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Exact Correlators of Giant Gravitons from Dual $N = 4$ SYM theory

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Abstract

A class of correlation functions of half-BPS composite operators are computed exactly (at finite N) in the zero coupling limit of $N = 4$ SYM theory. These have a simple dependence on the four-dimensional spacetime coordinates and are related to correlators in a one-dimensional Matrix Model with complex Matrices obtained by dimensional reduction of $N = 4$ SYM on a three-sphere. A key technical tool is Frobenius-Schur duality between symmetric and Unitary groups and the results are expressed simply in terms of $U(N)$ group integrals or equivalently in terms of Littlewood-Richardson coefficients. These correlation functions are used to understand the existence/properties of giant gravitons and related solutions in the string theory dual on $AdS_5 \times S^5$. Some of their properties hint at integrability in $N = 4$ SYM.

1 Introduction

The AdS/CFT correspondence [1, 2, 3] presents an opportunity to investigate new qualitative features of non-perturbative string theory using techniques in the dual conformal field theory. Finite N truncations in BPS spectra were studied as evidence of a stringy exclusion principle [4] (see also [5]). These were argued to provide evidence for non-commutative gravity in [6], and related developments appeared in [7, 8, 9].

In another line of development the work of [10] showed that in the presence of background fields, zero-branes can get polarized into higher branes. Giant gravitons were discovered in [11], combining heuristic expectations from non-commutativity and the Myers effect, and exhibiting time-dependent solutions to brane actions which describe branes extended in the spheres of the $AdS \times S$ backgrounds. Giant gravitons were further studied in [12, 13, 14, 15]. One of the results of these papers was that there are new giant gravitons which are large in the AdS directions. A number of puzzles were raised about the detailed correspondence between the spectrum of chiral primaries and the spectrum of gravitons : Kaluza-Klein gravitons, sphere giants and AdS giants. Some of these puzzles were addressed recently in [16] and the gauge theory dual of a sphere giant was proposed.

In this paper we will develop the dictionary between half-BPS operators and Yang Mills. We will observe that the space of half-BPS representations can be mapped to the space of Schur polynomials of $U(N)$, equivalently to the space of Young Diagrams characterizing representations of $U(N)$. We will compute correlation functions involving arbitrary Young diagrams. The map to Young Diagrams will reveal natural candidates for sphere giants, agreeing with [16], as well as candidates for AdS giants. In addition we will be lead to look for a simple generalization of sphere and AdS giant graviton solutions involving multiple windings. We will discuss the properties of the correlation functions in the light of the correspondence to giant gravitons.

2 Review and Notation.

We will recall some facts about half-BPS operators, free field contractions, symmetric groups and Unitary groups. The connection between Symmetric and Unitary groups was crucial in the development of the String theory of two-dimensional Yang Mills Theory [17, 18, 19, 20, 21, 22, 23, 24] and in certain aspects of Low-dimensional Matrix Models [25, 26]. Some key useful results are collected here. Many of the key derivations are reviewed in [27].

2.1 Half-BPS operators in $N = 4$ SYM

We will review some properties of half-BPS operators in $N = 4$ SYM [28, 29, 30, 31, 32]. A more complete list of references is in [33].

The half-BPS operators constructed from the 6 real scalars in the Yang-Mills theory lie in the $(0, l, 0)$ representation of the $SU(4) \sim SO(6)$ $\mathcal{R}$-symmetry group. This is the symmetric traceless representation of $SO(6)$ corresponding to Young Diagrams with one row of length l. These operators saturate a lower bound on their conformal dimensions which is related to their $\mathcal{R}$-symmetry charge. They include single trace as well as multiple trace operators. The single trace chiral primary operators are of the form $T_{i_1 \cdots i_6} Tr(X^{i_1} \cdots X^{i_n})$ where the X^i's, with $i = 1, ..., 6$, are the scalars of the theory transforming in the vector representation of the $SO(6)$ $\mathcal{R}$-symmetry group and the coefficients $T_{i_1 \cdots i_6}$ are symmetric and traceless in the $SO(6)$ indices.

The $\mathcal{N}$=4 theory can be decomposed into an $\mathcal{N}$=1 vector multiplet and three chiral multiplets. The scalars of the chiral multiplets are given in terms of the X^j's as $\Phi^j = X^j + iX^{j+3}$ for $j = 1, 2, 3$ where all fields transform in the adjoint. In this notation the $SO(6)$ $\mathcal{R}$-symmetry is partially hidden so that only a $U(3)$ action remains explicit. The $U(3)$ decomposition of the rank l symmetric traceless tensor representation of $SO(6)$ includes one copy of the rank l symmetric tensor representation of $U(3)$. Single trace operators of this form are of the form $\alpha_{j_1 \cdots j_l} Tr(\Phi^{j_1} \cdots \Phi^{j_l})$, where α is symmetric in the $U(3)$ indices, without the need for tracelessness. In this class of chiral primaries $Tr((\Phi^1)^l)$ appears once. Each single trace half-BPS representation contains precisely one such operator. We will henceforth drop the 1 and write this as $Tr((\Phi)^l)$, with the understanding that we have fixed a $U(3)$ index.

The argument can be generalized to multi-trace operators. A two-trace composite of scalars will take the form

$$T_{i_1 \cdots i_{l_1} i_{l_1+1} \cdots i_{l_2}} trX^{i_1} \cdots X^{i_{l_1}} trX^{i_{l_1+1}} \cdots X^{i_{l_2}}, \tag{1}$$

where T is a traceless symmetric tensor of $SO(6)$. This uses the fact that the correspondence between traceless symmetric reps (i.e of type $(0, l, 0)$) and the half-BPS condition holds irrespectively of how we choose to contract the gauge indices, as emphasized for example in [32, 34]. Using again the fact that the traceless symmetric tensor decomposes into $U(3)$ representations which include the symmetric representation built from the $l_1 + l_2$-fold tensor product, we know that $tr(\Phi^{l_1})tr(\Phi^{l_2})$ will appear once in this set of operators. While it is obvious that $tr(\Phi^{l_1})tr(\Phi^{l_2})$ preserves the same supersymmetry as $tr(\Phi^{l_1})$, we have now shown that every double trace half-BPS representation contains one operator of this form. By a similar argument, the multi-trace chiral primaries will include $Tr(\Phi^{l_1})Tr(\Phi^{l_2})\cdots$. More generally for a fixed R charge of n there will be operators of the form

$$(tr(\Phi^{l_1}))^{k_1} tr(\Phi^{l_2}))^{k_2} \cdots (tr(\Phi^{l_m}))^{k_m} \tag{2}$$

where the integers l_i, k_i form a partition of n

$$n = \sum_{i=1}^{m} l_i k_i \tag{3}$$

We have, therefore, a one-to-one correspondence then between half-BPS representations of charge n and partitions of n. A useful basis in this space of operators is given by Schur Polynomials of degree n for the unitary group $U(N)$. We will return to these in section 2.4.

We will associate a Schur polynomial in the complex matrix Φ to each short representation and we will compute correlators which are obtained by considering overlaps of such holomorphic polynomials and their conjugates involving Φ^*. Each observable by itself does not contain both Φ and Φ^*. This gives us a special relation between the weights of the operators involving Φ and those involving Φ^*. These special correlators are called extremal correlators in the literature. There exists a non-renormalization theorem [35, 36] protecting extremal correlators of the half-BPS chiral primaries (single and multi-trace) so that the weak coupling computation of the correlators can be safely extrapolated to strong coupling without change. To compute more general correlation functions, one would need $SO(6)$ descendants of $tr(\Phi^l)$ (and its multi-trace generalizations) which will typically involve all three complex Φ's and their conjugates.

2.2 Free fields and Combinatorics

The basic two-point function we will need follows from the free field correlator :

$$< \Phi_{ij}(x_1)\Phi^*_{kl}(x_2) >= \frac{\delta_{ik}\delta_{jl}}{(x_1 - x_2)^2} \tag{4}$$

Our main focus will be on the dependence of the correlators on the structure of the composites and N, so we will often suppress the spacetime dependences.

Free field contractions in $U(N)$ gauge theory will frequently lead to sums of the form

$$\sum_{i_1, i_2 \cdots i_n} \delta^{i_1}_{i_{\sigma(1)}} \delta^{i_2}_{i_{\sigma(2)}} \cdots \delta^{i_n}_{i_{\sigma(n)}} \tag{5}$$

where each index $i_1, \cdots i_n$ runs over integers from 1 to N. If σ is the identity then the above sum is N^n. If σ is a permutation with one cycle of length 2 and remaining cycles of length 1, then it is $N^{(n-1)}$. One checks that more generally the sum is $N^{C(\sigma)}$ where $C(\sigma)$ is the number of cycles in the permutation σ.

$$\sum_{i_1, i_2 \cdots i_n} \delta^{i_1}_{i_{\sigma(1)}} \delta^{i_2}_{i_{\sigma(2)}} \cdots \delta^{i_n}_{i_{\sigma(n)}} = N^{C(\sigma)} \tag{6}$$

Rather than writing out strings of delta functions carrying n different indices, we will use a multi-index $I(n)$ which is shorthand for a set of n indices. We will also use $I(\sigma(n))$ for a set of n indices with their labels shuffled by a permutation σ. With this notation, (6) takes the form

$$\sum_I \delta \begin{pmatrix} I(n) \\ I(\sigma(n)) \end{pmatrix} = N^{C(\sigma)} \tag{7}$$

We will often need a slight variation of this result,

$$\sum_I \delta \begin{pmatrix} I(\alpha(n)) \\ I(\beta(n)) \end{pmatrix} = N^{C(\beta\alpha^{-1})} = N^{C(\alpha^{-1}\beta)} \tag{8}$$

which follows trivially from the previous relation after noting that the left-hand-side is invariant under the replacement $n \to \alpha^{-1}(n)$, or by acting on the pair (α, β) with α^{-1} from the left, to reduce the LHS of (8) to that of (7).

2.3 Symmetric groups

After performing the sums over contractions we will obtain sums over permutations, or equivalently over elements of permutation groups. It will be useful to manipulate quantities which are formal sums over symmetric group elements. These live in the group algebra of S_n. An interesting function on the group algebra is the delta function which is 1 when the argument is the identity and 0 otherwise. This function has an expansion in characters

$$\delta(\rho) = \frac{1}{n!} \sum_R d_R \;\; \chi_R(\rho) \tag{9}$$

The sum is over representations R of S_n which are associated with Young Diagrams with n boxes. d_R is the dimension of a representation R and $\chi_R(\rho)$ the character, or trace, of the element $\rho \in S_n$ in the representation R.

Another useful fact is that characters, in an irreducible representation of the symmetric group R, of a product of an element $\mathcal{C}$ of the group algebra which commutes with everything with an arbitrary element σ can be factorized as follows :

$$\chi_R(\mathcal{C}\ \sigma) = \frac{\chi_R(\mathcal{C})\ \chi_R(\sigma)}{d_R}. \tag{10}$$

Elements $\mathcal{C}$ that we will run into involve either averages over the symmetric group of the form $\sum_{\alpha,\rho} f(\alpha\rho\alpha^{-1})\rho$, or $\sum_\rho g(\rho)\rho$ where $g(\rho)$ is a class function.

We will also need certain orthogonality properties. Consider

$$\sum_\sigma \chi_R(\sigma^{-1}) D_S(\sigma) \tag{11}$$

where $D_S(\sigma)$ is the matrix representing σ in the irreducible representation S. The matrix written down in (11) can be proved to commute with any permutation τ acting in the representation S. By Schur's Lemma it is therefore proportional to the identity. Hence

$$\sum_\sigma \chi_R(\sigma^{-1}) D_S(\sigma\ \alpha) = \sum_\sigma \chi_R(\sigma^{-1})\ \frac{\chi_S(\sigma)}{d_S}\ D_S(\alpha) \tag{12}$$

Now we can use the orthogonality of characters

$$\sum_\sigma \chi_R(\sigma^{-1})\ \chi_S(\sigma) = \delta_{RS}\ n! \tag{13}$$

in order to simplify (12)

$$\sum_\sigma \chi_R(\sigma^{-1})\ D_S(\sigma\alpha) = \frac{\delta_{RS}\ n!}{d_S} D_S(\alpha) \tag{14}$$

Taking a trace in the representation S we get

$$\sum_\sigma \chi_R(\sigma^{-1})\ \chi_S(\sigma\alpha) = \frac{\delta_{RS}\ n!}{d_S} \chi_S(\alpha) \tag{15}$$

2.4 Duality between Symmetric Groups and Unitary Groups

Denote by V the fundamental representation of $U(N)$. The space $Sym(V^{\otimes n})$ is a representation of $U(N)$ and also admits a commuting action of S_n. The action of $U(N)$ and S_n can thus be simultaneously diagonalised. This allows

Young Diagrams to be associated with both $U(N)$ and S_n representations. Some results following from this connection are summarized in the following. More details can be found in [37, 38] for example.

The Schur polynomials are characters of the unitary group in their irreducible representations.

$$\chi_R(U) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma)\ tr(\sigma\ U) \tag{16}$$

In the RHS the trace is being taken in $V^{\otimes n}$ and both σ and U are operators acting on this space. The action of σ is given by :

$$\sigma(v_{i_1} \otimes v_{i_2} \cdots \otimes v_{i_n}) = (v_{i_{\sigma(1)}} \otimes v_{i_{\sigma(2)}} \cdots \otimes v_{i_{\sigma(n)}}) \tag{17}$$

By inserting $U = 1$ we find a formula for the dimension of a representation of the unitary group :

$$Dim_N(R) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma) N^{C(\sigma)} \tag{18}$$

We will find it convenient, in this paper, to consider the extension of the Schur polynomials from Unitary to Complex Matrices, i.e

$$\chi_R(\Phi) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma) tr(\sigma \Phi) \tag{19}$$

These form a basis in the space of $U(N)$ invariant functions of the Matrix Φ where $U(N)$ acts on Φ by conjugation.

The multi-index notation introduced in section 2.1 is also useful in giving compact expressions for $tr(\sigma\Phi)$

$$tr(\sigma\Phi) = \sum_{i_1, i_2, \cdots, i_n} \Phi^{i_1}_{i_{\sigma(1)}} \Phi^{i_2}_{i_{\sigma(2)}} \cdots \Phi^{i_n}_{i_{\sigma(n)}} \tag{20}$$

$$= \sum_I \Phi \begin{pmatrix} I(n) \\ I(\sigma(n)) \end{pmatrix} \tag{21}$$

The fusion coefficients of $U(N)$ also have meaning in the context of symmetric groups. Let $g(R_1, R_2; S)$ be the multiplicity of the representation S in the tensor product of representations R_1 and R_2. Let n_{R_1}, n_{R_2}, and n_S be the number of boxes in the Young diagrams R_1, R_2, and S respectively. S_{n_S} contains the product $S_{n_{R_1}} \times S_{n_{R_2}}$ as a subgroup. As such the character in S of any permutation which takes the form $\sigma_1 \circ \sigma_2$, a product of two permutations where the first acts non-trivially on the first n_{R_1} elements and the second acts on the last n_{R_2} elements of n_S, can be decomposed into a product of characters :

$$\chi_S(\sigma_1 \circ \sigma_2) = \sum_{R_1 \in Rep(S_{n_1})} \sum_{R_2 \in Rep(S_{n_2})} g(R_1, R_2; S)\ \chi_{R_1}(\sigma_1)\ \chi_{R_2}(\sigma_2) \qquad (22)$$

We know such an expansion in characters of the product group must exist. It is a non-trivial result from the theory of symmetric and unitary groups, related to Frobenius-Schur duality, that the coefficients $g(R_1, R_2, S)$ appearing in the expansion are the multiplicities with which the representations S of $U(N)$ appear in the tensor product of the representations R_1 and R_2 [39]. These coefficients can be written in terms of $U(N)$ group integrals, and can also be computed using a combinatoric algorithm called the Littlewood-Richardson rule.

3 General Results

For two point functions, we find,

$$< \chi_R(\Phi)\chi_S(\Phi^*) >= \delta_{RS} \frac{Dim_N(R) n_R!}{d_R} \qquad (23)$$

In the above we have suppressed the x dependence but they can be restored by conformal invariance

$$< \chi_R(\Phi)(x_1)\chi_S(\Phi^*)(x_2) >= \delta_{RS} \frac{Dim_N(R) n_R!}{d_R} \frac{1}{(x_1 - x_2)^{2n_R}} \qquad (24)$$

For three point functions we have

$$< \chi_R(\Phi)\ \chi_S(\Phi)\ \chi_T(\Phi^*) >= g(R, S; T) \frac{n_T!\ Dim_N(T)}{d_T} \qquad (25)$$

Restoring the dependence on four dimensional space-time coordinates :

$$\begin{aligned} &< \chi_R(\Phi)(x_1)\ \chi_S(\Phi)(x_2)\ \chi_T(\Phi^*)(x_3) > \\ &\quad = g(R, S; T) \frac{n_T!\ Dim_N(T)}{d_T} \frac{1}{(x_1 - x_3)^{2n_R}(x_2 - x_3)^{2n_S}} \end{aligned} \qquad (26)$$

For $l \to 1$ we have

$$\begin{aligned} &< \chi_{R_1}(\Phi)\chi_{R_2}(\Phi) \cdots \chi_{R_n}(\Phi)\chi_S(\Phi^*) > \\ &\quad = g(R_1, R_2, \cdots R_n; S) \frac{n_S! Dim_N(S)}{d_S} \end{aligned} \qquad (27)$$

Restoring spacetime coordinates :

$$< \chi_{R_1}(\Phi)(x_1)\chi_{R_2}(\Phi)(x_2)\cdots\chi_{R_n}(\Phi)(x_l)\chi_S(\Phi^*)(y) > \\ = \frac{n_S!Dim(S)}{d_S}\frac{g(R_1,R_2,\cdots R_n;S)}{(x_1-y)^{2n_{R_1}}(x_2-y)^{2n_{R_2}}\cdots(x_l-y)^{2n_{R_l}}} \tag{28}$$

For correlators of this form where the conformal weights are related as above $n_S = n_{R_1} + \cdots n_{R_l}$, the kind of contractions we have described above are the most general.

For $l \to k$ we have

$$< \chi_{R_1}(\Phi)\chi_{R_2}(\Phi)\cdots\chi_{R_l}(\Phi)\ \chi_{S_1}(\Phi^*)\cdots\chi_{S_k}(\Phi^*) > \\ = \sum_S g(R_1,R_2\cdots R_l;S)\frac{n_S!Dim_N S}{d_S}g(S_1,S_2,\cdots S_k;S) \tag{29}$$

In this case we are capturing only a special case of the $l \to k$ multipoint function, where the spacetime coordinates of all the operators involving Φ^* coincide,

$$< \chi_{R_1}(\Phi)(x_1)\chi_{R_2}(\Phi)(x_2)\cdots\chi_{R_n}(\Phi)(x_l)\ \chi_{S_1}(\Phi^*)(y)\cdots\chi_{S_m}(\Phi^*)(y) > \\ = \sum_S \frac{n_S!DimS}{d_S}g(S_1,S_2,\cdots S_m;S)\frac{g(R_1,R_2\cdots R_n;S)g(S_1,S_2,\cdots S_m;S)}{(x_1-y)^{2n_{R_1}}\cdots(x_l-y)^{2n_{R_k}}} \tag{30}$$

The coefficient $g(R_1, R_2 \cdots R_n; S)$ is a positive integer. It can also be expressed in terms of Unitary group integrals :

$$g(R_1,R_2,\cdots R_n;S) = \int dU(\prod_{i=1}^n \chi_{R_i}(U))\chi_S(U^*) \tag{31}$$

It is obtained, in the case of $n = 2$ by the Littlewood-Richardson rule for fusing Young diagrams (see for example [40]). The product $\chi_{R_1}(U)\chi_{R_2}(U)$ can be expanded as $\sum_R g(R_1, R_2; R)\chi_R(U)$. Repeated use of this expansion in the integral (31) leads to

$$g(R_1,R_2\cdots R_n;S) = \sum_{S_1,S_2\cdots,S_{n-2}} (g(R_1,R_2;S_1)\ g(S_1,R_3;S_2)\cdots \\ \cdots g(S_{n-2},R_n;S)). \tag{32}$$

4 Two point functions

We will derive here formulae for the correlator $< \chi_R(\Phi)\chi_S(\Phi^*) >$. It follows trivially that this will only be non-zero when the number of boxes in R is the same as the number of boxes in S. So we have $n_R = n_S = n$. A more non-trivial fact is that the exact finite N correlator is proportional to δ_{RS}, i.e., it is non-zero only if R and S are the same Young Diagram.

We first convert the free field computation to sums over the symmetric group S_n.

$$
\begin{aligned}
&< \chi_R(\Phi)\chi_S(\Phi^*) > \\
&= \; < \sum_\sigma \frac{\chi_R(\sigma)}{n!} \; tr(\sigma\Phi) \sum_\tau \frac{\chi_S(\tau)}{n!} \; tr(\tau\Phi^*) > \\
&= \sum_{i_1,i_2\cdots i_n} \; \sum_{j_1,j_2\cdots j_n} \sum_\sigma \frac{\chi_R(\sigma)}{n!} \sum_\tau \frac{\chi_S(\tau)}{n!} \\
&\times \qquad < \Phi^{i_1}_{i_{\sigma(1)}} \Phi^{i_2}_{i_{\sigma(2)}} \cdots \Phi^{i_n}_{i_{\sigma(n)}} (\Phi^*)^{j_1}_{j_{\tau(1)}} (\Phi^*)^{j_2}_{j_{\tau(2)}} \cdots (\Phi^*)^{j_n}_{j_{\tau(n)}} > \\
&= \sum_{i_1,i_2\cdots i_n} \; \sum_{j_1,j_2\cdots j_n} \sum_\alpha \sum_{\sigma,\tau} \frac{\chi_R(\sigma)}{n!} \frac{\chi_S(\tau)}{n!} \\
&\times \qquad \delta^{i_1}_{j_{\alpha\tau(1)}} \delta^{i_2}_{j_{\alpha\tau(2)}} \cdots \delta^{i_n}_{j_{\alpha\tau(n)}} \quad \delta^{j_1}_{i_{\alpha^{-1}\sigma(1)}} \delta^{j_2}_{i_{\alpha^{-1}\sigma(2)}} \cdots \delta^{j_n}_{i_{\alpha^{-1}\sigma(n)}} \\
&= \sum_{i_1,i_2\cdots i_n} \; \sum_\alpha \sum_\sigma \frac{\chi_R(\sigma)}{n!} \sum_\tau \frac{\chi_S(\tau)}{n!} \; \delta^{i_{\tau^{-1}\alpha^{-1}(1)}}_{i_{\alpha^{-1}\sigma(1)}} \cdots \delta^{i_{\tau^{-1}\alpha^{-1}(n)}}_{i_{\alpha^{-1}\sigma(n)}} \\
&= \sum_{i_1,i_2\cdots i_n} \sum_\alpha \sum_\sigma \frac{\chi_R(\sigma)}{n!} \sum_\tau \frac{\chi_S(\tau)}{n!} \; \delta^{i_{\sigma^{-1}\alpha\tau^{-1}\alpha^{-1}(1)}}_{i_1} \cdots \delta^{i_{\sigma^{-1}\alpha\tau^{-1}\alpha^{-1}(n)}}_{i_n} \\
&= \sum_\alpha \sum_\sigma \frac{\chi_R(\sigma)}{n!} \sum_\tau \frac{\chi_S(\tau)}{n!} \; N^{C(\sigma^{-1}\alpha\tau^{-1}\alpha^{-1})} \qquad (33)
\end{aligned}
$$

To show how the notation developed in section 2 simplifies the calculations, we redo the same manipulations as above in a slightly more compact form. For higher point functions we will present the derivations exclusively in the more compact form.

$$
\begin{aligned}
&< \sum_{\sigma} \frac{\chi_R(\sigma)}{n!} \; tr(\sigma\Phi) \sum_{\tau} \frac{\chi_S(\tau)}{n!} \; tr(\tau\Phi^*) > \\
&= \sum_{I,J} \sum_{\sigma} \frac{\chi_R(\sigma)}{n!} \sum_{\tau} \frac{\chi_S(\tau)}{n!} < \Phi \begin{pmatrix} I(n) \\ I(\sigma(n)) \end{pmatrix} \Phi^* \begin{pmatrix} J(n) \\ J(\sigma(n)) \end{pmatrix} > \\
&= \sum_{I,J} \sum_{\sigma,\tau,\alpha} \frac{\chi_R(\sigma)}{n!} \frac{\chi_S(\tau)}{n!} \delta \begin{pmatrix} I(n) \\ J(\alpha\tau(n)) \end{pmatrix} \delta \begin{pmatrix} J(n) \\ I(\alpha^{-1}\tau(n)) \end{pmatrix} \\
&= \sum_{I} \sum_{\sigma,\tau,\alpha} \frac{\chi_R(\sigma)}{n!} \frac{\chi_S(\tau)}{n!} \delta \begin{pmatrix} I(\tau^{-1}\alpha^{-1}(n)) \\ I(\alpha^{-1}\sigma(n)) \end{pmatrix} \\
&= \sum_{I} \sum_{\sigma,\tau,\alpha} \frac{\chi_R(\sigma)}{n!} \frac{\chi_S(\tau)}{n!} \delta \begin{pmatrix} I(\sigma^{-1}\alpha\tau^{-1}\alpha^{-1}(n)) \\ I(n) \end{pmatrix} \\
&= \sum_{\sigma,\tau,\alpha} \frac{\chi_R(\sigma)}{n!} \frac{\chi_S(\tau)}{n!} N^{C(\sigma^{-1}\alpha\tau^{-1}\alpha^{-1})} \qquad (34)
\end{aligned}
$$

Having converted the sum over contractions to a sum over symmetric groups we will now use the connection between symmetric and Unitary groups to express the answer in terms of Dimensions of Unitary groups. Introducing a new summed permutation p constrained by a delta function, which simplifies the exponent of N, and then summing over τ we get,

$$
\begin{aligned}
&\sum_{\alpha,\sigma,\tau,p} \frac{\chi_R(\sigma)}{n!} \; \frac{\chi_S(\tau)}{n!} \; N^{C(p)} \; \delta(p^{-1}\sigma^{-1}\alpha\tau^{-1}\alpha^{-1}) \\
&= \frac{1}{(n!)^2} \sum_{\alpha,\sigma,\gamma} \chi_R(\sigma) \; \chi_S(\alpha^{-1}p^{-1}\sigma^{-1}\alpha) \; N^{C(p)} \\
&= \frac{1}{n!} \sum_{\sigma,p} \chi_R(\sigma) \; \chi_S(p^{-1}\sigma^{-1}) \; N^{C(p)} \\
&= \sum_{p} \frac{1}{d_R} \; \delta_{RS} \; \chi_S(p^{-1}) \; N^{C(p)} \\
&= n! \; \frac{Dim(R)}{d_R} \; \delta_{RS} \qquad (35)
\end{aligned}
$$

In the third line we used cyclic invariance of the trace to do the sum over α thus gaining a factor $n!$. To obtain the $\frac{\delta_{RS}}{d_R}$ in the fourth line, we used (13). In the final step we used (18).

The factor $\frac{n! Dim(R)}{d_R}$ will come up frequently, so we will call it f_R and make it more explicit. From group theory texts [38, 37], we find the dimensions :

$$d_R = \frac{n!}{\prod_{i,j} h_{i,j}}, \tag{36}$$

$$Dim(R) = \prod_{i,j} \frac{(N-i+j)}{h_{i,j}}. \tag{37}$$

The product runs over the boxes of the Young Diagram associated with R, with i labelling the rows and j labelling the column. The quantity $h_{i,j}$ is the hook-length associated with the box. The quantity f_R takes the simple form:

$$f_R = \prod_{i,j} (N - i + j) \tag{38}$$

This orthogonality of operators associated with Young Diagrams is reminiscent of *Unitary* Matrix Models. Indeed if we integrate the characters of $U(N)$ as with the unit-normalized Haar measure as in $\int dU \chi_R(U) \chi_S(U^\dagger) = \delta_{RS}$ we have orthogonality with a different normalization factor. The result in (35) involving supersymmetric observables constructed from *complex matrices* does not follow directly from group theory of the unitary group, but rather, as the derivation shows, by the connection between free field contractions of the complex matrices and symmetric group sums followed by the relations between Unitary and Symmetric groups. Further insight into the result (35) will be developed in the last section from a reduced Matrix Model of complex Matrices.

5 Three-point functions

We consider the following three-point function of Schur polynomials.

$$< \chi_{R_1}(\Phi) \chi_{R_2}(\Phi) \chi_S(\Phi^*) > \tag{39}$$

Representation R_1 has n_{R_1} boxes, R_2 has n_{R_2} and S has n_S. For a non-zero free field correlator, $n_S = n_{R_1} + n_{R_2}$. By using the expansion of Schur Polynomials in terms of characters of the symmetric group (19), this is

$$< \sum_{\sigma_1, \sigma_2, \tau} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!} \frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!} \; tr(\sigma_1 \Phi) \quad tr(\sigma_2 \Phi) \; \frac{\chi_S(\tau)}{n_S!} \; tr(\tau \Phi^*) > \tag{40}$$

where the sum over σ_1 runs over all permutations in $S_{n_{R_1}}$, σ_2 runs over all permutations in $S_{n_{R_2}}$, and τ runs over permutations in S_{n_S}. Expanding out the expressions of the form $tr(\sigma\Phi)$ using (20) we get

$$\sum_{\sigma_1,\sigma_2,\tau} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\frac{\chi_S(\tau)}{n_S!} < \Phi\begin{pmatrix} I_1(n_{R_1}) \\ I_1(\sigma_1(n_{R_1}))\end{pmatrix}\Phi\begin{pmatrix} I_2(n_{R_2}) \\ I_2(\sigma_2(n_{R_2}))\end{pmatrix} \times \Phi^*\begin{pmatrix} J(n_S) \\ J(\tau(n_S))\end{pmatrix} > \tag{41}$$

After doing the contractions we have a sum over an extra permutation α in S_{n_S},

$$\sum_{\sigma_1,\sigma_2,\tau,\alpha}\sum_{I_1,I_2,J} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\frac{\chi_S(\tau)}{n_S!}\delta\begin{pmatrix} I_1(n_{R_1})\, I_2(n_{R_2}) \\ J(\alpha\tau(n_S))\end{pmatrix} \times \delta\begin{pmatrix} J(\alpha(n_s)) \\ I_1(\sigma_1(n_1))\, I_2(\sigma_2(n_2))\end{pmatrix} \tag{42}$$

where we have used the compact form for products of delta functions explained in (7) and (8). It is convenient to combine the multi-indices (I_1, I_2) into a single multi-index I

$$\sum_{\sigma_1,\sigma_2,\tau,\alpha}\sum_{I,J} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\frac{\chi_S(\tau)}{n_S!}\delta\begin{pmatrix} I(n_S) \\ J(\alpha\tau(n_S))\end{pmatrix} \times \delta\begin{pmatrix} J(\alpha(n_s)) \\ I(\sigma_1\circ\sigma_2(n_S))\end{pmatrix} \tag{43}$$

Now we do the sum over the J multi-index, to get

$$\begin{aligned}&\sum_{\alpha,\sigma_1,\sigma_2,\tau}\sum_{I} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\frac{\chi_S(\tau)}{n_S!}\;\delta\begin{pmatrix} I(\tau^{-1}\alpha^{-1}(n_S)) \\ I(\alpha^{-1}(\sigma_1\circ\sigma_2)(n_S))\end{pmatrix} \\ &= \sum_{\alpha,\sigma_1,\sigma_2,\tau}\sum_{I} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\frac{\chi_S(\tau)}{n_S!}\;\delta\begin{pmatrix} I(n_S) \\ I(\alpha\tau\alpha^{-1}(\sigma_1\circ\sigma_2)(n_S))\end{pmatrix}\end{aligned} \tag{44}$$

We use (8) to rewrite this as

$$\sum_{\alpha,\sigma_1,\sigma_2,\tau} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\frac{\chi_S(\tau)}{n_S!}N^{C(\alpha\tau\alpha^{-1}(\sigma_1\circ\sigma_2))} \tag{45}$$

Introducing an extra sum over a symmetric group element and constraining it by insertion of a delta function over the symmetric group

$$\sum_{\alpha,\sigma_1,\sigma_2,\tau,p} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\frac{\chi_S(\tau)}{n_S!}N^{C(p)}\delta(p^{-1}\alpha\tau\alpha^{-1}(\sigma_1\circ\sigma_2)) \tag{46}$$

Performing the sum over τ

$$\sum_{\alpha,\sigma_1,\sigma_2,p} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\frac{\chi_S(\alpha^{-1}p(\sigma_1^{-1}\circ\sigma_2^{-1})\alpha)}{n_S!}N^{C(p)} \tag{47}$$

The sum over α is trivial and cancels the factor of $n_S!$ in the denominator

$$\sum_{\sigma_1,\sigma_2,p} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\ \chi_S((\sigma_1\circ\sigma_2)^{-1}p)N^{C(p)} \tag{48}$$

The sum $\sum_p N^{C(p)}p$ commutes with any element of S_{n_S} and therefore can be taken out of the character χ_S using (10) to obtain

$$\sum_{\sigma_1,\sigma_2,p} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\ \chi_S((\sigma_1\circ\sigma_2)^{-1})N^{C(p)}\frac{\chi_S(p)}{d_S}. \tag{49}$$

We expand the character of a permutation living in a product sub-group into a product of characters using (22),

$$\sum_{\sigma_1,\sigma_2,p} \frac{\chi_{R_1}(\sigma_1)}{n_{R_1}!}\frac{\chi_{R_2}(\sigma_2)}{n_{R_2}!}\ \sum_{R_1',R_2'} g(R_1',R_2';S)\ \chi_{R_1'}((\sigma_1)^{-1})$$
$$\times\ \chi_{R_2'}((\sigma_2)^{-1})\frac{\chi_S(p)}{d_S}N^{C(p)}. \tag{50}$$

Finally we use orthogonality of characters of the symmetric group (13) and (18) to get the answer

$$g(R_1,R_2;S)\frac{Dim(S)\ \ n_S!}{d_S} = g(R_1,R_2;S)f_S. \tag{51}$$

6 Multi-point functions

6.1 Derivation of result for $(l,1)$ multi-point functions

When we apply the manipulations of section 5 to the $(l,1)$ point function

$$<\chi_{R_1}(\Phi)\chi_{R_2}(\Phi)\cdots\chi_{R_n}(\Phi)\chi_S(\Phi^*)> \tag{52}$$

we are lead to an analog of (49) which is

$$\sum_{\sigma_1,\sigma_2,\cdots\sigma_l,p}\ \prod_{i=1}^{l}\frac{\chi_{R_i}(\sigma_1)}{n_{R_i}!}\ \ \chi_S((\sigma_1\circ\sigma_2\cdots\sigma_l)^{-1})\ N^{C(p)}\ \frac{\chi_S(p)}{d_S}. \tag{53}$$

To expand the character of a product $\chi_S((\sigma_1 \circ \sigma_2 \cdots \sigma_l)^{-1})$ we make repeated use of (22). This leads to

$$\chi_S(\prod_i \sigma_i^{-1}) = \sum_{R_1,R_2\cdots R_l} \sum_{S_1,S_2\cdots S_{l-2}} g(R_1,R_2;S_1)g(S_1,R_3;S_2)\cdots$$
$$\cdots g(S_{l-2},R_l;S)\prod_i \chi_{R_i}(\sigma_i^{-1}) \tag{54}$$

We can recognize this, using (32), as

$$\chi_S(\prod_i \sigma_i^{-1}) = \sum_{R_1,R_2\cdots R_l} g(R_1,R_2,\cdots R_l;S)\prod_i \chi_{R_i}(\sigma_i^{-1}) \tag{55}$$

The remaining steps proceed as in section 5, to give the answer (27) and (28) after introducing the spacetime dependences.

6.2 Derivation of the (l,k) multi-point function

The correlation function of interest is :

$$\sum_{\sigma_i\in S_{n_{R_i}}} \sum_{\tau_j\in S_{n_{S_j}}} < \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}} \; tr\;(\sigma_i\Phi)\prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}} \; tr\;(\sigma_j\Phi^*) > \tag{56}$$

When we introduce the spacetime coordinate dependences, all the Φ^* operators are at a point.

Unravelling the traces :

$$\sum_{\sigma_i\in S_{n_{R_i}}} \sum_{\tau_j\in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!}\prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!} \sum_{I_1,I_2,\cdots I_l}\sum_{J_1,J_2\cdots J_k}$$
$$< \Phi\begin{pmatrix} I_1(n_{R_1}) \\ I_1(\sigma_1(n_{R_1}))\end{pmatrix}\Phi\begin{pmatrix} I_2(n_{R_2}) \\ I_2(\sigma_2(n_{R_2}))\end{pmatrix}\cdots\;\Phi\begin{pmatrix} I_l(n_{R_l}) \\ I_l(\sigma_l(n_{R_l}))\end{pmatrix}$$
$$\times\;\Phi^*\begin{pmatrix} J_1(n_{S_1}) \\ J_1(\tau_1(n_{S_1}))\end{pmatrix}\Phi^*\begin{pmatrix} J_2(n_{S_2}) \\ J_2(\tau_2(n_{S_2}))\end{pmatrix}\cdots\;\Phi^*\begin{pmatrix} J_k(n_{S_k}) \\ J_k(\tau_k(n_{S_k}))\end{pmatrix} \tag{57}$$

We have used the multi-index notation introduced in section 2. After we do the contractions, we get a sum over permutations in S_{n_T} where $n_T =$

$\sum_{i=1}^{l} n_{R_i} = \sum_{j=1}^{k} n_{S_j}$. Performing the contractions we find

$$\sum_{\sigma_i \in S_{n_{R_i}}} \sum_{\tau_j \in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!} \prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!} \times \sum_I \sum_J \sum_{\alpha \in S_{n_T}} \delta\begin{pmatrix} I(n_T) \\ J(\alpha \prod_j \tau_j(n_T)) \end{pmatrix} \delta\begin{pmatrix} J(n_T) \\ I(\alpha^{-1} \prod_i \sigma_i(n_T)) \end{pmatrix} \tag{58}$$

In this equation we have replaced the set of multi-indices $(I_1, I_2 \cdots I_k)$ by a single multi-index I, which is convenient because we have permutations which mix the entire set of n_T indices.

Peforming the sum over the J multi-index we are left with

$$\sum_{\sigma_i \in S_{n_{R_i}}} \sum_{\tau_j \in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!} \prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!} \times \sum_I \sum_{\alpha \in S_{n_T}} \delta\begin{pmatrix} I(\ \prod_j \tau_j^{-1}\ \alpha^{-1}(n_T)\ ) \\ I(\ \alpha^{-1} \prod_i \sigma_i(n_T)\ ) \end{pmatrix} \tag{59}$$

Now we use the result (8) described in section 2, to get :

$$\sum_{\sigma_i \in S_{n_{R_i}}} \sum_{\tau_j \in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!} \prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!} \times \sum_{\alpha \in S_{n_T}} N^{C(\prod_i \sigma_i^{-1} \alpha \prod_j \tau_j^{-1} \alpha^{-1})} \tag{60}$$

It will be convenient to rewrite this by introducing an extra sum over a permutation p in S_{N_T}, and constrain the sum by inserting a delta function over the symmetric group.

$$\sum_{\sigma_i \in S_{n_{R_i}}} \sum_{\tau_j \in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!} \prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!} \times \sum_{\alpha \in S_{n_T}} \sum_{p \in S_{n_T}} N^{C(p)} \delta(p \prod_i \sigma_i^{-1} \alpha \prod_j \tau_j^{-1} \alpha^{-1}) \tag{61}$$

Now we expand the delta function into a sum over characters using (9).

$$\sum_{\sigma_i \in S_{n_{R_i}}} \sum_{\tau_j \in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!} \prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!} \sum_{\alpha \in S_{n_T}} \sum_{p \in S_{n_T}}$$
$$\times \sum_{T \in Rep(S_{n_T})} \frac{d_T}{n_T!} N^{C(p)} \chi_T(p^{-1}(\prod_i \sigma_i^{-1})\alpha(\prod_j \tau_j^{-1})\alpha^{-1}) \qquad (62)$$

We can expand the character in the last line into a product of characters by taking advantage of the fact that $\sum_p N^{C(p)} p$ as well as $\sum_\alpha \alpha \ \prod_j \tau_j^{-1} \ \alpha^{-1}$ are in the centre of the group algebra of S_{n_T}, and the equation (10).

$$\sum_{\sigma_i \in S_{n_{R_i}}} \sum_{\tau_j \in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!} \prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!} \sum_{\alpha \in S_{n_T}} \sum_{p \in S_{n_T}}$$
$$\times \sum_{T \in Rep(S_{n_T})} \frac{d_T}{n_T!} N^{C(p)} \chi_T(p) \ \frac{1}{d_T} \chi_T(\prod_i \sigma_i^{-1}) \frac{1}{d_T} \ \chi_T(\alpha(\prod_j \tau_j^{-1})\alpha^{-1})$$
$$= \sum_{\sigma_i \in S_{n_{R_i}}} \sum_{\tau_j \in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!} \prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!}$$
$$\times \sum_{p \in S_{n_T}} \sum_{T \in Rep(S_{n_T})} \frac{1}{d_T} N^{C(p)} \chi_T(p) \ \chi_T(\prod_i \sigma_i^{-1}) \chi_T(\prod_j \tau_j^{-1}) \qquad (63)$$

In the second line above we have done the sum over α, using the invariance of the character under conjugation, to get a factor of $n_T!$. We now apply the expansion (55) to $\chi_T(\prod_i \sigma_i^{-1} \ )$ and $\chi_T(\ \prod_j \tau_j^{-1})$ to get a product of characters.

$$\sum_{\sigma_i \in S_{n_{R_i}}} \sum_{\tau_j \in S_{n_{S_j}}} \prod_i \frac{\chi_{R_i}(\sigma_i)}{n_{R_i}!} \prod_j \frac{\chi_{S_j}(\tau_j)}{n_{S_j}!} \sum_{p \in S_{n_T}} \sum_{T \in Rep(S_{n_T})} \frac{1}{d_T} N^{C(p)} \chi_T(p)$$
$$\times \sum_{R'_1,\ldots,R'_l} \sum_{S'_1,\ldots,S'_k} g(R'_1, \cdots R'_l; T) g(S'_1, \ldots, S'_k; T) \prod_i \chi_{R'_i}(\sigma_i^{-1})$$
$$\times \prod_j \chi_{S'_j}(\tau_j^{-1}) \qquad (64)$$

Now we can use orthogonality of characters to do the sums over permutations σ_i, τ_j, to obtain $\prod_i \delta_{R_i,R'_i} \prod_j \delta_{S_j,S'_j}$, which allows us to the sums over

R'_i, S'_j trivially. We also recognize the sum $\sum_p N^{C(p)} \chi_T(p) = n_T! Dim(T)$. Putting these elements together we arrive at the final formula for the $(l \to k)$ multi-point function

$$\sum_T g(R_1, R_2, \cdots, R_l; T) \frac{n_T! \ Dim(T)}{d_T} g(S_1, S_2 \cdots, S_k; T). \tag{65}$$

7 Correlators of half-BPS composites and gravitons in $AdS_5 \times S^5$

Young Diagrams R associated with small $\mathcal{R}$-charge, i.e $n_R \ll N$ are associated with KK states. They can be written as sums of products of small numbers of traces. There is an approximate correspondence in the large N limit between single trace chiral primaries and single particle states in AdS via the AdS/CFT correspondence. Kaluza-Klein modes arising from dimensional reduction of ten-dimensional type IIB supergravity on S^5 couple to single trace chiral primaries on the boundary of AdS_5. Using the map given in [2, 3], one can compute correlators of the KK modes and compare to correlators of the single trace chiral primaries and one finds agreement in the large N limit for the three point functions, [41].

Among Young Diagrams with large numbers of boxes, two classes stand out. One involves large mostly-antisymmetric representations and another involves large mostly-symmetric representations. Among the large, mostly antisymmetric ones, there is a series of Young Diagrams which have one column of length L, where L is comparable to N and the remaining columns of length zero. We may denote these by the column lengths $\vec{c} = (L, \vec{0})$. These are proportional to the sub-determinants considered in [16] as duals of sphere giants

$$\chi_{(L,\vec{0})}(\Phi) \sim \frac{1}{L!} \epsilon_{i_1 \cdots i_L i_{L+1} \cdots i_N} \epsilon^{j_1 \cdots j_L j_{L+1} \cdots j_N} \Phi^{i_1}_{j_1} \cdots \Phi^{i_L}_{j_L} \tag{66}$$

This series has a cutoff of $L \leq N$ which corresponds to the fact that sphere giants have a maximal angular momentum of N.

More general representations among the mostly-antisymmetric ones have column lengths $\vec{c} = (L_1, L_2, \cdots L_k, \vec{0})$. All the $L_1 \cdots L_k$ are comparable to N and the number of non-zero column lengths k is small. If all of them are equal, a natural dual giant graviton is a simple generalization of the original sphere giant, where the three-brane wraps the S^3 in S^5 with a winding number k. An ansatz with winding number k leads to an effective probe Lagrangian of the same form as that for the original sphere giant, with the factor k scaling out of the Lagrangian. This leads to a solution with angular momentum and energy scaled up by a factor of k. When the L_i are not all equal, but differ by numbers small compared to N, we can think of the Young Diagram as obtained by fusing a large Young Diagram with all equal and large columns with a small Young Diagram. Thus the natural physical interpretation is in terms of composites involving giants and Kaluza-Klein gravitons.

The simplest large mostly-symmetric representations have one row of non-zero length L, so the vector $\vec{r}$ of row lengths is $\vec{r} = (L, \vec{0})$. Now L has no upper bound and this series has angular momenta increasing in units of 1. These properties suggest that these are duals of AdS giants. Indeed AdS giants have no upper bound on their angular momentum [14, 15, 13]. Duals to AdS giants at the classical level have been proposed in [15]. We find there is indeed a heuristic connection between the classical solutions considered there and these symmetric representations. The solutions in [15] have the form :

$$\Phi \sim \Phi_0 e^{i\omega t} \tag{67}$$

where $\Phi_0 \sim Diag(\eta, -\frac{\eta}{N-1} \cdots - \frac{\eta}{N-1})$. Viewing $\chi_R(\Phi)$ as wavefunctions we would expect a strong overlap between the candidate dual to the AdS giant and the corresponding solution. Indeed, approximating at large N the matrix Φ_0 with $Diag(\eta, \vec{0})$, we find that $\chi_R(\Phi_0)$ is zero unless R is the symmetric representation with one non-zero row length.

For these reasons, when R is described by the vector of row lengths $\vec{r} = (L, \vec{0})$, with L comparable to or much larger than N, we propose the spacetime dual to be the AdS giant. For $\vec{r} = (L, L, \cdots L, 0)$ with a small number k of non-zero entries, we associate it with a multiply wound AdS giant, i.e where the 3-brane multiply winds the S^3 in AdS_5. As in the sphere giant case, the multiple winding ansatz leads to an effective action which has the winding number k scaling out, leading to a scaling of the angular momentum and energy by k in agreement with the proposed gauge theory dual. As in the discussion of sphere giants, the Young Diagram with row lengths $\vec{r} = (L_1, L_2, \cdots, L_k, \vec{0})$ where the L_i are not all equal, can be considered to be duals of composite states involving AdS giants and KK gravitons.

The detailed test of this picture will emerge by comparison of correlators of giants computed in gauge theory and computations in string theory on $AdS_5 \times S^5$. We will write down gauge theory correlators involving the various types of gravitons, applying the general formulae derived in previous sections. We leave the computations on the string theory side to future work. Even a clear formulation of semiclassical gravity + probe calculations of these correlators remains to be articulated. Our remarks on the interpretation of the large N behaviour of the correlators we describe below are therefore somewhat heuristic.

7.1 Large antisymmetric representations and Sphere Giants

For the correlators that we shall consider in this and the next sections, we shall discuss two natural ways in which to normalize. For a correlator of the form $\langle \prod_{i=1}^{l} \chi_{R_i}(\Phi) \prod_{j=1}^{k} \chi_{S_j}(\Phi^*)\rangle$, we could normalize it by dividing by the norms of each individual character, i.e., by $\prod_{i=1}^{l} \parallel \chi_{R_i}(\Phi) \parallel$ and $\prod_{j=1}^{k} \parallel \chi_{S_j}(\Phi) \parallel$. Another possibility is to divide instead by the norm of the Φ operator $\parallel \prod_{i=1}^{l} \chi_{R_i}(\Phi) \parallel$ as well as by the norm of the Φ^* operator $\parallel \prod_{j=1}^{k} \chi_{S_j}(\Phi^*) \parallel$. The latter normalization is relevant if the correlator is to be interpreted as an overlap of the two states $\prod_{i=1}^{l} \chi_{R_i}(\Phi^*)|0\rangle$ and $\prod_{j=1}^{k} \chi_{S_j}(\Phi^*)|0\rangle$. In this case one can show using (29) that the correlator is bounded above by one. As a simple illustration consider the $m = 1$ case. From (29) we have

$$\parallel \prod_{i=1}^{l} \chi_{R_i}(\Phi) \parallel = \sqrt{\sum_T (g(R_1, R_2, ..., R_n; T))^2 \frac{n_T! \; dim_N T}{d_T}} \tag{68}$$

while $\parallel \chi_S(\Phi) \parallel = \sqrt{n_S! \; dim_N S / d_S}$. Dividing (27) by these two norms gives a normalized correlator which is less than one, since the sum over T always contains S in addition to additional positive terms. The general case follows in a similar manner.

Now we turn to some explicit examples of correlators involving large representations, i.e., representations with $\mathcal{O}(N)$ boxes in the associated Young Diagrams. As a first example we consider the three point correlators $\langle \chi_{L_1}(\Phi)\chi_{L_2}(\Phi)\chi_{L_3}(\Phi^*)\rangle$ where $L_3 = L_1 + L_2$ and the representations are the completely antisymmetric representations derived from the L_i^{th} tensor product representation. From (25), (35), (38) we can read off the normalized three point function as

$$\frac{\langle \chi_{L_1}(\Phi)\chi_{L_2}(\Phi)\chi_{L_3}(\Phi^*)\rangle}{\parallel \chi_{L_1}(\Phi) \parallel \parallel \chi_{L_2}(\Phi) \parallel \parallel \chi_{L_3}(\Phi) \parallel} = \sqrt{\frac{N!}{(N-L_3)!} \frac{(N-L_1)!}{N!} \frac{(N-L_2)!}{N!}}. \tag{69}$$

We have used the fact that the Littlewood-Richardson coefficient in this case is 1, as can be deduced from the rules in [40] for example.

Taking in particular $L_3 = N$ with $L_1 = (N/2 + L)$ and $L_2 = (N/2 - L)$ we obtain

$$\frac{\langle \chi_{(N/2-L)}(\Phi)\chi_{(N/2+L)}(\Phi)\chi_N(\Phi^*)\rangle}{\| \chi_{(N/2+L)}(\Phi) \|\|\| \chi_{(N/2-L)}(\Phi) \|\|\| \chi_N(\Phi) \|} = \sqrt{\frac{(N/2+L)!(N/2-L)!}{N!}}. \quad (70)$$

For $L = \mathcal{O}(1)$ this correlator represents the three point function of sphere giant gravitons, considered in [16]. For $L = 0$ the correlator is approximately $1/2^{N/2}$. This exponential decay in N suggests that an instanton mediates a transition between the single sphere giant with R-charge N and the pair of giants of R-charges $N/2$. The right-hand-side of (70) monotonically increases with L until at $L = (N/2 - 1)$ we get a correlator going like $1/N$. So in fact for all $0 \le L \le (N/2 - 1)$ the correlator is either exponentially or polynomially suppressed in N.

To normalize the correlator (69) as an overlap of two states as discussed above requires computing a four point function. This follows straightforwardly from (29) and the result is

$$\| \chi_{L_1}(\Phi)\chi_{L_2}(\Phi) \|^2 = \sum_{l=0}^{L_2} \frac{N!(N+1)!}{(N-L_1-l)!(N-L_2+l+1)!}. \quad (71)$$

We have used the fact that the tensor product of the representation with column lengths $(L_1, \vec{0})$ with $(L_2, \vec{0})$ (for $L_2 < L_1$) contains with unit multiplicity the representations $(L_1 + l, L_2 - l, \vec{0})$ with unit multiplicity, with k ranging from 0 to L_2. For the special case of $L_1 = L_2 = N/2$ the sum can be done and one finds the very simple expression $2^N N!$. Normalizing the three-point function (70) for $L = 0$ in this way leads to the exact answer of

$$\frac{\langle \chi_{N/2}(\Phi)\chi_{N/2}(\Phi)\chi_N(\Phi^*)\rangle}{\| (\chi_{N/2}(\Phi))^2 \|\|\| \chi_N(\Phi) \|} = \frac{1}{2^{N/2}}. \quad (72)$$

So even with this normalization we find an exponential decay in N of the overlap.

Another case of interest is the three point function of characters in the representations as in (69) with the L_3 representation replaced by a Young tableaux with two columns, one of length L_1 and the other of length L_2. One finds the the simple looking result

$$\frac{\langle \chi_{L_1}(\Phi)\chi_{L_2}(\Phi)\chi_{L_1,L_2}(\Phi^*)\rangle}{\| \chi_{L_1}(\Phi) \|\|\| \chi_{L_2}(\Phi) \|\|\| \chi_{L_1,L_2}(\Phi) \|} = \sqrt{\frac{N+1}{N-L_2+1}} \quad (73)$$

assuming that $L_2 \le L_1$. As L_2 ranges from 1 to L_1 one finds that the correlator either remains $\mathcal{O}(1)$ or becomes $\mathcal{O}(\sqrt{N})$ if $L_1 \approx N$. For L_1 and

L_2 both small, the three point functions involves KK states rather than giant gravitons. The Schur Polynomials are sums of traces and multi-traces with coefficients of $\mathcal{O}(1)$. The single traces are associated with single particle states in the leading large N limit and they have three point functions of which go like $\mathcal{O}(\frac{1}{N})$. The generic Schur polynomials have $\mathcal{O}(1)$ correlators because there are contributions from disconnected diagrams. For $L_2 = \mathcal{O}(1)$ with $L_1 = \mathcal{O}(N)$ this correlator may be associated with a sphere giant and a multi-particle Kaluza-Klein state. The fact that the correlator is $\mathcal{O}(1)$ might be interpreted as predicting that the KK state and the giant graviton do not form a bound state which would have appeared as a half-BPS fluctuation around the sphere giant (which has not been found in the literature). This statement should be interpreted with care since we do not fully understand the rules for computing correlators of giants in the spacetime approach.

In the opposite limit, $L_2 = L_1$, the result that the correlator is at least $\mathcal{O}(1)$ or actually growing with N for large L_1 is somewhat puzzling given our earlier argument that there should exist multiple-wrapped sphere giant gravitons. From a supergravity perspective however this probably should not be too suprising as these giants would all be on top of one another and therefore would overlap significantly. It is interesting to note that while the correlator that we are discussing does not have a nice large N limit, it does have a nice small N limit !

Had we normalized the correlator (73) using the overlap of states prescription then our previous arguments would have ruled out the growth in N that we found above. In fact for $L_1 = L_2 = N/2$ we find that the correlator decays as $1/N^{1/4}$ as opposed to the $\mathcal{O}(1)$ behaviour found above. Furthermore for $L_1 = L_2 = N$ we find that the correlator is $1/\sqrt{1+1/N} \approx 1$ as opposed to the $\sqrt{N}$ growth found in the other normalization.

Our last example is the overlap between a sphere giant and a multi-particle Kaluza-Klein state,

$$\frac{\langle (\chi_1(\Phi))^L \chi_L(\Phi^*) \rangle}{\| (\chi_1(\Phi))^L \| \| \chi_L(\Phi) \|} = \sqrt{\frac{1}{N^L} \frac{N!}{L!(N-L)!}}. \tag{74}$$

As before the subscript L denotes the totally antisymmetric representation and the subscript 1 the fundamental. This clearly decays exponentially for large L, which may be used as a prediction for the existence of a semiclassical giant graviton as a distinct object from multi-particle KK state.

7.2 Large Symmetric representations and AdS giants

Let us now consider some examples of correlators of characters in large symmetric representations, which we argued are duals of AdS giants. The first thing to consider is the large N behaviour of the overlap between a large number of single trace operators and a symmetric representation

$$\frac{\langle (tr(\Phi))^L \chi_L(\Phi^*)\rangle}{\| (tr(\Phi))^L \| \| \chi_L(\Phi^* \|} \tag{75}$$

In this section we are using χ_L to denote the character in the representation with one row of length L. The norm $\| (tr(\Phi))^L \|$ is easily seen to be $\sqrt{N^L L!}$. The above normalized correlator is then found to be :

$$\sqrt{\frac{f_L}{N^L L!}} = \frac{(N+L-1)!}{(N-1)!L!N^L} \tag{76}$$

For $N = L$ this is equal to $\frac{(2N-1)!}{N!N^N(N-1)!}$. This shows a rapid decay at large N going like N^{-N}. For $L \gg N$ we continue to get rapid decay. This tells us that the overlap between a large number of KK states and the proposed dual to the AdS giant is very small in the large N limit, so we may expect the semiclassical description of string theory on $AdS \times S$ with gravity coupled to branes to produce a dual to $\chi_L(\Phi)$ other than the naive large N extrapolation of multiparticle KK states. This is the semiclassical giant AdS graviton [11, 15, 13].

As in the case of the sphere giants, the following normalized three-point function is of interest

$$\frac{\langle \chi_{L_1}(\Phi)\chi_{L_2}(\Phi)\chi_{L_3}(\Phi^*)\rangle}{\| \chi_{L_1}(\Phi) \| \| \chi_{L_2}(\Phi) \| \| \chi_{L_3}(\Phi) \|} = \sqrt{\frac{(N+L_3-1)!}{(N-1)!}\frac{(N-1)!}{(N+L_1-1)!}\frac{(N-1)!}{(N+L_2-1)!}} \tag{77}$$

where $L_3 = L_1 + L_2$ and now all characters are in the symmetric representations. This looks roughly like the antisymmetric representations case (69) but differs in an essential way in that the L_i's now enter inside the factorials with plus signs instead of minus signs. This is one sign that there is no bound on the R-charge. This also gives rise to a significant difference in the behaviour of the correlator from (69). Take for example $L_1, L_2 = N/2$, then we find after expanding the factorials

$$\frac{\langle (\chi_{N/2}(\Phi))^2 \chi_N(\Phi^*)\rangle}{\| \chi_{N/2}(\Phi) \|^2 \| \chi_N(\Phi) \|} = (\frac{32}{27})^{N/2}, \tag{78}$$

i.e., exponential growth! In fact it is easy to show that this correlator grows with L_1 and L_2 when these two parameters are large, $\mathcal{O}(N)$ or larger say.

From our earlier general arguments we know that this exponential growth of the correlator in N would be removed in the overlap of states normalization. While the norm $\| \chi_{L_1}(\Phi)\chi_{L_2}(\Phi) \|$ can be evaluated and expressed as a sum analogous to the antisymmetric case of (71).

8 The Complex Matrix Model

Consider the reduced action [15].

$$\frac{R^3\Omega_3}{2g_{YM}^2}\int dtTr(\dot{\Phi}_1{}^2 + \dot{\Phi}_2{}^2 - \frac{1}{R^2}(\Phi_1^2 + \Phi_2^2)) \tag{79}$$

Here Ω_3 is the volume of S_3 at the boundary of AdS_5, where the metric is :

$$ds_{AdS}^2 = \frac{R^2}{cos^2(\rho)}(-dt^2 + d\rho^2 + sin^2(\rho)d\Omega_3) \tag{80}$$

This leads, up to an overall factor, to the Hamiltonian

$$H = \frac{Tr}{2}(P_1^2 + P_2^2 + \Phi_1^2 + \Phi_2^2) \tag{81}$$

with the angular momentum operator given by

$$J = Tr(P_1\Phi_2 - P_2\Phi_1) \tag{82}$$

and canonical commutation relations

$$[(P_1)_{ij}, (\Phi_1)_{kl}] = [(P_2)_{ij}, (\Phi_2)_{kl}] = -i\hbar\delta_{jk}\delta_{il} \tag{83}$$

We introduce complex matrices

$$\begin{aligned} Z &= \tfrac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2) \\ Z^\dagger &= \tfrac{1}{\sqrt{2}}(\Phi_1 - i\Phi_2) \end{aligned} \tag{84}$$

and the conjugates

$$\begin{aligned} \Pi &= \tfrac{1}{\sqrt{2}}(P_1 + iP_2) = -i\tfrac{\partial}{\partial Z^\dagger} \\ \Pi^\dagger &= \tfrac{1}{\sqrt{2}}(P_1 - iP_2) = -i\tfrac{\partial}{\partial Z} \end{aligned} \tag{85}$$

Since these obey relations,

$$\begin{aligned}[Z,\Pi] &= [Z^\dagger,\Pi^\dagger] = 0\\ [Z_{ij},\Pi^\dagger_{kl}] &= [Z^\dagger_{ij},\Pi_{kl}] = i\hbar\delta_{jk}\delta_{il}\end{aligned} \tag{86}$$

we can define creation-annihilation operator pairs

$$\begin{aligned}A &= \tfrac{1}{\sqrt{2}}(Z+i\Pi)\\ A^\dagger &= \tfrac{1}{\sqrt{2}}(Z^\dagger - i\Pi^\dagger)\end{aligned} \tag{87}$$

which obey the standard relation $[A_{ij},A^\dagger_{kl}] = \delta_{jk}\delta_{il}$. An independent set of creation-annihilation operator pairs can be defined,

$$\begin{aligned}B^\dagger &= \tfrac{1}{\sqrt{2}}(Z-i\Pi)\\ B &= \tfrac{1}{\sqrt{2}}(Z^\dagger + i\Pi^\dagger)\end{aligned} \tag{88}$$

which obey $[B,B^\dagger]=1$. It is also easily verified that

$$\begin{aligned}[A,B] &= [A,B^\dagger] = 0\\ [A^\dagger,B] &= [A^\dagger,B^\dagger] = 0\end{aligned} \tag{89}$$

In these variables, the Hamiltonian and angular momentum are

$$\begin{aligned}H &= Tr(A^\dagger A + B^\dagger B)\\ J &= Tr(A^\dagger A - B^\dagger B)\end{aligned} \tag{90}$$

Some eigenstates of the Hamiltonian, with their energies and momenta are listed below

$$\begin{aligned}&Tr((A^\dagger)^n)|0> && E=J=n\\ &Tr((B^\dagger)^m)|0> && E=-J=m\\ &Tr((A^\dagger)^n(B^\dagger)^m)|0> && E=n+m,\ J=n-m\end{aligned} \tag{91}$$

For chiral primaries with $E=|J|$ we have $n=0$ or $m=0$.

It is useful to diagonalize $A, A^\dagger$ by using the unitary symmetry,

$$\begin{aligned}A_{ij} &= \lambda_i\delta_{ij}\\ A^\dagger_{ij} &= \lambda^\dagger_i\delta_{ij}\end{aligned} \tag{92}$$

The measure, in terms of these variables, shows that we can treat the λ_i's as fermionic variables. The Hamiltonian for these fermionic oscillators is

$$H = \sum_i \lambda^\dagger_i\lambda_i \tag{93}$$

The fermionic wavefunctions are

$$\psi_F(\lambda_1, \lambda_2, \cdots \lambda_n) = e^{-\sum_i \bar{\lambda}_i \lambda_i} Det \begin{pmatrix} \lambda_1^{n_1} & \lambda_1^{n_2} & \cdots & \lambda_1^{n_N} \\ \lambda_2^{n_1} & \lambda_2^{n_2} & \cdots & \lambda_2^{n_N} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_N^{n_1} & \lambda_N^{n_2} & \cdots & \lambda_N^{n_N} \end{pmatrix} \tag{94}$$

For these states the energy and angular momentum are $E = J = \sum_i n_i$. The ground state corresponds to $n_1 = N-1, n_2 = N-2, \cdots n_N = 0$. The ground state wavefunction is a Van der Monde determinant

$$\Psi_0 = e^{-\sum_i \bar{\lambda}_i \lambda_i} \prod_{i<j} (\lambda_i - \lambda_j) \tag{95}$$

General excited states are described by Young diagrams with row lengths $\vec{r} = (r_1, r_2 \cdots r_N)$. The energy and angular momentum are

$$E = J = \sum_i r_i + i - 1 \tag{96}$$

The wavefunctions normalized by the Van der Monde can be recognized as Weyl's character formula and hence are the Schur Polynomials.

8.1 Hints on integrability

From (96) it is clear that all the states associated with Young Diagrams having n boxes have the same energy. As such there is a large degeneracy equal to the number of partitions of n. In (35) we found that the exact two-point functions at finite N are orthogonalized by the Schur Polynomials. This suggests that there are higher Hamiltonians which commute with (81) and which are diagonalized by the Schur polynomials and which have different eigenvalues for different Young diagrams. These higher Hamiltonians can be constructed in terms of $tr((AA^\dagger)^n)$ and $tr((BB^\dagger)^n)$. The 4 dimensional origin of these is a very intereating question. Presumably they will involve modifications of $N = 4$ Yang Mills with higher derivatives. These may be expected to be unrenormalizable, but perhaps they have a higher dimensional origin, generalizing the connection between $N = 4$ SYM and $(0, 2)$ six-dimensional superconformal theory or little string theory. [42, 43, 44, 45]

9 Summary and Outlook

We described a one-one mapping between the space of half-BPS representations to the space of $U(N)$ Young Diagrams, with holomorphic functions of one complex matrix playing an important role in the mapping. Using this map we identified a basis of composites which diagonalizes the two-point functions at finite N. The basis suggests natural candidates for giant gravitons, more precisely for sphere giants, AdS giants and composites of giants with Kaluza-Klein states.

We computed the normalized two and three point correlators of these observables, as well as a special class of higher point functions. The Frobenius-Schur duality between symmetric and Unitary groups played a useful role. It allowed us to use various results from $U(N)$ group theory to give exact answers for correlators, which could be developed in a large N expansion. Many of these correlators have simple interpretations in terms of giant sphere and AdS gravitons, multiply wound giants and composites of giants with KK states. However detailed interpretations will have to await a clear formulation of rules for computing interactions of giants from string theory in $AdS \times S$ or its semiclassical limit.

The holomorphic observables we considered were shown to have a distinguished role in a reduced $0+1$ Matrix Model related to $N=4$ SYM on a three sphere. The discussion of the Matrix Model also suggested hints of integrable structure in the $N=4$ theory.

Several generalizations of this work can be contemplated. A lot of information about correlators of descendants at finite N should be extracted. Most of our formulae involved dimensions and fusion coefficients of Unitary groups. When the descendants are involved there should be a nice generalization involving group theoretic quantities associated with $U(N)$ as well as the superconformal symmetry group. A generalization to maximally supersymmetric gauge theory with $O(N)$ and $Sp(N)$ gauge groups should also be possible.

Quarter-BPS operators, considered in detail recently [46, 47] are also of interest in the context of giant gravitons [48]. Many of the symmetric group techniques used here should be useful, but the final answers are unlikely to have simple relations to $U(N)$, since the observables of [46, 47] do not appear to be related to holomorphic gauge invariant functions of a complex Matrix. Whether another less obvious group or algebra plays an analogous role is a fascinating question.

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