robust H_∞ dynamic compensators and static state feedback controllers are established in terms of nonlinear matrix inequality (NLMI) approach, respectively. These conditions reduce the robust H_∞ control problems to a standard H_∞ problem for an auxiliary nonlinear system.

1. Introduction. Robust control system design based on H_∞ control problem for linear systems and nonlinear systems is a popular research area in the last decade. The H_∞ problems has been reduced to problems of solving Riccati-type equations (AREs) and Hamilton-Jacobi equations (HJEs) (or inequalities HJIs) in state-space framework, respectively (see, [1, 4, 7] and references therein). It has recently been emphasized [2] that H_∞ control problem in linear systems can be cast into the form of first-order matrix polynomial inequalities called linear matrix inequalities (LMIs) instead of AREs, which belong to the group of convex problem and thus can efficiently find feasible and solutions to them via the interior point methods [2]. Subsequently, motivated by LMI approach, the solution of nonlinear H_∞ control problem is characterized by means of nonlinear matrix inequalities (NLMIs) instead of the HJEs or HJIs in [8]. The H_∞ control problems in nonlinear systems with uncertainty are discussed in [9, 10, 12], but only *sufficient* condition for the existence of the robust H_∞ controller has been obtained by means of HJEs (or HJIs). Motivated by the discussion above, in this paper we focus on robust H_∞ control problem in nonlinear systems with uncertainty by NLMI approach. The objective of this paper is to obtain the *necessary and sufficient* conditions for the existence of so-called **strong robust H_∞ controllers** in terms of NLMI. This paper establishes the relationship between robust H_∞ control problems for nonlinear uncertain systems and standard H_∞ control problems for nonlinear systems, which presents a new idea to study robust H_∞ control problems for nonlinear uncertain systems. The results obtained in this paper is an extension of those in [8] for nonlinear systems without uncertainty. In addition, this paper can be regarded as an analogous of the results in [14, 15] where the

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robust H_∞ control problem for linear systems with uncertainties are considered by state feedback and dynamic output feedback, respectively. The main mathematical technique used in this paper is the partition of unity, with which the results in [14, 15] for linear uncertain system are further extended into its nonlinear counterpart.

Notations: $R^+ := [0,\infty)$; $\|\cdot\|$ denotes the Euclidean norm; B_r is open ball in some Euclidean space centered at the origin with radius $r > 0$ which is measured by the Euclidean norm; A^T is the transpose of matrix A; $\mathbf{X}$ (or $\mathbf{X_0}$) is the open subset of some Euclidean space which contains the origin; by $P > 0$ (or $P \geq 0$) for some Hermitian matrix we mean that the matrix is (semi-definite) definite positive; C^k ($k > 0$) denotes the set of all functions which are continuously differentiable k times; C^0, therefore, is the set of all continuous functions.

2. Preliminaries. Consider the class of nonlinear time-invariant system described

$$\begin{aligned} \dot{x} &= A(x)x + B(x)w, \\ z &= C(x)x + D(x)w, \end{aligned} \tag{1}$$

where state vector $x \in \mathbf{X}$, $\mathbf{X} \subset R^n$ is assumed to be a convex open bounded subset of the origin, $w \in R^p$ and $z \in R^q$ are input vector and output vector, respectively. $A(x)$, $B(x)$, $C(x)$ and $D(x)$ are assumed to be C^0 matrix-valued functions of appropriate dimensions.

Definition 2.1. *System (1) with initial condition $x(0) = 0$ is said to have L_2-gain less than or equal to some positive constant γ if*

$$\int_0^T \|z(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2 dt \tag{2}$$

for all $T \geq 0$ and $w(t) \in L_2[0,T]$ as long as the trajectory $x = x(t) \in \mathbf{X}$ for $t \in [0,T]$. □

In order to show the main results, we introduce the following lemmas.

Lemma 2.1. *(see [11]) Let X, Y, Z be symmetric constant matrices in $R^{n\times n}$ with $X \geq 0$, $Y < 0$ and $Z \geq 0$ such that for all nonzero $\xi \in R^n$*

$$(\xi^T Y \xi)^2 - 4(\xi^T X \xi \xi^T Z \xi) > 0.$$

Then there exists a positive constant λ such that

$$\lambda^2 X + \lambda Y + Z < 0.$$

□

Similar to the celebrated bounded-real lemma for linear systems, we have the following analogous result for nonlinear systems (see [8]).

Lemma 2.2. *If system (1) satisfies the following conditions.*

(i) There exists a positive definite C^0 matrix-valued function such that for all $x \in \mathbf{X}$

$$\begin{pmatrix} A^T(x)P(x)+P(x)A(x) & P(x)B(x) & C^T(x) \\ B^T(x)P(x) & -I & D^T(x) \\ C(x) & D(x) & -I \end{pmatrix} < 0. \tag{3}$$

(ii) There exists a positive definite C^1 function $V : \mathbf{X} \to R$ such that $\frac{\partial V(x)}{\partial x} = 2x^T P(x)$.

Then system (1) has L_2-gain ≤ 1 and is asymptotically stable. □

REMARK 2.1. *If conditions (i) and (ii) hold, then we say that system (1) has strong H_∞ performance (see [8]). If system (1) is linear system, then (3) is a linear matrix inequality (LMI) on constant positive definite matrix P. Furthermore, if LMI (3) holds, then condition (ii) is unnecessary for we can choose $V(x) = x^T Px$.* □

In addition, we will use the following elementary inequality:

LEMMA 2.3. *Suppose $\sum_{i=1}^N a_i = 1$ with $a_i \geq 0$ and $\mu_i \geq 0$ $(i = 1, 2, \cdots, N)$, then*

$$(\sum_{i=1}^N \mu_i a_i)^2 \leq \sum_{i=1}^N \mu_i^2 a_i.$$

3. Robust H_∞-Control Problem. Consider the following nonlinear systems with uncertainty.

$$\begin{aligned} \dot{x} &= [A(x) + \Delta A(x,t)]x + B_1(x)w + [B_2(x) + \Delta B_2(x,t)]u, \\ z &= C_1(x)x + D_{12}(x)u, \\ y &= C_2(x)x + D_{21}(x)w, \end{aligned} \tag{4}$$

where the dimensions of x, w, u, z and y are n, p_1, p_2, q_1 and q_2 respectively. For all $x \in \mathbf{X}$, $A(x), B_i(x), C_i(x)$ and $D_{ij}(x)$ $(i, j = 1, 2)$ are C^0 matrix-valued functions. $\Delta A(x,t)$ and $\Delta B(x,t)$ satisfy the following assumption.

ASSUMPTION 3.1. *Let*

$$(\,\Delta A(x,t) \quad \Delta B(x,t)\,) = E(x)G(x,t)\,(\,H_1(x) \quad H_2(x)\,),$$

where $E(x) \in R^{n\times \tilde{p}_1}$, $H_i(x)$ $(i = 1, 2)$ are known C^0 matrix-valued functions, $G(x,t) \in \Omega$ is unknown matrix-valued functions with

$$\Omega := \{G(x,t) | G^T(x,t)G(x,t) \leq I \quad x \in \mathbf{X},\ t \in R^+, \\ \text{the elements of } G(x,t) \text{ Lebesgue measurable}\}.$$

For system (4), we are interested in constructing the form of the dynamic compensator as follows

$$\begin{aligned} \dot{\xi} &= \hat{A}(\xi)\xi + \hat{B}(\xi)y, \\ u &= \hat{C}(\xi)\xi + \hat{D}(\xi)y, \end{aligned} \tag{5}$$

where $\xi \in \mathbf{X_0}$ and $\mathbf{X_0}$ is a convex open subset in R^k $(k \leq n)$ containing the origin. In this case, the closed-loop systems (4) and (5) can be rewritten as follows:

$$\begin{aligned} \dot{x}_c &= A_c(x_c)x_c + B_c(x_c)w \\ z &= C_c(x_c)x_c + D_c(x_c)w \end{aligned} \tag{6}$$

where $x_c = \begin{pmatrix} x \\ \xi \end{pmatrix}$,

$$\begin{aligned}
A_c &= A^a + \Delta A^a + (B_2^a + \Delta B_2^a)FC_2^a, \quad B_c = B_1^a + (B_2^a + \Delta B_2^a)FD_{21}^a, \\
C_c &= C_1^a + D_{12}^a F C_2^a, \quad D_c = D_{12}^a F D_{21}^a, \quad A^a = \begin{pmatrix} A(x) & 0 \\ 0 & 0 \end{pmatrix}, \\
B_1^a &= \begin{pmatrix} B_1(x) \\ 0 \end{pmatrix}, \quad B_2^a = \begin{pmatrix} B_2(x) & 0 \\ 0 & I \end{pmatrix}, \quad C_1^a = \begin{pmatrix} C_1(x) & 0 \end{pmatrix}, \\
C_2^a &= \begin{pmatrix} C_2(x) & 0 \\ 0 & I \end{pmatrix}, \quad \Delta A^a = \begin{pmatrix} \Delta A(x,t) & 0 \\ 0 & 0 \end{pmatrix}, \quad \Delta B_2^a = \begin{pmatrix} \Delta B_2(x,t) & 0 \\ 0 & 0 \end{pmatrix}, \\
D_{12}^a &= \begin{pmatrix} D_{12}(x) & 0 \end{pmatrix}, \quad D_{21}^a = \begin{pmatrix} D_{21}(x) \\ 0 \end{pmatrix}, \quad F = \begin{pmatrix} \hat{D}(\xi) & \hat{C}(\xi) \\ \hat{B}(\xi) & \hat{A}(\xi) \end{pmatrix}.
\end{aligned} \tag{7}$$

Inspired by [8], we concentrate on so-called **strong robust H_∞-performance** in this paper. We give the definition of strong robust H_∞-performance as follows.

DEFINITION 3.1. *The closed-loop system (6) is said to have strong robust H_∞-performance if there exists a C^0 positive definite matrix-valued function $P_c(x_c) = P_c^T(x_c) > 0$ which satisfies the following inequality*

$$M(x_c,t) := \begin{pmatrix} A_c^T(x_c)P_c(x_c) + P_c(x_c)A_c(x_c) & P_c(x_c)B_c(x_c) & C_c^T(x_c) \\ B_c^T(x_c)P_c(x_c) & -I & D_c^T(x_c) \\ C_c(x_c) & D_c(x_c) & -I \end{pmatrix} < 0 \tag{8}$$

and

$$\frac{\partial V_c(x_c)}{\partial x_c} = 2x_c^T P_c(x_c) \tag{9}$$

for all $x_c \in \mathbf{X_c} = \mathbf{X} \times \mathbf{X_0} \subset R^n \times R^k$, all uncertainty $G \in \Omega$ and some C^1 positive definite function $V_c : \mathbf{X_c} \to \mathbf{R}^+$. □

REMARK 3.1. *From (8) and (9), the existence of Lyapunov function $V_c(x_c)$ is then derived in [3]. Similar to Remark 2.1, if system (6) is linear system with uncertainty, then (8) is a linear matrix inequality (LMI) on constant positive definite matrix P_c. In this case, additional requirement (9) is unnecessary.* □

Next, we give the necessary and sufficient conditions of existence of the dynamic compensator (5) such that the resulting closed-loop system (6) has strong robust H_∞ performance.

THEOREM 3.1. *There exists a dynamic compensator (5) such that the resulting closed-loop systems have strong robust H_∞ performance if and only if there exists a C^0 (or C^∞) positive function $\epsilon = \epsilon(x_c)$ for all $x_c \in \mathbf{X_c}$ such that the resulting closed-loop systems of (5) and the following systems (10) have strong H_∞ performance.*

$$\begin{aligned} \dot{x} &= A(x)x + \begin{pmatrix} B_1(x) & \frac{1}{\sqrt{\epsilon}}E(x) \end{pmatrix} \tilde{w} + B_2(x)u, \\ \tilde{z} &= \begin{pmatrix} C_1(x) \\ \sqrt{\epsilon}H_1(x) \end{pmatrix} x + \begin{pmatrix} D_{12}(x) \\ \sqrt{\epsilon}H_2(x) \end{pmatrix} u, \\ y &= C_2(x)x + \begin{pmatrix} D_{21}(x) & 0 \end{pmatrix} \tilde{w}, \end{aligned} \tag{10}$$

where disturbance input $\tilde{w} \in R^{p_1+\tilde{p}_1}$.

Proof. Necessity: Let

$$M_\Delta(x_c,t) := \begin{pmatrix} \Delta\Gamma_a & P_c\Delta B_2^a F D_{21}^a & 0 \\ D_{21}^{a^T} F^T \Delta B_2^{a^T} P_c & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$M_0(x_c) := \begin{pmatrix} \Gamma_a & \Gamma_{a1} & \Gamma_{a2}^T \\ \Gamma_{a1}^T & -I & (D_{12}^a F D_{21}^a)^T \\ \Gamma_{a2} & D_{12}^a F D_{21}^a & -I \end{pmatrix},$$

where

$$\Delta\Gamma_a := P_c(\Delta A^a + \Delta B_2^a F C_2^a) + (\Delta A^a + \Delta B_2^a F C_2^a)^T P_c,$$

$$\Gamma_a := P_c(A^a + B_2^a F C_2^a) + (A^a + B_2^a F C_2^a)^T P_c,$$

$$\Gamma_{a1} := P_c(B_1^a + B_2^a F D_{21}^a), \quad \Gamma_{a2} := C_1^a + D_{12}^a F C_2^a.$$

Then for all $x_c \in \mathbf{X_c}$, $t \in R^+$, one gets

$$M(x_c,t) = M_\Delta(x_c,t) + M_0(x_c) < 0. \tag{11}$$

That is,

$$M_0(x_c) < -M_\Delta(x_c,t).$$

For the convenience, let $E_0 := \begin{pmatrix} E \\ 0 \end{pmatrix}$, $H_{i0} := \begin{pmatrix} H_i & 0 \end{pmatrix}$, $i = 1, 2$.

$$M_1(x_c) := \begin{pmatrix} P_c E_0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} P_c E_0 \\ 0 \\ 0 \end{pmatrix}^T,$$

$$M_2(x_c) := \begin{pmatrix} (H_{10} + H_{20} F C_2^a)^T \\ (H_{20} F D_{21}^a)^T \\ 0 \end{pmatrix} \begin{pmatrix} (H_{10} + H_{20} F C_2^a)^T \\ (H_{20} F D_{21}^a)^T \\ 0 \end{pmatrix}^T.$$

Suppose η is a column vector with appropriate dimension. Then for all $\eta \neq 0$, we have

$$\eta^T M_0(x_c)\eta < -\eta^T M_\Delta(x_c,t)\eta.$$

Using similar technique as [11], we have that for each $x_c \in \mathbf{X_c}$,

$$[\eta^T M_0(x_c)\eta]^2 > 4\eta^T M_1(x_c)\eta\eta^T M_2(x_c)\eta.$$

Then it follows from Lemma 2.1 that for every $v \in \mathbf{X_c}$, there exists a positive constant ϵ_v such that

$$\epsilon_v^2 M_1(v) + \epsilon_v M_0(v) + M_2(v) < 0.$$

It follows from the continuity of every element of $M_i(x_c)$ ($i = 0, 1, 2$) for all $x_c \in \mathbf{X_c}$ that there exists a closed ball $B(v)$ centered at v such that for all $x_c \in B(v)$

$$\epsilon_v^2 M_1(x_c) + \epsilon_v M_0(x_c) + M_2(x_c) < 0,$$

then there is a partition of unity $\{B_i, \phi_i\}$, where $B_i = B_i(v_i)$, the union of their interiors covers $\mathbf{X_c}$, ϕ_i is a smooth map on $\mathbf{R^{n+k}}$ supported on B_i with $\phi_i \geq 0$ and $\sum_{i=1}^{\infty} \phi_i(v) = 1$.

Let

$$\epsilon = \epsilon(x_c) = \sum_{i=1}^{\infty} \epsilon_{v_i}\phi_i(x_c).$$

Then $\epsilon(x_c)$ is well-defined and smooth on $\mathbf{X_c}$ since for any given $x_c \in \mathbf{X_c}$, there is a definite number of index i such that $x_c \in B_i$, that is, there exist some positive integers $n_N > n_{N-1} > \cdots > n_1$ such that $\sum_{i=1}^{N} \phi_{n_i}(x_c) = 1$. Then for each $x_c \in \mathbf{X_c}$, noticing the semi-definite positive of matrices $M_1(x_c)$ and $M_2(x_c)$, it follows from Lemma 2.3 that we have

$$\begin{aligned}
&\epsilon^2(x_c)M_1(x_c) + \epsilon(x_c)M_0(x_c) + M_2(x_c)\\
&= [\sum_{i=1}^{\infty} \epsilon_{v_i}\phi_i(x_c)]^2 M_1(x_c) + \sum_{i=1}^{\infty} \epsilon_{v_i}\phi_i(x_c)M_0(x_c) + M_2(x_c)\\
&= [\sum_{i=1}^{N} \epsilon_{v_{n_i}}\phi_{n_i}(x_c)]^2 M_1(x_c) + \sum_{i=1}^{N} \epsilon_{v_{n_i}}\phi_{n_i}(x_c)M_0(x_c) + M_2(x_c)\\
&\leq \sum_{i=1}^{N} \phi_{n_i}(x_c)[\epsilon_{v_{n_i}}^2 M_1(x_c) + \epsilon_{v_{n_i}} M_0(x_c) + M_2(x_c)]\\
&< 0.
\end{aligned}$$

Therefore, there exists a C^0 (or strongly C^∞) positive function $\epsilon = \epsilon(x_c)$ in $\mathbf{X_c}$ such that the following inequality holds for all $x_c \in \mathbf{X_c}$.

$$\epsilon^2 M_1(x_c) + \epsilon M_0(x_c) + M_2(x_c) < 0. \tag{12}$$

That is,

$$
\begin{pmatrix} \Gamma_{11} & \Gamma_{12} & (C_1^a + D_{12}^a F C_2^a)^T \\ \Gamma_{12}^T & \Gamma_{22} & (D_{12}^a F D_{21}^a)^T \\ C_1^a + D_{12}^a F C_2^a & D_{12}^a F D_{21}^a & -I \end{pmatrix} < 0, \tag{13}
$$

where

$$
\begin{aligned}
\Gamma_{11} &:= P_c(A^a + B_2^a F C_2^a) + (A^a + B_2^a F C_2^a)^T P_c + \epsilon^{-1} P_c E_{10} E_{10}^T P_c \\
&\quad + \epsilon^{-1}(H_{10} + H_{20} F C_2^a)^T (H_{10} + H_{20} F C_2^a), \\
\Gamma_{12} &:= P_c(B_1^a + B_2^a F D_{21}^a) + \epsilon^{-1}(H_{10} + H_{20} F C_2^a)^T H_{20} F D_{21}^a, \\
\Gamma_{22} &:= -I + \epsilon^{-1}(H_{20} F D_{21}^a)^T H_{20} F D_{21}^a.
\end{aligned}
$$

It is readily derived from Schur complement (see [6]) that (13) is equivalent to

$$
\begin{pmatrix} \Gamma_a & P_c \Omega_{21}^T & \Omega_{13}^T \\ \Omega_{21} P_c & -I & \Omega_{32}^T \\ \Omega_{31} & \Omega_{32} & -I \end{pmatrix} < 0, \tag{14}
$$

where

$$
\begin{aligned}
\Omega_{21} &:= \begin{pmatrix} (B_1^a + B_2^a F D_{21}^a)^T \\ \frac{1}{\sqrt{\epsilon}} E_0^T \end{pmatrix}, \\
\Omega_{31} &:= \begin{pmatrix} \sqrt{\epsilon}(H_{10} + H_{20} F C_2^a) \\ C_1^a + D_{12}^a F C_2^a \end{pmatrix}, \\
\Omega_{32} &:= \begin{pmatrix} \sqrt{\epsilon} H_{20} F D_{21}^a & 0 \\ D_{12}^a F D_{21}^a & 0 \end{pmatrix}.
\end{aligned} \tag{15}
$$

It follows from (14) and Lemma 2.2 that the closed-loop systems (10) and (5) have strong H_∞ performance. This completes the proof of necessity.

Sufficiency: If the closed-loop systems (10) and (5) have strong H_∞ performance, it follows from the above proof that (14) holds for some C^0 positive function $\epsilon = \epsilon(x_c)$. Therefore, it follows from (11) and (12) that we have

$$
\begin{aligned}
&M(x_c, t) \\
&= M_\Delta(x_c, t) + M_0(x_c) \\
&< M_\Delta(x_c, t) - \epsilon M_1(x_c) - \epsilon^{-1} M_2(x_c) \\
&= \begin{pmatrix} P_c E_0 \\ 0 \\ 0 \end{pmatrix} G \begin{pmatrix} (H_{10} + H_{20} F C_2^a)^T \\ (H_{20} F D_{21}^a)^T \\ 0 \end{pmatrix}^T + \begin{pmatrix} (H_{10} + H_{20} F C_2^a)^T \\ (H_{20} F D_{21}^a)^T \\ 0 \end{pmatrix} G^T \begin{pmatrix} P_c E_0 \\ 0 \\ 0 \end{pmatrix} \\
&\quad - \epsilon M_1(x_c) - \epsilon^{-1} M_2(x_c) \\
&\le 0.
\end{aligned} \tag{16}
$$

This completes the proof of sufficiency. □

REMARK 3.2. *Theorem 3.1 presents a new approach to discuss robust H_∞ control problem and establishes the relationship between uncertain system (4) and auxiliary system as in (10). One of the main contribution of this paper is to transform the robust H_∞ control problem into a H_∞ control problem via auxiliary system (10). For computing NLMI, much discussions are made in [8] and can be applied to auxiliary system (10).*

REMARK 3.3. *The partition of unity is the key technique used in the proof of the necessity for Theorem 3.1 which presents an approach to construct parameter $\epsilon(x_c)$ from local definition ϵ_v, see the above proof.*

REMARK 3.4. *In the sufficient condition of Theorem 3.1, $\epsilon(x_c)$ can be loosened to any well-defined function. For convenience of control design, we can choose $\epsilon(x_c)$ as a constant scalar $\epsilon > 0$.*

Next, we consider strong robust H_∞ control problem of systems (5) via the following static state feedback controller:

$$u = K(x)x. \tag{17}$$

Similar to Theorem 3.1, we have the result as follows:

THEOREM 3.2. *There exists a state feedback controller (17) such that the resulting closed-loop systems have strong robust H_∞ performance if and only if there exists a C^0 (or C^∞) positive function $\epsilon = \epsilon(x)$ for all $x \in \mathbf{X}$ such that the resulting closed-loop systems of (17) and systems (10) have strong H_∞ performance.* □

As an application of Theorem 3.1 and 3.2 to the linear case of system (4), we present the results as follows:

$$\begin{aligned} \dot{x} &= [A + \Delta A(x,t)]x + B_1 w + [B_2 + \Delta B_2(x,t)]u, \\ z &= C_1 x + D_{12} u, \\ y &= C_2 x + D_{21} w, \end{aligned} \tag{18}$$

where the dimensions of all parameters are the same as above, and matrices A, B_i, C_i, $(i = 1, 2)$, D_{12}, D_{21}, E_1 and E_2 are constant with appropriate dimensions.

From Assumption 3.1, we have the following results.

THEOREM 3.3. *There exists a linear dynamic compensator*

$$\begin{aligned} \dot{\xi} &= \hat{A}\xi + \hat{B}y, \\ u &= \hat{C}\xi + \hat{D}y \end{aligned} \tag{19}$$

such that the resulting closed-loop systems have robust H_∞ performance if and only if there exists a positive constant ϵ such that the resulting closed-loop systems of (19) *and the following systems (20) have H_∞ performance.*

$$\dot{x} = Ax + \begin{pmatrix} B_1 & \frac{1}{\sqrt{\epsilon}}E \end{pmatrix} \tilde{w} + B_2 u,$$

$$(20) \qquad \tilde{z} = \begin{pmatrix} C_1 \\ \sqrt{\epsilon} H_1 \end{pmatrix} x + \begin{pmatrix} D_{12} \\ \sqrt{\epsilon} H_2 \end{pmatrix} u,$$

$$y = C_2 x + \begin{pmatrix} D_{21} & 0 \end{pmatrix} \tilde{w},$$

where disturbance input $\tilde{w} \in R^{p_1+\tilde{p}_1}$. □

REMARK 3.5. *The controller (19) or linear static state feedback controller $u = Kx$ can be obtained by standard LMI Toolbox (see [5]) if ϵ is given.*

REMARK 3.6. *Theorem 3.3 can be regarded as an extension of the main results for linear systems in [14, 15] where algebraic Riccati inequality approach is applied.*

4. Conclusion. In this paper, we have considered the robust H_∞ control problem of the nonlinear uncertain systems with the matched conditions. In terms of NLMI, we have obtained the necessity and sufficient conditions for the existence of robust H_∞ dynamic compensators or static state feedback controller, under which the corresponding closed-loop systems have strong robust H_∞-performance, respectively. This paper establishes the relationship between robust H_∞ control problems for nonlinear uncertain systems and standard H_∞ control problems for nonlinear systems, which presents a new idea to study robust H_∞ control problems for nonlinear uncertain systems.

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