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Rigidity of area-minimizing two-spheres in three-manifolds

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We give a sharp upper bound for the area of a minimal two-sphere in a three-manifold (M, g) with positive scalar curvature. If equality holds, we show that the universal cover of (M, g) is isometric to a cylinder.

1. Introduction

A classical result in differential geometry due to Toponogov [14] states that every simple closed geodesic γ on a two-dimensional surface (Σ, g) satisfies

$$\text{length}(\gamma)^2 \inf_{\Sigma} K \leq 4\pi^2,$$

where K denotes the Gaussian curvature of Σ. Moreover, equality holds if and only if (Σ, g) is isometric to the standard sphere S^2 up to scaling (see also [8] for a different proof).

We next consider a three-manifold (M, g) with positive scalar curvature. By a theorem of Schoen and Yau [13], any area-minimizing surface in M is homeomorphic to either S^2 or $\mathbb{RP}^2$. The case of area-minimizing projective planes was studied in [3]. In particular, if Σ is an embedded $\mathbb{RP}^2$ of minimal area, then the area of Σ can be estimated from above by

$$\text{area}(\Sigma, g) \inf_{M} R \leq 12\pi$$

(cf.[3, Theorem 1]). Moreover, equality holds if and only if (M, g) is isometric to $\mathbb{RP}^3$ up to scaling. A survey of related rigidity results involving scalar curvature can be found in [4].

In this paper, we consider the case of area-minimizing two-spheres. We shall assume throughout that (M, g) is a compact three-manifold with $\pi_2(M) \neq 0$. We denote by $\mathscr{F}$ the set of all smooth maps $f : S^2 \to M$ which represent a non-trivial element of $\pi_2(M)$. We define

$$\mathscr{A}(M, g) = \inf\{\text{area}(S^2, f^*g) : f \in \mathscr{F}\}. \tag{1.1}$$

We now state the main result of this paper:

Theorem 1.1. *We have*

$$\mathscr{A}(M,g)\inf_M R \leq 8\pi, \tag{1.2}$$

where R denotes the scalar curvature of (M,g). Moreover, if equality holds, then the universal cover of (M,g) is isometric to the standard cylinder $S^2 \times \mathbb{R}$ up to scaling.

Inequality (1.2) follows directly from the formula for the second variation of area. We now describe the proof of the rigidity statement. By scaling, we may assume that $\mathscr{A}(M,g) = 4\pi$ and $\inf_M R = 2$. It follows from results of Meeks and Yau [12] that the infimum in (1.1) is attained by a smooth immersion $f \in \mathscr{F}$ (see also [9]). Using the implicit function theorem, we construct a one-parameter family of immersed two-spheres with constant mean curvature. Using the formula for the second variation of area, we are able to show that these surfaces all have area $\mathscr{A}(M,g) = 4\pi$. Consequently, these spheres are all round and totally geodesic. This allows us to construct a local isometry from the cylinder $S^2 \times \mathbb{R}$ into M. The use of constant mean curvature surfaces is motivated in part by work of Bray [1] and Huisken and Yau [11] (see also [2, 10]).

We note that Cai and Galloway [5] have obtained a similar rigidity theorem for minimal tori in three-manifolds of nonnegative scalar curvature. The proof in [5] uses a different argument based on a deformation of the metric to strictly positive scalar curvature. The arguments in this paper can be adapted to give an alternative proof of Theorem 1 in [5]. See also [7] for related work in this direction.

2. Proof of (1.2)

Let us consider a smooth immersion $f : S^2 \to M$. Since f has trivial normal bundle, there exists a globally defined unit normal field ν. In other words, for each point $x \in S^2$, $\nu(x) \in T_{f(x)}M$ is a unit vector which is orthogonal to the image of $df_x : T_xS^2 \to T_{f(x)}M$. The following result is a consequence of the Gauss–Bonnet theorem.

Proposition 2.1. *For any immersion $f : S^2 \to M$, we have*

$$\int_{S^2} (R - 2\,Ric(\nu,\nu) - |II|^2)\, d\mu_{f^*g} \leq 8\pi,$$

where II denotes the second fundamental form of f.

Proof. By the Gauss equations, we have

$$R - 2\,\mathrm{Ric}(\nu,\nu) - |I\!I|^2 = 2K - H^2,$$

where H and K denote the mean curvature and the Gaussian curvature, respectively. This implies

$$\int_{S^2} (R - 2\,\mathrm{Ric}(\nu,\nu) - |I\!I|^2)\, d\mu_{f^*g} \leq 2 \int_{S^2} K\, d_{f^*g} = 8\pi$$

by the Gauss–Bonnet theorem. □

We next consider a map $f \in \mathscr{F}$ which attains the infimum in (1.1). The existence of a minimizer is guaranteed by the following result.

Proposition 2.2. *There exists a smooth map $f \in \mathscr{F}$ such that area $(S^2, f^*g) = \mathscr{A}(M,g)$. Moreover, the map f is an immersion.*

Proposition 2.2 is an immediate consequence of Theorem 7 in [12] (see also [9, Theorem 4.2]). In fact, Meeks and Yau show that either f is an embedding or a two-to-one covering map whose image is an embedded $\mathbb{RP}^2$. We will not use this stronger statement here.

Let $f \in \mathscr{F}$ be a smooth immersion with area$(S^2, f^*g) = \mathscr{A}(M,g)$. Using the formula for the second variation of area, we obtain

$$\int_{S^2} (\mathrm{Ric}(\nu,\nu) + |I\!I|^2)\, u^2\, d\mu_{f^*g} \leq \int_{S^2} |\nabla u|^2_{f^*g}\, d\mu_{f^*g}$$

for every smooth function $u : S^2 \to \mathbb{R}$. Choosing $u = 1$ gives

$$\int_{S^2} (\mathrm{Ric}(\nu,\nu) + |I\!I|^2)\, d\mu_{f^*g} \leq 0.$$

Using Proposition 2.1, we obtain

$$\begin{aligned} \mathrm{area}(S^2, f^*g)\, \inf_M R &\leq \int_{S^2} (R + |I\!I|^2)\, d\mu_{f^*g} \\ &\leq 8\pi + 2\int_{S^2} (\mathrm{Ric}(\nu,\nu) + |I\!I|^2)\, d\mu_{f^*g} \\ &\leq 8\pi. \end{aligned} \tag{2.1}$$

This completes the proof of (1.2).

3. The case of equality

In this section, we analyze the case of equality. Suppose that

$$\mathscr{A}(M,g) \inf_M R = 8\pi.$$

After rescaling the metric if necessary, we may assume that $\mathscr{A}(M,g) = 4\pi$ and $\inf_M R = 2$. By Proposition 2.2, we can find a smooth immersion $f \in \mathscr{F}$ such that $\text{area}(S^2, f^*g) = 4\pi$.

Proposition 3.1. *Let $f \in \mathscr{F}$ be a smooth immersion such that area $(S^2, f^*g) = 4\pi$. Then the surface $\Sigma = f(S^2)$ is totally geodesic. Moreover, we have $R = 2$ and $Ric(\nu,\nu) = 0$ at each point on Σ.*

Proof. By assumption, we have $\text{area}(S^2, f^*g) = 4\pi$ and $\inf_M R = 2$. Consequently, the inequalities in (2.1) are all equalities. In particular, we have

$$\int_{S^2} (R + |II|^2)\, d\mu_{f^*g} = 8\pi \tag{3.1}$$

and

$$\int_{S^2} (\text{Ric}(\nu,\nu) + |II|^2)\, d\mu_{f^*g} = 0. \tag{3.2}$$

It follows from (3.2) that the constant functions lie in the nullspace of the Jacobi operator $L = \Delta_{f^*g} + \text{Ric}(\nu,\nu) + |II|^2$. This implies

$$\text{Ric}(\nu,\nu) + |II|^2 = 0$$

at each point on Σ. Moreover, since $\text{area}(S^2, f^*g) = 4\pi$ and $\inf_M R = 2$, the identity (3.1) implies that $R = 2$ and $|II|^2 = 0$ at each point on Σ. This completes the proof. $\square$

Proposition 3.2. *Let $f \in \mathscr{F}$ be a smooth immersion such that area $(S^2, f^*g) = 4\pi$. Then there exists a positive real number δ_1 and a smooth map $w : S^2 \times (-\delta_1, \delta_1) \to \mathbb{R}$ with the following properties:*

- *For each point $x \in S^2$, we have $w(x,0) = 0$ and $\frac{\partial}{\partial t} w(x,t)\big|_{t=0} = 1$.*
- *For each $t \in (-\delta_1, \delta_1)$, we have $\int_{S^2} (w(\cdot,t) - t)\, d\mu_{f^*g} = 0$.*

- *For each* $t \in (-\delta_1, \delta_1)$*, the surface*

$$\Sigma_t = \{\exp_{f(x)}(w(x,t)\nu(x)) : x \in S^2\}$$

has constant mean curvature.

Proof. The Jacobi operator associated with the minimal immersion $f : S^2 \to M$ is given by $L = \Delta_{f^*g} + \mathrm{Ric}(\nu,\nu) + |II|^2$. Using Proposition 3.1, we conclude that $L = \Delta_{f^*g}$. Hence, the assertion follows from the implicit function theorem. □

For each $t \in (-\delta_1, \delta_1)$, we define a map $f_t : S^2 \to M$ by $f_t(x) = \exp_{f(x)}(w(x,t)\,\nu(x))$. Clearly, $f_0(x) = f(x)$ for all $x \in S^2$. To fix notation, we denote by $\nu_t(x) \in T_{f_t(x)}M$ the unit normal vector to the surface $\Sigma_t = f_t(S^2)$ at the point $f_t(x)$. We assume that ν_t depends smoothly on x and t, and $\nu_0(x) = \nu(x)$ for all $x \in S^2$. Moreover, we denote by II_t the second fundamental form of f_t.

Lemma 3.3. *There exists a positive real number* $\delta_2 < \delta_1$ *with the following property: if* $t \in (-\delta_2, \delta_2)$ *and* $u : S^2 \to \mathbb{R}$ *is a smooth function satisfying* $\int_{S^2} u\,d\mu_{f_t^*g} = 0$*, then*

$$\int_{S^2} |\nabla u|^2_{f_t^*g}\,d\mu_{f_t^*g} - \int_{S^2} (Ric(\nu_t,\nu_t) + |II_t|^2)\,u^2\,d\mu_{f_t^*g} \geq 0.$$

Proof. We can find a uniform constant $c > 0$ such that

$$\int_{S^2} |\nabla u|^2_{f_t^*g}\,d\mu_{f_t^*g} \geq c \int_{S^2} u^2\,d\mu_{f_t^*g}$$

for each $t \in (-\delta_1, \delta_1)$ and every smooth function $u : S^2 \to \mathbb{R}$ satisfying $\int_{S^2} u\,d\mu_{f_t^*g} = 0$. Moreover, it follows from Proposition 3.1 that

$$\sup_{S^2}(\mathrm{Ric}(\nu_t,\nu_t) + |II_t|^2) \to 0$$

as $t \to 0$. Putting these facts together, the assertion follows. □

Lemma 3.4. *For each* $t \in (-\delta_1, \delta_1)$*, we have*

$$\int_{S^2} (Ric(\nu_t,\nu_t) + |II_t|^2)\,d\mu_{f_t^*g} \geq 0.$$

Proof. Since f minimizes area in its homotopy class, we have

$$\text{area}(S^2, f_t^* g) \geq \text{area}(S^2, f^* g) = 4\pi.$$

Moreover, we have $\inf_M R = 2$. Applying Proposition 2.1 to the map $f_t : S^2 \to M$, we obtain

$$\begin{aligned} 8\pi &\leq \text{area}(S^2, f_t^* g) \inf_M R \\ &\leq \int_{S^2} (R + |II_t|^2)\, d\mu_{f_t^* g} \\ &\leq 8\pi + 2 \int_{S^2} (\text{Ric}(\nu_t, \nu_t) + |II_t|^2)\, d\mu_{f_t^* g}. \end{aligned}$$

From this, the assertion follows. □

By assumption, the surface Σ_t has constant mean curvature. The mean curvature vector of Σ_t can be written in the form $-H(t)\, \nu_t$, where $H(t)$ is a smooth function of t. For each $t \in (-\delta_1, \delta_1)$, the lapse function $\rho_t : S^2 \to \mathbb{R}$ is defined by

$$\rho_t(x) = \left\langle \nu_t(x), \frac{\partial}{\partial t} f_t(x) \right\rangle. \tag{3.3}$$

Clearly, $\rho_0(x) = 1$ for all $x \in S^2$. By continuity, we can find a positive real number $\delta_3 < \delta_2$ such that $\rho_t(x) > 0$ for all $x \in S^2$ and all $t \in (-\delta_3, \delta_3)$. The lapse function $\rho_t : S^2 \to \mathbb{R}$ satisfies the Jacobi equation

$$\Delta_{f_t^* g} \rho_t + (\text{Ric}(\nu_t, \nu_t) + |II_t|^2)\, \rho_t = -H'(t) \tag{3.4}$$

(cf.[10, equation (1.2)]).

Proposition 3.5. *We have area*$(S^2, f_t^* g) = 4\pi$ *for all* $t \in (-\delta_3, \delta_3)$.

Proof. Let $\overline{\rho}_t$ denote the mean value of the lapse function $\rho_t : S^2 \to \mathbb{R}$ with respect to the induced metric $f_t^* g$; that is,

$$\overline{\rho}_t = \frac{1}{\text{area}(S^2, f_t^* g)} \int_{S^2} \rho_t \, d\mu_{f_t^* g}.$$

It follows from Lemma 3.3 that

$$\int_{S^2} |\nabla \rho_t|^2_{f_t^* g}\, d\mu_{f_t^* g} - \int_{S^2} (\text{Ric}(\nu_t, \nu_t) + |II_t|^2)\, (\rho_t - \overline{\rho}_t)^2\, d\mu_{f_t^* g} \geq 0$$

for all $t \in (-\delta_2, \delta_2)$. Moreover, Lemma 3.4 implies that

$$\overline{\rho}_t^2 \int_{S^2} (\mathrm{Ric}(\nu_t, \nu_t) + |II_t|^2) \, d\mu \geq 0$$

for all $t \in (-\delta_1, \delta_1)$. Adding both inequalities yields

$$\int_{S^2} |\nabla \rho_t|^2_{f_t^* g} \, d\mu_{f_t^* g} + \int_{S^2} (\mathrm{Ric}(\nu_t, \nu_t) + |II_t|^2) \, \rho_t \, (2 \, \overline{\rho}_t - \rho_t) \, d\mu_{f_t^* g} \geq 0 \tag{3.5}$$

for all $t \in (-\delta_2, \delta_2)$.

In the next step, we multiply Equation (3.4) by $2 \, \overline{\rho}_t - \rho_t$ and integrate. This gives

$$\begin{aligned} &\int_{S^2} |\nabla \rho_t|^2_{f_t^* g} \, d\mu_{f_t^* g} + \int_{S^2} (\mathrm{Ric}(\nu_t, \nu_t) + |II_t|^2) \, \rho_t \, (2 \, \overline{\rho}_t - \rho_t) \, d\mu_{f_t^* g} \\ &\quad = -H'(t) \int_{S^2} (2 \, \overline{\rho}_t - \rho_t) \, d\mu_{f_t^* g} = -H'(t) \int_{S^2} \rho_t \, d\mu_{f_t^* g}. \end{aligned} \tag{3.6}$$

Putting these facts together, we obtain

$$H'(t) \int_{S^2} \rho_t \, d\mu_{f_t^* g} \leq 0 \tag{3.7}$$

for each $t \in (-\delta_2, \delta_2)$. Therefore, we have $H'(t) \leq 0$ for all $t \in (-\delta_3, \delta_3)$. Since $H(0) = 0$, it follows that $H(t) \geq 0$ for all $t \in (-\delta_3, 0]$ and $H(t) \leq 0$ for all $t \in [0, \delta_3)$. Using the identity

$$\frac{d}{dt} \mathrm{area}(S^2, f_t^* g) = \int_{S^2} \Big\langle H(t) \, \nu_t, \frac{\partial}{\partial t} f_t \Big\rangle \, d\mu_{f_t^* g} = H(t) \int_{S^2} \rho_t \, d\mu_{f_t^* g},$$

we obtain

$$\mathrm{area}(S^2, f_t^* g) \leq \mathrm{area}(S^2, f^* g) = 4\pi$$

for all $t \in (-\delta_3, \delta_3)$. Since f minimizes area in its homotopy class, we conclude that $\mathrm{area}(S^2, f_t^* g) = 4\pi$ for all $t \in (-\delta_3, \delta_3)$. □

Proposition 3.6. *For each $t \in (-\delta_3, \delta_3)$, the surface Σ_t is totally geodesic, and we have $R = 2$ and $Ric(\nu_t, \nu_t) = 0$ at each point on Σ_t. Moreover, the lapse function $\rho_t : S^2 \to \mathbb{R}$ is constant.*

Proof. By Proposition 3.5, we have $\mathrm{area}(S^2, f_t^* g) = 4\pi$. Hence, it follows from Proposition 3.1 that Σ_t is totally geodesic, and $R = 2$ and $\mathrm{Ric}(\nu_t, \nu_t) = 0$ at each point on Σ_t. Substituting this into (3.4), we obtain $\Delta_{f_t^* g} \rho_t = 0$. Therefore, the function $\rho_t : S^2 \to \mathbb{R}$ is constant, as claimed. □

Corollary 3.7. *The normal vector field ν_t is a parallel vector field near Σ. In particular, each point on Σ has a neighborhood which is isometric to a Riemannian product.*

Proof. By Proposition 3.6, the lapse function $\rho_t : S^2 \to \mathbb{R}$ is constant. This implies

$$\begin{aligned}&\Big\langle D_{\frac{\partial f_t}{\partial x_i}} \nu_t, \frac{\partial f_t}{\partial t}\Big\rangle - \Big\langle D_{\frac{\partial f_t}{\partial t}} \nu_t, \frac{\partial f_t}{\partial x_i}\Big\rangle \\ &= \frac{\partial}{\partial x_i}\Big\langle \nu_t, \frac{\partial f_t}{\partial t}\Big\rangle - \frac{\partial}{\partial t}\Big\langle \nu_t, \frac{\partial f_t}{\partial x_i}\Big\rangle = \frac{\partial}{\partial x_i}\rho_t(x) = 0\end{aligned}$$

for each point $x \in S^2$. Moreover, we have

$$\begin{aligned}&\Big\langle D_{\frac{\partial f_t}{\partial x_i}} \nu_t, \frac{\partial f_t}{\partial x_j}\Big\rangle - \Big\langle D_{\frac{\partial f_t}{\partial x_j}} \nu_t, \frac{\partial f_t}{\partial x_i}\Big\rangle \\ &= \frac{\partial}{\partial x_i}\Big\langle \nu_t, \frac{\partial f_t}{\partial x_j}\Big\rangle - \frac{\partial}{\partial x_j}\Big\langle \nu_t, \frac{\partial f_t}{\partial x_i}\Big\rangle = 0\end{aligned}$$

for all $x \in S^2$. Putting these facts together, we obtain

$$\Big\langle D_{\frac{\partial f_t}{\partial x_i}} \nu_t, V\Big\rangle - \Big\langle D_V \nu_t, \frac{\partial f_t}{\partial x_i}\Big\rangle = 0$$

for each point $x \in S^2$ and all vectors $V \in T_{f(x)}M$. In particular, we have

$$\Big\langle D_{\frac{\partial f_t}{\partial x_i}} \nu_t, \nu_t\Big\rangle - \Big\langle D_{\nu_t} \nu_t, \frac{\partial f_t}{\partial x_i}\Big\rangle = 0$$

for each point $x \in S^2$. Since the vector field ν_t has unit length, we conclude that $D_{\nu_t}\nu_t = 0$. On the other hand, it follows from Proposition 3.6 that the surfaces Σ_t are totally geodesic. This implies $D_{\frac{\partial f_t}{\partial x_i}} \nu_t = 0$ for each point $x \in S^2$. Thus, we conclude that the normal vector field ν_t is parallel. This completes the proof of Corollary 3.7. □

We now consider the product $S^2 \times \mathbb{R}$, where S^2 is equipped with the induced metric f^*g. We define a map $\Phi : S^2 \times \mathbb{R} \to M$ by $\Phi(x, t) = \exp_{f(x)} (t\,\nu(x))$. It follows from Corollary 3.7 that the restriction $\Phi|_{S^2 \times (-\delta,\delta)}$ is a local isometry if $\delta > 0$ is sufficiently small.

Proposition 3.8. *The map $\Phi : S^2 \times \mathbb{R} \to M$ is a local isometry.*

Proof. We first show that $\Phi|_{S^2\times[0,\infty)}$ is a local isometry. Suppose this is false. Let τ be the largest positive real number with the property that $\Phi|_{S^2\times[0,\tau]}$

is a local isometry. We now define a map $\tilde{f} : S^2 \to M$ by $\tilde{f}(x) = \Phi(x, \tau)$. Clearly, $\tilde{f}$ is homotopic to f; consequently, $\tilde{f}$ represents a non-trivial element of $\pi_2(M)$. Moreover, we have $\text{area}(S^2, \tilde{f}^*g) = \text{area}(S^2, f^*g) = 4\pi$. Therefore, $\tilde{f}$ has minimal area among all maps in $\mathscr{F}$. By Corollary 3.7, each point on the surface $\tilde{\Sigma} = \tilde{f}(S^2)$ has a neighborhood which is isometric to a Riemannian product. Hence, if $\delta > 0$ is sufficiently small, then the map $\Phi|_{S^2 \times [0,\tau+\delta)}$ is a local isometry. This contradicts the maximality of τ.

Therefore, the restriction $\Phi|_{S^2 \times [0,\infty)}$ is a local isometry. An analogous argument shows that $\Phi|_{S^2 \times (-\infty,0]}$ is a local isometry. This completes the proof of Proposition 3.8. □

Since $\Phi : S^2 \times \mathbb{R} \to M$ is a local isometry, it follows that Φ is a covering map (cf.[6, Section 1.11]). Consequently, the universal cover of (M, g) is isometric to $S^2 \times \mathbb{R}$, equipped with the standard metric. This completes the proof of Theorem 1.1.

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