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Discrete Symmetries of the Superpotential and Calculation of Disk Invariants

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Abstract

The integrality of Ooguri-Vafa disk invariants is verified using discrete symmetries of the superpotential of the mirror Landau-Ginzburg theory to calculate quantum corrections to the boundary variables. We show that these quantum corrections are completely determined if we assume that the discrete symmetry of the superpotential also holds in terms of the quantum corrected variables. We discuss the case of local $\mathbb{P}^2$ blown up at three points and local $\mathbb{F}_2$ blown up at two points in detail.

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Contents

1 Introduction

The calculation of topological string amplitudes with boundaries has received a lot of attention recently[1, 2, 3, 4]. Besides being of mathematical interest, these amplitudes capture exactly certain terms in the effective $\mathcal{N} = 1$ four dimensional theory [5]. The calculation of disk amplitudes has been carried out both using mirror symmetry [1, 2] and directly using localization techniques similar to the ones used for calculating closed string Gromov-Witten invariants [6, 8, 7]. These calculations have verified the integrality of open string invariants defined in [9]. Also, large N duality with Chern-Simons theory [10] has lead to the verification, in certain cases, of the general structure of amplitudes as predicted in [11, 9], for all genera [12]. More recently it has been proposed, and verified in certain cases, that disk amplitudes can be obtained from genus zero closed topological string amplitudes of a Calabi-Yau fourfold [3].

The most extensive check of the integrality of open string invariants was carried out in [2]. Fig. 1(a) shows the toric data of the geometries discussed in that paper. As in the closed string case, it is necessary to express the disk amplitude in the "flat coordinates" of the boundary theory in order to obtain integer valued invariants. The flat coordinates of the boundary theory are related to the classical area of the disk through closed string quantum corrections. It was shown in [2] that these quantum corrections to the classical area of the disk can be expressed as integrals over certain cycles on the Riemann surface which parameterizes the position of the mirror D-brane.

In this paper, we show that quantum corrections to the boundary variables can be calculated using discrete symmetries of the superpotential of the Landau-Ginzburg theory which is mirror to the Calabi-Yau threefold. Fig. 1(c) gives the toric data of the geometries we will discuss in this paper, local $\mathbb{P}^2$ blown up at three points and local $\mathbb{F}_2$ blown up at two points (both with four Kähler parameters). Taking suitable limits of the Kähler parameters, we can blow down exceptional curves and therefore obtain results for the blown down geometries depicted in a) and b) with no further effort. We will show that quantum corrections obtained using the discrete symmetries lead to integer invariants.

2 Mirror manifolds, quantum corrections, and discrete symmetries

2.1 Mirror manifolds from LG Superpotential

In this section, we will follow [14, 2] to derive the equations for the Calabi-Yau threefolds mirror to local $\mathbb{P}^2$ blown up at three points and local $\mathbb{F}_2$ blown up at two points. The toric diagram for these is shown in Fig. 2.

Local $\mathcal{B}_3$: Consider the non-compact CY which is the total space of the anticanonical bundle over $\mathcal{B}_3$. The linear sigma model charges for this

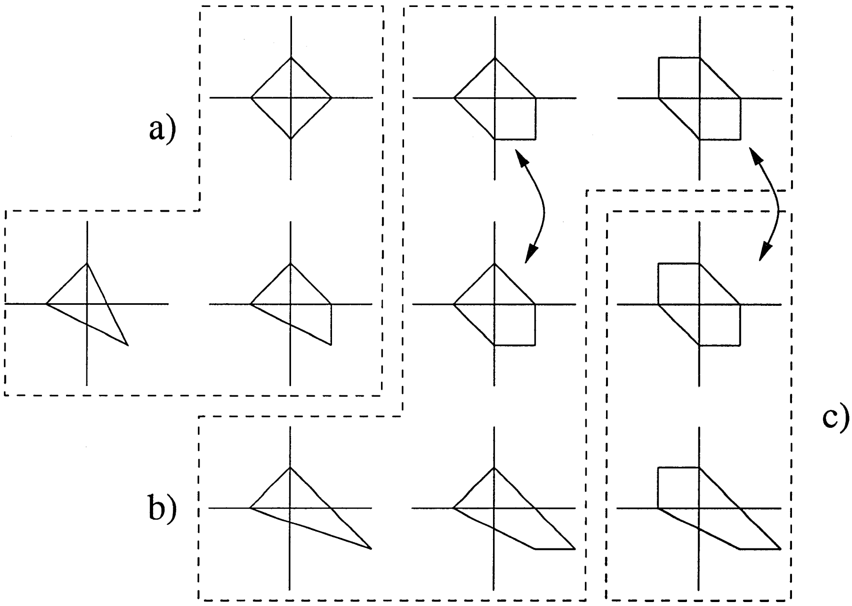

Figure 1: a) Toric data for $\mathbb{F}_0$ ($\mathbb{P}_1 \times \mathbb{P}_1$), $\mathbb{P}^2$, and $\mathbb{F}_1$ ($\mathbb{P}^2$ blown up at one point), b) and c) blowups of $\mathbb{F}_0$ and of $\mathbb{F}_1$, $\mathbb{F}_2$ and its blowups.

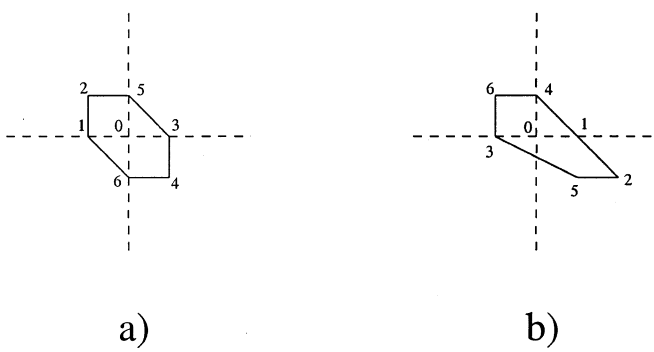

Figure 2: a) $\mathbb{P}^2$ blown up at three points, b) $\mathbb{F}_2$ blown up at two points. We will denote these surface by $\mathcal{B}_3$ and $\mathcal{F}_2$ respectively. The numbers denote the order in which the vertices appear in the linear sigma model charge vectors.

Calabi-Yau space are given by [15]

$$\begin{aligned} l^{(1)} &= (-2,1,0,1,0,0,0),\ \ l^{(2)}=(-2,0,1,0,1,0,0)\,, \\ l^{(3)} &= (-1,1,-1,0,0,1,0),\ \ l^{(4)}=(-1,-1,1,0,0,0,1)\,. \end{aligned} \tag{1}$$

The superpotential of the mirror Landau-Ginzburg theory is given by [13]

$$W = \sum_{i=0}^{6} e^{-Y_i}\,. \tag{2}$$

The Y_i are related to the chiral fields Φ_i of the linear sigma model via $\mathrm{Re}(Y_i) = |\Phi_i|^2$. The D-term constraints on the Φ_i at low energies, as encoded in the charges (1), translate into relations among the mirror variables Y_i. For $\mathcal{B}_3$, expressing the superpotential in terms of the independent variables Y_0, Y_1 and Y_2 yields

$$W(Y_0,Y_1,Y_2)=x_0+x_1+x_2+e^{-t_1}\frac{x_0^2}{x_1}+e^{-t_2}\frac{x_0^2}{x_2}+e^{-t_3}\frac{x_0x_2}{x_1}+e^{-t_4}\frac{x_0x_1}{x_2}, \tag{3}$$

where $x_i = e^{-Y_i}$ and the t_i are the Kähler parameters of the original Calabi-Yau. The periods of the Landau-Ginzburg theory are given by [16, 13, 14]

$$\Pi = \int e^{-W}\prod_i \frac{dx_i}{x_i}\,, \tag{4}$$

where the measure of integration reflects the fact that the Y_i are the fundamental fields. Because the superpotential is homogeneous of degree one, we can rewrite the above integral in the following way,

$$\begin{aligned} \Pi &= \int e^{-x_0\{W(0,Y_1-Y_0,Y_2-Y_0)-zw\}}dx_0\,dz\,dw\,\frac{dx_1dx_2}{x_1x_2} \\ &= \int \delta(W(0,Y_1-Y_0,Y_2-Y_0)-zw)dz\,dw\frac{d\tilde{x}_1d\tilde{x}_2}{\tilde{x}_1\tilde{x}_2}\,,\ \ \tilde{x}_i=e^{-\tilde{Y}_i}=e^{-Y_i+Y_0}\,. \end{aligned} \tag{5}$$

Thus, the periods of the mirror LG theory are equivalent to the integral of the holomorphic 3-form over a 3-cycle in a Calabi-Yau 3-fold [14],

$$\Pi = \int \Omega\,,\quad \Omega = dzdw\frac{d\tilde{x}_1d\tilde{x}_2}{\tilde{x}_1\tilde{x}_2}/df\,, \tag{6}$$

where $f = W(0,\tilde{Y}_1,\tilde{Y}_2) - zw$ and $f = 0$ defines the Calabi-Yau. The Calabi-Yau mirror to local $\mathcal{B}_3$ is hence given by

$$P_{\mathcal{B}_3}(u,v) = 1+e^u+e^v+e^{-t_1-u}+e^{-t_2-v}+e^{-t_3-u+v}+e^{-t_4+u-v}=zw\,, \tag{7}$$

where we have substituted u, v for $-\tilde{Y}_1, -\tilde{Y}_2$ respectively.

Local $\mathcal{F}_2$: Now consider the Calabi-Yau which is the total space of the anticanonical bundle over $\mathbb{F}_2$ blown up at two points. In this case, the linear sigma model charges are

$$\begin{aligned} l^{(1)} &= (-2,1,0,1,0,0,0)\,,\ l^{(2)} = (0,-2,1,0,1,0,0)\,, \\ l^{(3)} &= (-1,1,-1,0,0,1,0)\,,\ l^{(4)} = (-1,-1,1,0,0,0,1)\,. \end{aligned} \tag{8}$$

From Eq(6), it follows that the mirror Calabi-Yau in this case is given by

$$P_{\mathcal{F}_2}(u,v) = 1+e^u+e^v+e^{-t_1-u}+e^{-t_2+2u-v}+e^{-t_3-u+v}+e^{-t_4+u-v} = zw, \tag{9}$$

and the holomorphic 3-form by

$$\Omega = dudv\frac{dz}{z}\,. \tag{10}$$

2.2 Open string topological amplitudes

Open string topological amplitudes calculate certain superpotential terms in the effective $\mathcal{N}=1$ theory on the world-volume of a D6-brane wrapped on a Lagrangian 3-cycle of a Calabi-Yau threefold. These terms in the effective theory are of the form [5, 2]

$$h\int d^2\theta(\mathrm{Tr}W^2)^{h-1}(\mathcal{W}^2)^g\,, \tag{11}$$

where g is the genus, h is the number of boundaries, W is the gaugino superfield and $\mathcal{W}$ is the $\mathcal{N}=2$ graviphoton multiplet.

The open string amplitudes get contributions from the BPS domain walls in four dimensions, which correspond to D4-branes ending on the D6-branes. These D4-branes are classified by the relative homology classes in the CY3-fold with boundary on the Lagrangian cycle on which the D6-brane is wrapped. It was shown in [1, 2] that the Lagrangian cycle in the A-model geometry maps to a holomorphic 2-cycle Σ in the B-model geometry of the mirror Calabi-Yau. Hence, under mirror symmetry, the D6-brane on a non-compact 3-cycle becomes a D5-brane wrapped on a non-compact 2-cycle. This mirror D-brane is parametrized by z and its position is given by

$$\begin{aligned} w &= 0\,, \\ P(u,v) &= 0\,. \end{aligned} \tag{12}$$

Thus, the holomorphic 2-cycle Σ, given by $P(u,v)=0$, parameterizes the position of the mirror D-brane. The superpotential in this case is given by

[1, 2]

$$W_{\mathcal{N}=1} = \int_{u_0}^{u} v(u)du\,, \tag{13}$$

where $v(u)$ is obtained by solving $P(u,v)=0$ and u_0 is a fixed point on Σ.

In order to obtain invariants associated with disks ending on the Lagrangian cycle of the D6-brane, the above expression for the superpotential has to be compared with the general form of the genus zero amplitude predicted in [9],

$$W_{\mathcal{N}=1} = \sum_{\Sigma \in H_2(X,\mathbf{Z})} \sum_{m \in \mathbf{Z}} \sum_{n=1}^{\infty} \frac{N_{\Sigma,m}}{n^2} q^{n\Sigma} e^{nm\widehat{u}}\,. \tag{14}$$

Here, $q^{\Sigma} = e^{\int_{\Sigma}\omega}$ and $N_{\Sigma,m}$ is the number of D4-branes which wrap the curve Σ and end on the D6-brane by winding m times around the non-trivial 1-cycle of the Lagrangian 3-cycle on which the D6-brane is wrapped. In the above expression, ω is the quantum corrected Kähler form for the Calabi-Yau and $\widehat{u}$ is the quantum corrected area of the disk.

2.3 PF equations, mirror map, and relations between quantum corrections

In this section, we show, based on a recent paper [3] by Mayr, how relations between the ratios $\log \frac{z_i}{q_i}$, where $\log(z_i)$ are the classical and $\log(q_i)$ the quantum corrected Kähler parameters, can be determined without solving the Picard-Fuchs equations.

Consider a Calabi-Yau manifold with linear sigma model charges $l_i^{(\alpha)}$. The Picard-Fuchs differential operators are then given by

$$\mathcal{D}_{\alpha} = \prod_{l_i^{(\alpha)}>0} (\frac{\partial}{\partial a_i})^{l_i^{(\alpha)}} - \prod_{l_i^{(\alpha)}<0} (\frac{\partial}{\partial a_i})^{-l_i^{(\alpha)}}\,, \tag{15}$$

where a_i are complex structure parameters, but not all of them are independent. Only the following combinations lead to independent complex structure parameters of the mirror Calabi-Yau,

$$z_{\alpha} = \prod_i a_i^{l_i^{(\alpha)}}\,. \tag{16}$$

Here the number of z_α is equal to the number of Kähler parameters of the original Calabi-Yau. In terms of the z_α the above differential operators are given by [3]

$$\mathcal{D}_\alpha = \prod_{l_i^{(\alpha)}>0} \prod_{j=0}^{l_i^{(\alpha)}-1} (\sum_\beta l_i^{(\beta)}\theta_\beta - j) - z_\alpha \prod_{l_i^{(\alpha)}<0} \prod_{j=0}^{-l_i^{(\alpha)}-1} (\sum_\beta l_i^{(\beta)}\theta_\beta - j)\,, \tag{17}$$

where $\theta_\beta = z_\beta \frac{\partial}{\partial z_\beta}$. Using the quantum corrected Kähler parameter $-\log(q)$ (the solution of the PF equations) we define

$$R_\beta = \log(z_\beta/q_\beta)\,. \tag{18}$$

The R_β satisfy the following differential equation [3],

$$\mathcal{D}_\alpha R_\beta = z_\alpha A^\alpha_\beta\,, \tag{19}$$

where the A^α_β are defined by the linear part of the differential operator $\mathcal{D}_\alpha$ [3], $\mathcal{D}^{lin}_\alpha = z_\alpha \sum A^\alpha_\beta \theta_\beta$. We have here used the fact that if there is more than one positive number in the vector $l^{(\alpha)}$, as is the case in all the examples we will be considering, then the first term in Eq(17) does not contribute to $\mathcal{D}^{lin}_\alpha$. If in addition, there is more than one negative number, then A^α_β vanishes. If the vector $l^{(\alpha)}$ has only one negative number, say $l_s^{(\alpha)}$, then

$$A^\alpha_\beta = (\prod_{j=1}^{-l_s^{(\alpha)}-1} j)\, l_s^{(\beta)}\,. \tag{20}$$

Thus, we see that if $\sum_\beta a_\beta A^\alpha_\beta = 0$ for all α,

$$\mathcal{D}_\alpha(\sum_\beta a_\beta R_\beta) = 0 \;\;, \forall\, \alpha\,. \tag{21}$$

Since R_β is a power series in z_α, we get the following relation,

$$\sum_\beta a_\beta R_\beta = 0\,. \tag{22}$$

We use this relation to verify that the quantum corrections to the boundary variables obtained using discrete symmetries of the superpotential do not lead to inconsistent results.

Let's apply this method to our two examples. The charges for local $\mathcal{B}_3$ are given in Eq(1). From Eq(20), it then follows that

$$A^1_\beta = A^2_\beta = l_1^{(\beta)} = (-2,-2,-1,-1)\,,\; A^3_\beta = A^4_\beta = 0\,. \tag{23}$$

Therefore, the solution to the equation $\sum_\beta a_\beta A^\alpha_\beta = 0$ is given by $a_\beta = (a_1, a_2, a_3, -2a_1 - 2a_2 - a_3)$ yielding the following relations,

$$z_1 = q_1(\frac{z_4}{q_4})^2\,,\ z_2 = q_2(\frac{z_4}{q_4})^2\,,\ z_3 = q_3(\frac{z_4}{q_4})\,. \tag{24}$$

For local $\mathcal{F}_2$, we get, using the charges (9),

$$\begin{aligned} A^1_\beta &= (-2,0,-1,-1)\,, \\ A^2_\beta &= (1,-2,1,-1)\,, \\ A^3_\beta &= A^4_\beta = 0\,. \end{aligned} \tag{25}$$

We now solve $\sum_\beta a_\beta A^\alpha_\beta = 0$ for all α by $a_\beta = (a_1, -\frac{a_1+2a_4}{2}, -2a_1 - a_4, a_4)$, yielding the two relations

$$z_1 = q_1\sqrt{\frac{z_2}{q_2}}(\frac{z_3}{q_3})^2\,,\ z_4 = q_4\frac{z_2}{q_2}\frac{z_3}{q_3}\,. \tag{26}$$

We will obtain these relations again using the discrete symmetries of the superpotential, and ultimately be able to verify them by considering the explicit logarithmic solutions to the PF equations.

2.4 Discrete symmetries of the LG superpotential and quantum corrected variables

Consider the LG theory which is the mirror of the Calabi-Yau twofold $\mathcal{O}(-2) \mapsto \mathbb{P}^1$. The superpotential is given by

$$W(\Sigma, Y_0, Y_1, Y_2) = \Sigma(Y_1 + Y_2 - 2Y_0 - t) + e^{-Y_0} + e^{-Y_1} + e^{-Y_2}\,. \tag{27}$$

We see that there is a $\mathbb{Z}_2$ symmetry which interchanges Y_1 and Y_2. After integrating out Σ, we get

$$W = e^{-Y_0} + e^{-Y_1} + e^{Y_1 - 2Y_0 - t}\,. \tag{28}$$

Now, the $\mathbb{Z}_2$ symmetry corresponds to

$$\begin{aligned} e^{-Y_1} &\mapsto e^{Y_1 - 2Y_0 - t}\,, \\ x_1 &\mapsto \frac{x_0^2 e^{-t}}{x_1}\,,\quad x_i = e^{-Y_i}\,. \end{aligned} \tag{29}$$

Although this model does not correspond to a Calabi-Yau threefold, we can obtain a similar superpotential from a degenerate limit of the superpotential mirror to the local $\mathbb{P}^1 \times \mathbb{P}^1$. Here, we are just using this superpotential as an example to illustrate the method we will use later.

Let us define quantum corrected variables

$$\widehat{x}_i = S_i(q)x_i\,, \quad q = e^{-T}\,, \tag{30}$$

where T is the quantum corrected area of the $\mathbb{P}^1$. The superpotential in terms of these new variables is given by

$$W = \frac{\widehat{x}_0}{S_0(q)} + \frac{\widehat{x}_1}{S_1(q)} + e^{-t}\frac{S_1}{S_0^2(q)}\frac{\widehat{x}_0^2}{\widehat{x}_1}\,. \tag{31}$$

If we now assume that the $\mathbb{Z}_2$ symmetry of the superpotential still exists in terms of the new variables, i.e.

$$\widehat{x}_1 \mapsto e^{-T}\frac{\widehat{x}_0^2}{\widehat{x}_1}\,, \tag{32}$$

we get the following relation between $S_0(q)$ and $S_1(q)$,

$$\frac{S_1(q)}{S_0(q)} = \sqrt{\frac{q}{z}}\,, \quad z = e^{-t}\,. \tag{33}$$

We will use this basic $x \mapsto 1/x$ symmetry for the Calabi-Yau threefold cases to compute quantum corrections.

2.4.1 Examples

In this subsection, we derive the quantum corrections to the boundary variables for certain cases discussed in [2] using the $\mathbb{Z}_2$ symmetry of the superpotential.

$\mathcal{O}(-3)$ over $\mathbb{P}^2$: The superpotential of the mirror theory is given by [13]

$$W = x_0 + x_1 + x_2 + e^{-t}\frac{x_0^3}{x_1 x_2}\,. \tag{34}$$

Let the quantum corrected variables be

$$\widehat{x}_i = \frac{x_i}{S_i(q)}\,. \tag{35}$$

Since x_1 and x_2 can be interchanged without affecting the superpotential, $S_1(q) = S_2(q)$. Also, we see that the following transformation leaves the superpotential invariant,

$$x_1 \mapsto e^{-t}\frac{x_0^3}{x_1 x_2}\,. \tag{36}$$

Assuming this symmetry holds in terms of the quantum corrected variables, i.e.

$$\widehat{x}_1 \mapsto e^{-T}\frac{\widehat{x}_0^3}{\widehat{x}_1\widehat{x}_2}\,, \tag{37}$$

implies that

$$\frac{S_1(q)}{S_0(q)} = (\frac{q}{z})^{\frac{1}{3}}\,. \tag{38}$$

In the phase in which $x_0 = 1$ and therefore $S_0(q) = 1$, the above equation gives the same result as in [2].

local $\mathbb{P}^1 \times \mathbb{P}^1$: In this case, the superpotential of the mirror LG theory is given by [13, 14]

$$W = x_0 + x_1 + x_2 + e^{-t_1}\frac{x_0^2}{x_1} + e^{-t_2}\frac{x_0^2}{x_2}\,. \tag{39}$$

The transformations

$$x_1 \mapsto e^{-t_1}\frac{x_0^2}{x_1}\,,\;\; x_2 \mapsto e^{-t_2}\frac{x_0^2}{x_2} \tag{40}$$

implemented in terms of the quantum corrected variables $\widehat{x}_i = S_i(q_1,q_2)x_i$ read

$$\widehat{x}_1 \mapsto e^{-T_1}\frac{\widehat{x}_0^2}{\widehat{x}_1}\,,\;\; \widehat{x}_2 \mapsto e^{-T_2}\frac{\widehat{x}_0^2}{\widehat{x}_2}\,. \tag{41}$$

Invariance under this transformation implies

$$S_1(q_1,q_2) = \sqrt{\frac{q_1}{z_1}}\,,\;\; S_2(q_1,q_2) = \sqrt{\frac{q_2}{z_2}}\,. \tag{42}$$

local $\mathbb{P}^2$ blown up at one point:

The superpotential of the mirror LG theory is given by [13, 2]

$$W = x_0 + x_1 + x_2 + e^{-t_b-t_f}\frac{x_0^3}{x_1x_2} + e^{-t_f}\frac{x_0^2}{x_2}\,. \tag{43}$$

The superpotential is invariant under the transformation

$$x_2 \mapsto \frac{1}{x_2}(z_bz_f\frac{x_0^3}{x_1} + z_fx_0^2)\,. \tag{44}$$

In terms of quantum corrected variables $\widehat{x}_i = S_i(q_b, q_f)x_i$ we require that the superpotential be invariant under the transformation,

$$\widehat{x}_2 \mapsto \frac{1}{\widehat{x}_2}(q_b q_f \frac{\widehat{x}_0^3}{\widehat{x}_1} + q_f \widehat{x}_0^2) \,. \tag{45}$$

This requirement uniquely fixes S_i to be

$$\frac{S_1(q_b, q_f)}{S_0(q_b, q_f)} = \frac{q_b}{z_b} \,, \; \frac{S_2(q_b, q_f)}{S_0(q_b, q_f)} = \sqrt{\frac{q_f}{z_f}} \,. \tag{46}$$

In the phase $x_0 = 1$, $S_0 = 1$, and the above relations determine S_1 and S_2. Using the relation $\frac{q_f}{z_f} = (\frac{q_b}{z_b})^2$, derived eg. from Eq(22), we see that $S_1 = S_2$.

3 Integrality of Ooguri-Vafa open string invariants

3.1 Local $\mathcal{F}_2$

The linear sigma model charges for this case are [15]

$$\begin{aligned} l^{(1)} &= (-2,1,0,1,0,0,0)\,, \; l^{(2)} = (0,-2,1,0,1,0,0)\,, \\ l^{(3)} &= (-1,1,-1,0,0,1,0)\,, \; l^{(4)} = (-1,-1,1,0,0,0,1)\,. \end{aligned} \tag{47}$$

The mirror to this theory is a Landau-Ginzburg model with superpotential [13]

$$W = x_0 + x_1 + x_2 + z_1\frac{x_0^2}{x_1} + z_2\frac{x_1^2}{x_2} + z_3\frac{x_0 x_2}{x_1} + z_4\frac{x_0 x_1}{x_2} \,, \;\; z_i = e^{-t_i} \,. \tag{48}$$

The above superpotential is invariant under the transformation

$$x_2 \mapsto \frac{1}{x_2}(\frac{z_2 x_1^2 + z_4 x_0 x_1}{1 + z_3 \frac{x_0}{x_1}}) \,. \tag{49}$$

In terms of the quantum corrected variables $\widehat{x}_i = S_i x_i$, we require that the superpotential be invariant under the transformation,

$$\widehat{x}_2 \mapsto \frac{1}{\widehat{x}_2}(\frac{q_2 \widehat{x}_1^2 + q_4 \widehat{x}_0 \widehat{x}_1}{1 + q_3 \frac{\widehat{x}_0}{\widehat{x}_1}}) \,. \tag{50}$$

This implies that

$$\frac{S_1}{S_0} = \frac{q_3}{z_3} \,, \; \frac{S_2}{S_0} = \sqrt{\frac{q_4 q_3}{z_4 z_3}} \,. \tag{51}$$

It also yields the following relation,

$$\frac{q_4}{z_4} = \frac{q_2}{z_2}\frac{q_3}{z_3}\,, \tag{52}$$

consistent with the result (26) derived above.

We consider the phase in which the $x_0 = 1$ and set $\widehat{x}_1 = -e^{\widehat{u}}, \widehat{x}_2 = -e^{\widehat{v}}$ [2]. The equation parameterizing the position of the brane in the mirror geometry that we must solve is

$$1 - \frac{e^{\widehat{u}}}{S_1} - \frac{e^{\widehat{v}}}{S_2} - S_1 e^{-t_1-\widehat{u}} - \frac{S_2}{S_1^2} e^{-t_2+2\widehat{u}-\widehat{v}} + \frac{S_1}{S_2} e^{-t_3+\widehat{v}-\widehat{u}} + \frac{S_2}{S_1} e^{-t_4+\widehat{u}-\widehat{v}} = 0\,. \tag{53}$$

The logarithmic solutions to the PF equations have the general form[1]

$$\Pi_i = \partial_{\rho_i}\Pi_0\big|_{\vec{\rho}=0}\,, \tag{54}$$

where

$$\begin{aligned} \Pi_0(\vec{z}) &= \sum_{\vec{n}} c(\vec{n},\vec{\rho})\vec{z}^{\,\vec{n}+\vec{\rho}}\big|_{\vec{\rho}=0}\,, \\ c(\vec{n},\vec{\rho}) &= \frac{1}{\prod_i \Gamma(\sum_\alpha l_i^\alpha(n_\alpha+\rho_\alpha)+1)}\,. \end{aligned} \tag{55}$$

For the case we are considering, these solutions can be expressed in terms of

$$\begin{aligned} A &= \sum_{m,n,p,r} \frac{(-1)^{p+r}\Gamma(2m+p+r)}{\Gamma(m-2n+p-r+1)\Gamma(n-p+r+1)m!n!p!r!} z_1^m z_2^n z_3^p z_4^r \\ B &= \sum_n \frac{\Gamma(2n)}{n!^2} z_2^n \end{aligned} \tag{56}$$

as

$$\begin{aligned} -T_1 = \log(q_1) &= \log(z_1) + 2A - B\,, \\ -T_2 = \log(q_2) &= \log(z_2) + 2B\,, \\ -T_3 = \log(q_3) &= \log(z_3) + A - B\,, \\ -T_4 = \log(q_4) &= \log(z_4) + A + B\,. \end{aligned} \tag{57}$$

[1]In applying these formulae, the identity $\frac{d}{dx}\frac{1}{\Gamma(x)}\Big|_{x=1-n} = (-1)^{n+1}\Gamma(n)$ for n a positive integer proves useful.

To express the solution to Eq(53) purely in terms of B-model variables, we need to invert the mirror map,

$$\begin{aligned} z_1 &= q_1(1 - 2q_1 + q_2 + 3q_1^2 - 4q_1q_2 + 4q_1q_4 - 2q_3q_4 + \dots)\,, \\ z_2 &= \frac{q_2}{(1+q_2)^2}\,; \end{aligned} \tag{58}$$

z_3 and z_4 can be obtained from the above and the relations among the parameters.

Of the two solutions for $e^{\widehat{v}}$, we choose the one that reduces to $1 - e^{\widehat{u}}$ in the large radius limit $z_\alpha \to 0$, since this is the relation we know to hold classically,

$$\widehat{v} = \log\left[\frac{S_2e^{\widehat{u}} - \frac{S_2}{S_1}e^{2\widehat{u}} - S_1S_2z_1 + \sqrt{(S_2x - \frac{S_2}{S_1}e^{2\widehat{u}} - S_1S_2z_1)^2 - 4(\frac{e^{\widehat{u}}}{S_1} - z_3)(S_2^2z_2e^{3\widehat{u}}z_2 + S_2^2z_4e^{2\widehat{u}})}}{2(e^{\widehat{u}} - S_1z_3)}\right].$$

Having expressed $\widehat{v}$ completely in terms of flat coordinates of the B-model variables, we are now ready to calculate the instanton numbers $N_{\vec{k},m}$ using the relation [1, 2]

$$\widehat{v} = \sum_{\vec{k}\in H_2(X,\mathbb{Z})}\sum_{m\in\mathbb{Z}}\sum_{n=1}^{\infty}\frac{m}{n}N_{\vec{k},m}q_1^{nk_1}q_2^{nk_2}q_3^{nk_3}q_4^{nk_4}e^{nm\widehat{u}}\,. \tag{59}$$

Below, we tabulate the functions $I_{\vec{k}}(\widehat{x})$, which we define as

$$I_{\vec{k}}(\widehat{x}) = \sum_{m\neq 0} N_{\vec{k},m}\,\widehat{x}^m\,. \tag{60}$$

The $N_{\vec{k},m}$ can easily be determined from these via Taylor expansion, and pass the integrality test. We give the $I_{\vec{k}}(\widehat{x})$ for $\sum_{i=1}^4 k_i = 1, 2$ below and for $\sum_{i=1}^4 k_i = 3, 4, 5$ in the appendix.

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(1000)	$\frac{1}{x}$	(0002)	$-\frac{x^2}{(1+x)(1-x)^3}$
(0100)	$-\frac{x}{1-x}$	(1100)	$-\frac{2x}{1-x}$
(0010)	$-\frac{1}{x}$	(1010)	0
(0001)	$\frac{x}{1-x}$	(1001)	$\frac{2x}{1-x}$
(2000)	0	(0110)	$\frac{x}{1-x}$
(0200)	$-\frac{x^3}{(1-x)^3(1+x)}$	(0101)	$\frac{x^2}{(1-x)^3}$
(0020)	0	(0011)	$-\frac{x}{1-x}$

3.2 Local $\mathcal{B}_3$

Here we consider the non-compact Calabi-Yau which is the total space of the anticanonical bundle on toric del Pezzo $\mathcal{B}_3$. The linear sigma model with moduli space this Calabi-Yau has the charge vectors

$$\begin{aligned} l^{(1)} &= (-2,1,0,1,0,0,0)\,,\ l^{(2)} = (-2,0,1,0,1,0,0)\,, \\ l^{(3)} &= (-1,1,-1,0,0,1,0)\,,\ l^{(4)} = (-1,-1,1,0,0,0,1)\,. \end{aligned} \tag{61}$$

The superpotential of the mirror theory is given by

$$W = x_0 + x_1 + x_2 + e^{-t_1}\frac{x_0^2}{x_1} + e^{-t_2}\frac{x_0^2}{x_2} + e^{-t_3}\frac{x_0x_2}{x_1} + e^{-t_4}\frac{x_0x_1}{x_2}\,. \tag{62}$$

The above superpotential is invariant under the transformation

$$x_1 \mapsto \frac{1}{x_1}\Big(\frac{z_1x_0^2 + z_3x_0x_2}{1+z_3\frac{x_0}{x_2}}\Big)\,. \tag{63}$$

In terms of the quantum corrected variables $\widehat{x}_i = S_i(q)x_i$ we require

$$\widehat{x}_i \mapsto \frac{1}{\widehat{x}_2}\Big(\frac{q_1\widehat{x}_0^2 + q_3\widehat{x}_0\widehat{x}_2}{1+q_4\frac{\widehat{x}_0}{\widehat{x}_2}}\Big)\,. \tag{64}$$

This implies that

$$\frac{S_1}{S_0} = \sqrt{\frac{q_1}{z_1}}\,,\ \frac{S_2}{S_0} = \frac{q_4}{z_4}\,, \tag{65}$$

and we also get a relation

$$\frac{q_1}{z_1} = \frac{q_4}{z_4}\frac{q_3}{z_3}\,. \tag{66}$$

This relation agrees with those determined in section 2.3. Using these relations, we only need to determine z_4 in terms of the q_α.

The general formula (55) allows us to express the logarithmic solutions to the PF equations as

$$\begin{aligned}\log(q_1) &= \log(z_1) + 2A\,, \\ \log(q_2) &= \log(z_2) + 2A\,, \\ \log(q_3) &= \log(z_3) + A\,, \\ \log(q_4) &= \log(z_4) + A\,,\end{aligned} \tag{67}$$

where A is given by

$$A = \sum_{m,n,p,r} \frac{(-1)^{p+r}\Gamma(2m+2n+p+r)}{\Gamma(m+p-r+1)\Gamma(n-p+r-1)m!n!p!r!}\,. \tag{68}$$

By inverting the solution $\log(z_4) + A$ of the PF equation, we get

$$z_4 = q_4(1-q_1+q_1^2-q_1^3+q_1^4-q_2-q_1q_2-2q_1^2q_2-2q_1^3q_2-3q_1^4q_2+q_2^2+\cdots). \tag{69}$$

In the phase in which $x_0 = 1$, the Riemann surface parameterizing the position of the mirror brane is given by

$$S_1(q)-e^{\widehat{u}}-e^{\widehat{v}}-q_1e^{-\widehat{u}}-q_2e^{-\widehat{v}}+q_3e^{\widehat{v}-\widehat{u}}+q_4e^{\widehat{u}-\widehat{v}}=0,\ \widehat{x}_1=-e^{\widehat{u}},\widehat{x}_2=-e^{\widehat{v}}. \tag{70}$$

Solving for $\widehat{v}$ yields

$$\widehat{v} = \log_{\frac{S_1(q)-e^{\widehat{u}}-q_1e^{-\widehat{u}}+\sqrt{(S_1(q)-e^{\widehat{u}}-q_1e^{-\widehat{u}})^2-4(1-q_3e^{-\widehat{u}})(q_2-q_4e^{\widehat{u}})}}{2(1-q_3e^{-\widehat{u}})}}\,. \tag{71}$$

To obtain the disk invariants, we compare this solution with

$$\widehat{v} = \sum_{\vec{k}\in H_2(X,\mathbb{Z})}\sum_{m\in\mathbb{Z}}\sum_{n=1}^{\infty}\frac{m}{n}N_{\vec{k},m}q_1^{nk_1}q_2^{nk_2}q_3^{nk_3}q_4^{nk_4}e^{nm\widehat{u}}\,, \tag{72}$$

and define as before,

$$I_{\vec{k}}(\widehat{x}) = \sum_{m\neq 0} N_{\vec{k},m}\widehat{x}^m\,. \tag{73}$$

The functions $I_{\vec{k}}(x)$ for $\sum_{i=1}^{4} k_i = 1, 2$ are given in the table below. The results for $\sum_{i=1}^{4} k_i = 3, 4, 5$ are given in the appendix.

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(1000)	$\frac{1}{x}$	(0002)	$-\frac{x^2}{(1+x)(1-x)^3}$
(0100)	$-\frac{x}{1-x}$	(1100)	$\frac{1}{x} - \frac{2x}{1-x}$
(0010)	$-\frac{1}{x}$	(1010)	0
(0001)	$\frac{x}{1-x}$	(1001)	$\frac{2x}{1-x}$
(2000)	0	(0110)	$-\frac{1}{x} + \frac{x}{1-x}$
(0200)	$-\frac{x}{(1+x)(1-x)^3}$	(0101)	$\frac{x}{(1-x)^3}$
(0020)	0	(0011)	$-\frac{x}{1-x}$

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A Local $\mathcal{F}_2$

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(3000)	0	(2100)	$-\frac{3x}{1-x}$	(2010)	0
(2001)	$\frac{3x}{1-x} - \frac{1}{x}$	(1200)	$-\frac{2x^2}{(1-x)^3}$	(1110)	$\frac{2x}{1-x}$
(1101)	$\frac{2x(1+x)}{(1-x)^3}$	(1020)	0	(1011)	$\frac{1-x-2x^2}{x(1-x)}$
(1002)	$-\frac{2x}{(1-x)^3}$	(0300)	$-\frac{x^4(1+x^2)}{(1-x)^5(1+x+x^2)}$	(0210)	$\frac{x^2}{(1-x)^3}$
(0201)	$\frac{x^3(1+x)}{(1-x)^5}$	(0120)	0	(0111)	$-\frac{x(1+x)}{(1-x)^3}$
(0102)	$-\frac{2x^3}{(1-x)^5}$	(0030)	0	(0021)	0
(0012)	$\frac{x}{(1-x)^3}$	(0003)	$\frac{x^3(1+x)}{(1-x)^5(1+x+x^2)}$		

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(4000)	0	(3100)	$-\frac{4x}{1-x} + \frac{1}{x}$	(3010)	0
(3001)	$-\frac{1+x-2x^2-4x^3}{(1-x)x^2}$	(2200)	$-\frac{x(3+4x+3x^2)}{(1-x)^3(1+x)}$	(2110)	$\frac{3x}{1-x} - \frac{1}{x}$
(2101)	$\frac{2x(9-8x+4x^2)}{(1-x)^3}$	(2020)	0	(2011)	$-\frac{3x}{1-x} + \frac{1}{x^2} + \frac{2}{x}$
(2002)	$-\frac{x(15-2x-11x^2+8x^3)}{(1-x)^3(1+x)}$	(1300)	$-\frac{2x^3(1+x)}{(1-x)^5}$	(1210)	$\frac{2x(1+x)}{(1-x)^3}$
(1201)	$\frac{2x^2(1+4x+x^2)}{(1-x)^5}$	(1120)	0	(1111)	$-\frac{2x(7-6x+3x^2)}{(1-x)^3}$
(1102)	$-\frac{6x^2(1+x)}{(1-x)^5}$	(1030)	0	(1021)	0

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(1012)	$\frac{2x(6-7x+3x^2)}{(1-x)^3}$	(1003)	$\frac{4x^2}{(1-x)^5}$	(0400)	$-\frac{x^5(1+2x+4x^2+2x^3+x^4)}{(1-x)^7(1+x)^3}$
(0310)	$\frac{x^3(1+x)}{(1-x)^5}$	(0301)	$\frac{x^4(1+3x+x^2)}{(1-x)^7}$	(0220)	$-\frac{x^2}{(1-x)^3(1+x)}$
(0211)	$-\frac{x^2(1+4x+x^2)}{(1-x)^5}$	(0202)	$-\frac{2x^4(2+7x+12x^2+7x^3+2x^4)}{(1-x)^7(1+x)^3}$	(0130)	0
(0121)	$\frac{x}{(1-x)^3}$	(0112)	$\frac{3x^2(1+x)}{(1-x)^5}$	(0103)	$\frac{5x^4}{(1-x)^7}$
(0040)	0	(0031)	0	(0022)	$-\frac{x}{(1-x)^3(1+x)}$
(0013)	$-\frac{2x^2}{(1-x)^5}$	(0004)	$-\frac{2x^4(1+3x+x^2)}{(1-x)^7(1+x)^3}$		

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{K}}(x)$
(5000)	0	(4100)	$-\frac{5x}{1-x}+\frac{1}{x^2}+\frac{2}{x}$	(4010)	0
(4001)	$-\frac{1+x+x^2-3x^3-5x^4}{(1-x)x^3}$	(3200)	$-\frac{10(2-2x+x^2)}{(1-x)^3}$	(3110)	$-\frac{1+x-2x^2-4x^3}{(1-x)x^2}$
(3101)	$-2\frac{1-3x-37x^2+54x^3-25x^4}{(1-x)^3x}$	(3020)	0	(3011)	$\frac{1+x+x^2-3x^3-4x^4}{x^3-x^4}$
(3002)	$5(-\frac{2}{(1-x)^3}-\frac{6}{1-x}+\frac{1}{x}+8)$	(2300)	$-\frac{x^2(3+8x+3x^2)}{(1-x)^5}$	(2210)	$\frac{2x(9-8x+4x^2)}{(1-x)^3}$
(2201)	$\frac{x(3+23x+8x^2+8x^3)}{(1-x)^5}$	(2120)	0	(2111)	$2\frac{1-3x-4x^2(9-13x+6x^2)}{(1-x)^3x}$
(2102)	$-\frac{x(15+17x+5x^2+5x^3)}{(1-x)^5}$	(2030)	0		

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(2021)	0	(2012)	$-\frac{6}{x}+\frac{10x(6-9x+4x^2)}{(1-x)^3}$	(2003)	$\frac{x(12-3x+5x^2)}{(1-x)^5}$
(1400)	$-\frac{2x^4(1+3x+x^2)}{(1-x)^7}$	(1310)	$\frac{2x^2(1+4x+x^2)}{(1-x)^5}$	(1301)	$\frac{2x^3(1+9x+9x^2+x^3)}{(1-x)^7}$
(1220)	$-\frac{2x}{(1-x)^3}$	(1211)	$-\frac{2x(1+10x+4x^2+3x^3)}{(1-x)^5}$	(1202)	$-\frac{12x^3(1+3x+x^2)}{(1-x)^7}$
(1130)	0	(1121)	$\frac{x(12-14x+6x^2)}{(1-x)^3}$	(1112)	$\frac{4x(3+4x+x^2+x^3)}{(1-x)^5}$
(1103)	$\frac{20x^3(1+x)}{(1-x)^7}$	(1040)	0	(1031)	0
(1022)	$-\frac{2}{(1-x)^3}-\frac{4}{1-x}+\frac{1}{x}+6$				

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(1013)	$-\frac{2x(5-x+2x^2)}{(1-x)^5}$	(1004)	$-\frac{10x^3}{(1-x)^7}$	(0500)	$-\frac{x^6(1+x+5x^2+5x^4+x^5+x^6)}{(1-x)^9(1+x+x^2+x^3+x^4)}$
(0410)	$\frac{x^4(1+3x+x^2)}{(1-x)^7}$	(0401)	$\frac{x^5(1+6x+6x^2+x^3)}{(1-x)^9}$	(0320)	$-\frac{2x^3}{(1-x)^5}$
(0311)	$-\frac{x^3(1+9x+9x^2+x^3)}{(1-x)^7}$	(0302)	$-\frac{2x^5(3+8x+3x^2)}{(1-x)^9}$	(0230)	0
(0221)	$\frac{3x^2(1+x)}{(1-x)^5}$	(0212)	$\frac{6x^3(1+3x+x^2)}{(1-x)^7}$	(0203)	$\frac{14x^5(1+x)}{(1-x)^9}$
(0140)	0	(0131)	0	(0122)	$-\frac{x(1+4x+x^2)}{(1-x)^5}$
(0113)	$-\frac{10x^3(1+x)}{(1-x)^7}$				

$\vec{k}$	$I_{\vec{k}}(x)$
(0104)	$-\frac{14x^5}{(1-x)^9}$
(0050)	0
(0041)	0
(0032)	0
(0023)	$\frac{x(1+x)}{(1-x)^5}$
(0014)	$\frac{5x^3}{(1-x)^7}$
(0005)	$\frac{x^5(5+2x+2x^2+5x^3)}{(1-x)^9(1+x+x^2+x^3+x^4)}$

B Local $\mathcal{B}_3$

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(3000)	0	(2100)	$\frac{1}{x^2}+\frac{2}{x}-\frac{3x}{1-x}$	(2010)	0
(2001)	$-\frac{1}{x}+\frac{3x}{1-x}$	(1200)	$\frac{1}{x}-\frac{x(6x^2-14x+10)}{(1-x)^3}$	(1110)	$-\frac{1}{x^2}+\frac{4x}{1-x}$
(1101)	$\frac{6x^3-14x^2+12x}{(1-x)^3}$	(1020)	0	(1011)	$\frac{1}{x}-\frac{2x}{1-x}$
(1002)	$-\frac{2x}{(1-x)^3}$	(0300)	$-\frac{x(1+x^2)}{(1-x)^5(1+x+x^2)}$	(0210)	$-\frac{1}{x}+\frac{x(4x^2-9x+6)}{(1-x)^3}$
(0201)	$\frac{x(1+x)}{(1-x)^5}$	(0120)	0	(0111)	$-\frac{4x^3-9x^2+7x}{(1-x)^3}$
(0102)	$-\frac{2x^2}{(1-x)^5}$	(0030)	0	(0021)	0
(0012)	$\frac{x}{(1-x)^3}$	(0003)	$\frac{x^3(1+x)}{(1-x)^5(1+x+x^2)}$		

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(4000)	0	(3100)	$\frac{1}{x^3}+\frac{2}{x^2}+\frac{3}{x}-\frac{4x}{1-x}$
(3010)	0	(3001)	$-\frac{1+x-2x^2-4x^3}{(1-x)x^2}$
(2200)	$\frac{2+x(1+x)(6-26x-15x^2+61x^3-32x^4)}{(1-x)^3x^2(1+x)}$	(2110)	$-\frac{1+x+x^2-3x^3-3x^4}{(1-x)x^3}$
(2101)	$-\frac{6}{x}+\frac{10x(6-9x+4x^2)}{(1-x)^3}$	(2020)	0
(2011)	$-\frac{3x}{1-x}+\frac{1}{x^2}+\frac{2}{x}$	(2002)	$-\frac{x(15-2x-11x^2+8x^3)}{(1-x)^3(1+x)}$
(1300)	$\frac{1}{x}-\frac{2x(15-40x+48x^2-27x^3+6x^4)}{(1-x)^5}$	(1210)	$-\frac{3+3x-27x^2-7x^3+54x^4-30x^5}{(1-x)^3x^2}$
(1201)	$\frac{2x(20-43x+50x^2-27x^3+6x^4)}{(1-x)^5}$	(1120)	0
(1111)	$36-\frac{8}{(1-x)^3}-\frac{28}{1-x}+\frac{7}{x}$	(1102)	$-\frac{2x(5-x+2x^2)}{(1-x)^5}$
(1030)	0	(1021)	0

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(1012)	$\frac{2x(6-7x+3x^2)}{(1-x)^3}$	(1003)	$\frac{4x^2}{(1-x)^5}$
(0400)	$-\frac{x(1+2x+4x^2+2x^3+x^4)}{(1-x)^7(1+x)^3}$	(0310)	$-\frac{1}{x}+\frac{x(20-57x+70x^2-40x^3+9x^4)}{(1-x)^5}$
(0301)	$\frac{x(1+3x+x^2)}{(1-x)^7}$	(0220)	$\frac{1-4x^2-3x^3+6x^4+3x^5-4x^6}{(1-x)^3x^2(1+x)}$
(0211)	$-\frac{x(26-62x+73x^2-40x^3+9x^4)}{(1-x)^5}$	(0202)	$-\frac{2x^2(2+7x+12x^2+7x^3+2x^4)}{(1-x)^7(1+x)^3}$
(0130)	0	(0121)	$-\frac{1}{x}+\frac{x(6-9x+4x^2)}{(1-x)^3}$
(0112)	$\frac{3x(2-x+x^2)}{(1-x)^5}$	(0103)	$\frac{5x^3}{(1-x)^7}$
(0040)	0	(0031)	0
(0022)	$-\frac{x}{(1-x)^3(1+x)}$	(0013)	$-\frac{2x^2}{(1-x)^5}$
(0004)	$-\frac{2x^4(1+3x+x^2)}{(1-x)^7(1+x)^3}$		

$\vec{k}$	$I_{\vec{k}}(x)$	$\vec{k}$	$I_{\vec{k}}(x)$
(5000)	0	(4100)	$\frac{1+x+x^2+x^3-4x^4-5x^5}{(1-x)x^4}$
(4010)	0	(4001)	$-\frac{1+x+x^2-3x^3-5x^4}{(1-x)x^3}$
(3200)	$\frac{4+4x+9x^2-91x^3-21x^4+195x^5-110x^6}{(1-x)^3x^3}$	(3110)	$-\frac{1+x+x^2+x^3-4x^4-4x^5}{(1-x)x^4}$
(3101)	$-\frac{9+13x-93x^2-89x^3+290x^4-150x^5}{(1-x)^3x^2}$	(3020)	0
(3011)	$\frac{1+x+x^2-3x^3-4x^4}{x^3-x^4}$	(3002)	$5(-\frac{2}{(1-x)^3}-\frac{6}{1-x}+\frac{1}{x}+8)$
(2300)	$p1$	(2210)	$-2\frac{3+2x+6x^2-60x^3+4x^4+100x^5-60x^6}{(1-x)^3x^3}$
(2201)	$p2$	(2120)	0
(2111)	$2\frac{6+8x-60x^2-33x^3+149x^4-80x^5}{(1-x)^3x^2}$	(2102)	$-\frac{x(177-415x+485x^2-265x^3+60x^4)}{(1-x)^5}$
(2030)	0		

$$p1 = \frac{4}{x^2} + \frac{30}{x} - \frac{x(300 - 948x + 1217x^2 - 720x^3 + 165x^4)}{(1-x)^5}$$

$$p2 = -\frac{20 - 100x - 265x^2 + 1160x^3 - 1597x^4 + 965x^5 - 225x^6}{(1-x)^5 x}$$

$\vec{k}$	$I_{\vec{k}}(x)$
(2021)	0
(2003)	$\frac{x(12-3x+5x^2)}{(1-x)^5}$
(1310)	$-2\frac{3+5x-70x^2+15x^3+317x^4-556x^5+370x^6-90x^7}{(1-x)^5x^2}$
(1220)	$\frac{2+3x^2-29x^3+9x^4+37x^5-24x^6}{(1-x)^3x^3}$
(1202)	$-\frac{6x(5+6x^2-2x^3+x^4)}{(1-x)^7}$
(1121)	$-\frac{3+3x-27x^2-7x^3+54x^4-30x^5}{(1-x)^3x^2}$
(1103)	$\frac{4x^2(7+x+2x^2)}{(1-x)^7}$
(1031)	0

$\vec{k}$	$I_{\vec{k}}(x)$
(2012)	$-\frac{6}{x}+\frac{10x(6-9x+4x^2)}{(1-x)^3}$
(1400)	$\frac{1}{x}-2\frac{x(35-133x+260x^2-287x^3+186x^4-66x^5+10x^6)}{(1-x)^7}$
(1301)	$2\frac{x(50-147x+281x^2-297x^3+189x^4-66x^5+10x^6)}{(1-x)^7}$
(1211)	$2\frac{13-65x-104x^2+575x^3-826x^4+509x^5-120x^6}{(1-x)^5 x}$
(1130)	0
(1112)	$\frac{12x(14-34x+40x^2-22x^3+5x^4)}{(1-x)^5}$
(1040)	0
(1022)	$-\frac{2}{(1-x)^3}-\frac{4}{1-x}+\frac{1}{x}+6$

$\vec{k}$	$I_{\vec{k}}(x)$
(0104)	$-\frac{14x^4}{(1-x)^9}$
(0050)	0
(0041)	0
(0032)	0
(0023)	$\frac{x(1+x)}{(1-x)^5}$
(0014)	$\frac{5x^3}{(1-x)^7}$
(0005)	$\frac{x^5(5+2x+2x^2+5x^3)}{(1-x)^9(1+x+x^2+x^3+x^4)}$

$\vec{k}$	$I_{\vec{k}}(x)$
(1013)	$-\frac{2x(5-x+2x^2)}{(1-x)^5}$
(1004)	$-\frac{10x^3}{(1-x)^7}$
(0500)	$-\frac{x(1+x+5x^2+5x^4+x^5+x^6)}{(1-x)^9(1+x+x^2+x^3+x^4)}$
(0410)	$-\frac{1-7x-29x^2+168x^3-368x^4+429x^5-287x^6+104x^7-16x^8}{(1-x)^7 x}$
(0401)	$\frac{x(1+6x+6x^2+x^3)}{(1-x)^9}$
(0320)	$\frac{2}{x^2}+\frac{10}{x}-\frac{x(59-195x+257x^2-155x^3+36x^4)}{(1-x)^5}$
(0311)	$-\frac{x(70-230x+437x^2-467x^3+299x^4-105x^5+16x^6)}{(1-x)^7}$
(0302)	$-\frac{2x^2(3+8x+3x^2)}{(1-x)^9}$
(0230)	0
(0221)	$-\frac{6}{x}+\frac{3x(28-86x+110x^2-65x^3+15x^4)}{(1-x)^5}$
(0212)	$\frac{x(20-11x+27x^2-11x^3+5x^4)}{(1-x)^7}$
(0203)	$\frac{14x^3(1+x)}{(1-x)^9}$
(0140)	0
(0131)	0
(0122)	$-\frac{x(26-62x+73x^2-40x^3+9x^4)}{(1-x)^5}$
(0113)	$-\frac{2x^2(8-x+3x^2)}{(1-x)^7}$

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