Classical solutions in string field theory around the tachyon vacuum

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Abstract

In a previous paper [hep-th/0012251] we proposed a simple class
of actions for string field theory around the tachyon vacuum. In this
paper we search for classical solutions describing D-branes of different

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dimensions using the ansatz that the solutions factorize into the direct product of a matter state and a universal ghost state. We find closed form expressions for the matter state describing D-branes of all dimensions. For the space filling D25-brane the state is the matter part of the zero angle wedge state, the "sliver", built in [hep-th/0006240]. For the other D-brane solutions the matter states are constructed using a solution generating technique outlined in [hep-th/0008252]. The ratios of tensions of various D-branes, requiring evaluation of determinants of infinite dimensional matrices, are calculated numerically and are in very good agreement with the known results.

Contents

1 Introduction and summary

Cubic open string field theory [1] has turned out to be a powerful tool in studying various conjectures [2, 3] about tachyon condensation on bosonic D-branes [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. One aspect of the tachyon conjectures that remains to be confirmed is the expected absence of physical open string excitations around the tachyon vacuum. In a previous paper we proposed that a simple class of cubic actions represent string field theory built upon the tachyon vacuum [22]. As opposed to the conventional cubic SFT where the kinetic operator is the BRST operator Q_B, here the kinetic operator Q is non-dynamical and is built solely out of worldsheet ghost fields.[2] In this class of actions the absence of physical open string states around the vacuum is manifest. Gauge invariance holds in this class of actions, and therefore basic consistency requirements are expected to be satisfied.

One major confirmation of the physical correctness of the proposed actions would be the construction of classical solutions which describe the known D-brane configurations. An indirect argument for the existence of these solutions was given in [22] where it was also shown that under certain assumptions the proposed action reproduces in a rather nontrivial fashion the correct ratios of tensions of D-branes of different dimensions.

In this paper we give a direct construction of the classical solutions representing various D-branes and verify that the ratios of their tensions agree with the known answer. We use an ansatz where the solution Ψ representing a D-brane has a factorized form $\Psi_m \otimes \Psi_g$, with Ψ_m and Ψ_g being string fields built solely out of matter and ghost operators respectively.[3] Such factorized form is clearly compatible with the structure of the relevant string field equation since the kinetic operator Q does not mix matter and ghost sectors,[4] and moreover, as is familiar, the star product also factors into the matter and ghost sectors. More explicitly, given string fields $A = A_m \otimes A_g$ and $B = B_m \otimes B_g$, we have $A * B = (A_m *^m B_m) \otimes (A_g *^g B_g)$, where $*^m$ and $*^g$ denote multiplication rules in the matter and ghost sectors respectively. While the matter factor Ψ_m is clearly different for the various D-branes, we assume that the ghost factor Ψ_g is common to all the D-branes.

With this ansatz, and the specific form of the action proposed in [22], the string field theory equations of motion $Q\Psi + \Psi * \Psi = 0$ factorizes into

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[2]A subset of this class of actions was discussed previously in ref.[23].

[3]We wish to thank W. Taylor for emphasizing this factorization property to us.

[4]This is not true, of course, for the standard BRST operator.

a matter part and a ghost part, with the matter part yielding the equation $\Psi_m *^m \Psi_m = \Psi_m$. We now ask how we can find solutions of this equation. In fact, any solution of $\Psi * \Psi = \Psi$ where the ghost number zero field Ψ factors as $\Psi_m \otimes \Psi_g$ provides a solution of $\Psi_m *^m \Psi_m = \Psi_m$. There are at least two known translationally invariant solutions of $\Psi * \Psi = \Psi$. One solution is provided by the identity string field $\mathcal{I}$, and the second is provided by the "sliver", the zero angle wedge state Ξ constructed in [10]. This state was constructed in background independent language; it only requires the total Virasoro operators of the full matter and ghost CFT. For reasons which will become clear later, we identify the matter part Ξ_m of the sliver, with suitable normalization, as the matter part of the solution describing the D25-brane. More recently, Kostelecky and Potting [16] investigated solutions of the equation $\Psi_m *^m \Psi_m = \Psi_m$ by using an explicit representation of the $*$-product in terms of Neumann coefficients for free (matter) scalars. In addition to the matter part of the identity $\mathcal{I}$, they found one nontrivial solution $\mathcal{T}_m$. Thus it is natural to ask: what is the relationship between the matter component Ξ_m of the sliver and the state $\mathcal{T}_m$ found in ref.[16]. We study this question using the level truncation scheme, and find very good evidence that the two states are really the same. Thus we can use either description for representing the D-25-brane. Our understanding of the sliver Ξ_m shows that the solution describing the D25-brane belongs to the universal subspace of the state space [24], the space generated by the action of matter Virasoro generators and ghost oscillators on the SL(2,R) invariant vacuum. Furthermore, the sliver string field Ξ_m provides a "simple closed-form" solution for the matter factor representing the D25-brane in the SFT of [22]. By "closed form" we mean that the operational definition of Ξ_m is explicit. Even more, its geometrical meaning is clear. By "simple" we mean that the exact calculation of Ξ_m to any given level requires only a finite number of operations.[5]

We then use the observations of [16] to construct the string field Ψ_m describing a lower dimensional D-brane starting from the expression for the matter string field representing the space filling D25 brane. The key point noted there is that the properties satisfied by the matter Neumann coefficients V_{mn}^{rs} $(m, n \geq 1)$ that guarantee the existence of the translational invariant solution are also satisfied by the extended Neumann coefficients $V_{mn}'^{rs}$ $(m, n \geq 0)$ defined by Gross and Jevicki [25] by adding a new pair of oscillators to represent the center of mass position and momentum op-

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[5]In this vein one would say that the description of this state as $\mathcal{T}_m$ [16] is of closed form, as it is given by an explicit formula in terms of an exactly calculable infinite dimensional matrix. The formula is not simple in that it involves inverses and square roots of this infinite matrix, so even the finite level truncation of $\mathcal{T}_m$ can only be constructed approximately with finite number of operations.

erators. Thus the same method used for generating translational invariant solution can be used to generate lump solutions. In implementing this procedure one requires the background dependent description $\mathcal{T}_m$ of the D25 string field as a function of matter Neumann coefficients. We carry out the construction thus obtaining "closed-form" expressions for the matter string fields representing lower dimensional branes. The ratio of tensions of these D-branes has an analytic expression in terms of the Neumann coefficients, but explicit computation of this ratio involves evaluating determinants of infinite dimensional matrices. We calculate this ratio in the level truncation using the known expression for the Neumann coefficients. While the convergence to the answer is relatively slow as a function of the level L, the relative simplicity of our expressions allows numerical computations up to levels of the order of several thousands! A fit of the data obtained at various levels suggests that corrections vanish as inverse powers of $\ln(L)$. The numerical results at large values of L as well as an extrapolation of these results to $L = \infty$ via a fit using a cubic polynomial in $1/\ln(L)$ gives results very close to the expected answer. We consider these results to be strong evidence for the correctness of the SFT we proposed in [22].

Our concrete implementation of the procedure suggested in [16] actually finds families of solutions corresponding to lower dimensional branes. The solutions have gaussian profiles in the directions transverse to the brane. We find that for a D-$(25 - k)$ brane there is a k-parameter family of solutions, with the parameters controlling the width of the lumps in different transverse directions. We believe that all these solutions are gauge equivalent. This is necessary for the identification with D-branes, since a physical D-brane has no moduli other than its position in the transverse space. One indirect piece of evidence to this effect is that the ratios of tensions converge to the correct values for any solution in the family. Some more direct but still incomplete arguments are given in appendix C. In this context it will be interesting to explore if the width of the lump in conventional string field theory, studied in ref.[9], changes when we use a gauge different from Siegel gauge.

Since besides the sliver state Ξ_m, the identity state $\mathcal{I}_m$ also squares to itself under the $*^m$ product, it is natural to ask why we identify the sliver and not the identity as the solution representing the D25-brane. While we do not have a concrete proof that $\mathcal{I}_m$ cannot be the matter part of the D25-brane solution, we offer the following observations. First of all, as we have discussed, starting from the sliver state we can construct lump solutions of arbitrary co-dimension with correct ratios of tensions as expected of D-branes. If we apply the same procedure to $\mathcal{I}_m$, we get back $\mathcal{I}_m$ and not a lower dimensional brane. Thus, for example, there is no obvious candidate for a 24-brane solution with tension 2π times the tension associated with the

state $\mathcal{I}_m$, as would be expected of a D24-brane solution if $\mathcal{I}_m$ represented the D25-brane. This clearly makes Ξ_m a much stronger candidate than $\mathcal{I}_m$ for the D25-brane solution. $\mathcal{I}_m$ suffers from the further complication that its normalization properties are much worse than those of Ξ_m. Whereas the normalization of Ξ_m involves an infinite dimensional determinant which is finite at least up to any given level (although it could vanish as the level goes to infinity), the normalization of $\mathcal{I}_m$ involves a determinant which vanishes at any finite level.

In the proposal of [22] the explicit form of the kinetic operator $\mathcal{Q}$ was not fixed. In fact, we discussed two classes of such operators. In the first class, exemplified by $\mathcal{Q} = c_0$, the operator does not annihilate the identity string field $\mathcal{I}$. In the second class, exemplified by $\mathcal{Q} = c_0 + \frac{1}{2}(c_2 + c_{-2})$, the operator does annihilate the identity string field $\mathcal{I}$. Both yield gauge invariant actions without physical open strings around the tachyon vacuum. A proper understanding of the ghost factor representing the D25 brane (and in fact all other D-branes, since we assume that this factor is universal) would be expected to yield some information on $\mathcal{Q}$, since the ghost equation is of the form $\mathcal{Q}\Psi_g = -\Psi_g *^g \Psi_g$. Given that the matter part of the string field for the D25-brane is the sliver state Ξ_m, we expect Ψ_g to be closely related to the ghost part Ξ_g of the state. Since Ξ_g is of ghost number zero and Ψ_g must be of ghost number one we conjecture that $\Psi_g = \mathcal{C}\Xi_g$ where $\mathcal{C}$ is a ghost number one operator built solely out of ghosts. It may turn out that both $\mathcal{Q}$ and $\mathcal{C}$ are determined by demanding the existence of a non-trivial solution to the field equation. Knowledge of $\mathcal{C}$ and $\mathcal{Q}$ would amount to a complete specification of the SFT action, and a complete knowledge of the string fields representing D-branes.

The rest of the paper is organized as follows. In section 2 we discuss the factorization properties of the field equations, and give the construction of the matter part of the D25-brane solution in the oscillator representation. We also produce numerical evidence that this solution is identical to the matter part of the sliver state constructed in ref.[10]. In section 3 we construct the lump solutions, compute the ratio of tensions of lump solutions of different dimensions numerically and show that the result is in very good agreement with the known results. We conclude in section 4 by listing some of the open questions. Appendix A contains a list of Neumann coefficients needed for our analysis. Appendix B discusses the transformation of the 3-string vertex when we go from the momentum basis to the oscillator basis. It also contains the precise relationship between our variables and those used in ref.[25], and some properties of the Neumann coefficients which are important for our analysis. Appendix C explores the possibility that a parameter appearing in the construction of the lump solution is a gauge artifact. In

appendix D we derive some properties of the sliver state.

2   Construction of the D25-brane solution

We begin with the string field theory action:

$$\mathcal{S}(\Psi) \equiv -\frac{1}{g_0^2}\left[\frac{1}{2}\langle\,\Psi\,,\,\mathcal{Q}\,\Psi\,\rangle + \frac{1}{3}\langle\,\Psi\,,\,\Psi*\Psi\,\rangle\right],\qquad(2.1)$$

where Ψ is the string field represented by a state of ghost number one in the combined matter-ghost state space, g_0 is the open string coupling constant, $\mathcal{Q}$ is an operator made purely of ghost fields and satisfying various requirements discussed in ref.[22], $\langle\,,\,\rangle$ denotes the BPZ inner product, and $*$ denotes the usual $*$-product of the string fields. This action is supposed to describe the string field theory action around the tachyon vacuum. Although the action is formally background independent, for practical computation (*e.g.* choosing a basis in the state space for expanding Ψ) we need to use a conformal field theory (CFT), and we take this to be the CFT describing the D25-brane in flat space-time.

2.1   Factorization property of the field equations

If (2.1) really describes the string field theory around the tachyon vacuum, then the equations of motion of this field theory:

$$\mathcal{Q}\Psi = -\Psi*\Psi\,,\qquad(2.2)$$

must have a space-time independent solution describing the D25-brane, and also lump solutions of all codimensions describing lower dimensional D-branes. We shall look for solutions of the form:

$$\Psi = \Psi_m \otimes \Psi_g\,,\qquad(2.3)$$

where Ψ_g denotes a state obtained by acting with the ghost oscillators on the SL(2,R) invariant vacuum of the ghost CFT, and Ψ_m is a state obtained by acting with matter oscillators on the SL(2,R) invariant vacuum of the matter CFT. Let us denote by $*^g$ and $*^m$ the star product in the ghost and matter sector respectively. Eq.(2.2) then factorizes as

$$\mathcal{Q}\Psi_g = -\Psi_g *^g \Psi_g\,,\qquad(2.4)$$

and

$$\Psi_m = \Psi_m *^m \Psi_m \,. \tag{2.5}$$

Such a factorization is possible since $\mathcal{Q}$ is made purely of ghost operators. Note that we have used the freedom of rescaling Ψ_g and Ψ_m with λ and λ^{-1} to put eqs.(2.4), (2.5) in a convenient form.

In looking for the solutions describing D-branes of various dimensions we shall assume that Ψ_g remains the same for all solutions, whereas Ψ_m is different for different D-branes. Given two static solutions of this kind, described by Ψ_m and Ψ'_m, the ratio of the energy associated with these two solutions is obtained by taking the ratio of the actions associated with the two solutions. For a string field configuration satisfying the equation of motion (2.2), the action (2.1) is given by

$$\mathcal{S}|_\Psi = -\frac{1}{6g_0^2} \langle \Psi , \mathcal{Q}\,\Psi \rangle \,. \tag{2.6}$$

Thus with the ansatz (2.3) the action takes the form:

$$\mathcal{S}|_\Psi = -\frac{1}{6g_0^2} \langle \Psi_g \,|\, \mathcal{Q}\Psi_g \rangle_g \langle \Psi_m | \Psi_m \rangle_m \equiv K \langle \Psi_m | \Psi_m \rangle_m \,, \tag{2.7}$$

where $\langle\ |\ \rangle_g$ and $\langle\ |\ \rangle_m$ denote BPZ inner products in ghost and matter sectors respectively. $K = -(6g_0^2)^{-1}\langle \Psi_g | \mathcal{Q}\Psi_g \rangle_g$ is a constant factor calculated from the ghost sector which remains the same for different solutions. Thus we see that the ratio of the action associated with the two solutions is

$$\frac{\mathcal{S}|_{\Psi'}}{\mathcal{S}|_\Psi} = \frac{\langle \Psi'_m | \Psi'_m \rangle_m}{\langle \Psi_m | \Psi_m \rangle_m} \,. \tag{2.8}$$

The ghost part drops out of this calculation.

The analysis in the rest of this section will focus on the construction of a space-time independent solution to eq.(2.5) representing a D25-brane. As pointed out in the introduction, there are two ways of doing this. One method [10] gives a description of this state in terms of matter Virasoro generators and the other method [16] describes this state in terms of the oscillators of the matter fields. Since the second method can be generalized to describe lump solutions, we first describe this method in detail, and then compare this with the first description.

2.2   A solution for the D25 brane

Following ref.[25, 26, 27] we represent the star product of two states $|A\rangle$ and $|B\rangle$ in the matter CFT as[6]

$$|A *^m B\rangle_3 = {}_1\langle A|\,{}_2\langle B|V_3\rangle\,, \tag{2.9}$$

where the three string vertex $|V_3\rangle$ is given by

$$|V_3\rangle = \int d^{26}p_{(1)} d^{26}p_{(2)} d^{26}p_{(3)} \delta^{(26)}(p_{(1)} + p_{(2)} + p_{(3)}) \exp(-E)|0,p\rangle_{123}\,, \tag{2.10}$$

and

$$E = \frac{1}{2} \sum_{\substack{r,s \\ m,n \geq 1}} \eta_{\mu\nu} a_m^{(r)\mu\dagger} V_{mn}^{rs} a_n^{(s)\nu\dagger} + \sum_{\substack{r,s \\ n \geq 1}} \eta_{\mu\nu} p_{(r)}^\mu V_{0n}^{rs} a_n^{(s)\nu\dagger} + \frac{1}{2} \sum_r \eta_{\mu\nu} p_{(r)}^\mu V_{00}^{rr} p_{(r)}^\nu\,. \tag{2.11}$$

Here $a_m^{(r)\mu}$, $a_m^{(r)\mu\dagger}$ are non-zero mode matter oscillators acting on the r-th string state normalized so that

$$[a_m^{(r)\mu}, a_n^{(s)\nu\dagger}] = \eta^{\mu\nu}\delta_{mn}\delta^{rs}\,, \qquad m,n \geq 1\,. \tag{2.12}$$

$p_{(r)}$ is the 26-component momentum of the r-th string, and $|0,p\rangle_{123} \equiv |p_{(1)}\rangle \otimes |p_{(2)}\rangle \otimes |p_{(3)}\rangle$ is the tensor product of Fock vacuum of the three strings, annihilated by the non-zero mode annihilation operators $a_m^{(r)\mu}$, and eigenstate of the momentum operator of the rth string with eigenvalue $p_{(r)}^\mu$. $|p\rangle$ is normalized as

$$\langle p|p'\rangle = \delta^{26}(p + p')\,. \tag{2.13}$$

The coefficients V_{mn}^{rs} for $0 \leq m,n < \infty$ can be calculated by standard methods [25, 27] and have been given in appendix A.[7] Some properties of V_{mn}^{rs} have been discussed in appendix B. Since we are interested in this section in space-time translational invariant solutions, we can ignore the momentum dependent factors in the vertex. and the relevant form of E is:

$$E = \frac{1}{2} \sum_{r,s} \eta_{\mu\nu} a^{(r)\mu\dagger} \cdot V^{rs} \cdot a^{(s)\nu\dagger}\,, \tag{2.14}$$

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[6]Whenever we use explicit operator representations of the string product string fields will be denoted as kets or bras as appropriate.

[7]In our conventions we take $\alpha' = 1$.

where the dots represent sums over mode numbers, and V_{mn}^{rs} for $m, n \geq 1$ is written as the V^{rs} matrix. For the analysis of lumps, however, we will need the full vertex.

Some appreciation of the properties reviewed in Appendix B is necessary for structural reasons. Equation (B.15), in particular, gives

$$V^{rs} = \frac{1}{3}(C + \omega^{s-r}U + \omega^{r-s}\bar{U}),\tag{2.15}$$

where $\omega = e^{2\pi i/3}$, U and C are regarded as matrices with indices running over $m, n \geq 1$,

$$C_{mn} = (-1)^m \delta_{mn}, \quad m, n \geq 1,\tag{2.16}$$

and U satisfies ((B.17))

$$\bar{U} \equiv U^* = CUC, \qquad U^2 = \bar{U}^2 = 1, \qquad U^\dagger = U, \qquad \bar{U}^\dagger = \bar{U}.\tag{2.17}$$

The superscripts r, s are defined $\mathrm{mod}(3)$, and (2.15) manifestly implements the cyclicity property $V^{rs} = V^{(r+1)(s+1)}$. Also note the transposition property $(V^{rs})^T = V^{sr}$. Finally, eqs.(2.15), (2.17) allow one to show that

$$[CV^{rs}, CV^{r's'}] = 0 \quad \forall \ \ r, s, r', s',\tag{2.18}$$

and

$$(CV^{12})(CV^{21}) = (CV^{21})(CV^{12}) = (CV^{11})^2 - CV^{11},$$
$$(CV^{12})^3 + (CV^{21})^3 = 2(CV^{11})^3 - 3(CV^{11})^2 + 1.\tag{2.19}$$

Equations (2.15) up to (2.19) are all that we shall need to know about the matter part of the relevant star product (as given in eqs. (2.9), (2.10) and (2.14)) to construct the translationally invariant solution. In fact, since (2.18), (2.19) follow from (2.15) and (2.17), these two equations are really all that is strictly needed. Such structure will reappear in the next section with matrices that also include $m = 0$ and $n = 0$ entries, and thus will guarantee the existence of a solution constructed in the same fashion as the solution to be obtained below.

We are looking for a space-time independent solution of eq.(2.5). The strategy of ref.[16] is to take a trial solution of the form:[8]

$$|\Psi_m\rangle = \mathcal{N}^{26} \exp\left(-\frac{1}{2}\eta_{\mu\nu} \sum_{m,n \geq 1} S_{mn}\, a_m^{\mu\dagger} a_n^{\nu\dagger}\right)|0\rangle,\tag{2.20}$$

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[8]We caution the reader that although in this section and in section 3 we shall follow the general strategy described in [16], our explicit formulæ differ from theirs in several instances.

where $|0\rangle$ is the SL(2,R) invariant vacuum of the matter CFT, $\mathcal{N}$ is a normalization factor, and S_{mn} is an infinite dimensional matrix with indices m, n running from 1 to ∞. We shall take S to be twist invariant:[9]

$$CSC = S. \tag{2.21}$$

We shall check in the end that the solution constructed below is indeed twist invariant.

If we define

$$\Sigma = \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix}, \qquad \mathcal{V} = \begin{pmatrix} V^{11} & V^{12} \\ V^{21} & V^{22} \end{pmatrix}, \tag{2.22}$$

and

$$\chi^{\mu T} = \left(a^{(3)\mu\dagger} V^{31}, \quad a^{(3)\mu\dagger} V^{32} \right), \qquad \chi^{\mu} = \begin{pmatrix} V^{13} a^{(3)\mu\dagger} \\ V^{23} a^{(3)\mu\dagger} \end{pmatrix}, \tag{2.23}$$

then using eqs. (2.9), (2.10), (2.14) we get

$$|\Psi_m * \Psi_m\rangle_3 = \mathcal{N}^{52} \det\{(1 - \Sigma\mathcal{V})^{-1/2}\}^{26} \tag{2.24}$$
$$\times \exp\left[-\frac{1}{2}\, \eta_{\mu\nu}\{\chi^{\mu T}[(1 - \Sigma\mathcal{V})^{-1}\Sigma]\chi^{\nu} + a^{(3)\mu\dagger} \cdot V^{33} \cdot a^{(3)\nu\dagger}\}\right]|0\rangle_3.$$

In deriving eq.(2.24) we have used the general formula [16]

$$\langle 0| \exp\left(\lambda_i a_i - \frac{1}{2} P_{ij} a_i a_j\right) \exp\left(\mu_i a_i^\dagger - \frac{1}{2} Q_{ij} a_i^\dagger a_j^\dagger\right)|0\rangle \tag{2.25}$$

$$= \det(K)^{-\frac{1}{2}} \exp\left(\mu^T K^{-1}\lambda - \frac{1}{2}\lambda^T Q\, K^{-1}\lambda - \frac{1}{2}\mu^T K^{-1}P\mu\right), \quad K \equiv 1 - PQ.$$

In using this formula we took the a_i to be the list of oscillators $(a_m^{(1)}, a_m^{(2)})$ with $m \geq 1$. (2.24) then follows from (2.25) by identifying P with Σ, Q with $\mathcal{V}$, μ with χ and setting λ to 0.

Demanding that the exponents in the expressions for $|\Psi_m\rangle$ and $|\Psi_m * \Psi_m\rangle$, given in eqs.(2.20) and (2.24) respectively, match, we get

$$S = V^{11} + (V^{12}, V^{21})(1 - \Sigma\mathcal{V})^{-1}\Sigma \begin{pmatrix} V^{21} \\ V^{12} \end{pmatrix}, \tag{2.26}$$

where we have used the cyclicity property of the V matrices and the mod 3 periodicity of the indices r and s to write the equation in a convenient form. To proceed, we assume that

$$[CS, CV^{rs}] = 0 \quad \forall \quad r, s. \tag{2.27}$$

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[9]Due to this property the BPZ conjugate of the state $|\Psi_m\rangle$ is the same as its hermitian conjugate. Otherwise we need to keep track of extra $-$ signs coming from the fact that the BPZ conjugate of $a_m^\dagger$ is $(-1)^{m+1}a_m$.

We shall check later that the solution obeys these conditions. We can now write eq.(2.26) in terms of

$$T \equiv CS = SC, \qquad M^{rs} \equiv CV^{rs}, \tag{2.28}$$

and because of (2.18), (2.27) we can manipulate the equation as if T and M^{rs} are numbers rather than infinite dimensional matrices. We first multiply (2.26) by C and write it as:

$$T = X + (M^{12}, M^{21})(1 - \Sigma \mathcal{V})^{-1} \begin{pmatrix} cTM^{21} \\ TM^{12} \end{pmatrix}, \tag{2.29}$$

where

$$X = M^{11} = CV^{11}. \tag{2.30}$$

We then note that since the submatrices commute:

$$(1 - \Sigma \mathcal{V})^{-1} = \begin{pmatrix} 1 - TX & -TM^{12} \\ -TM^{21} & 1 - TX^{-1} \end{pmatrix} \tag{2.31}$$

$$= ((1 - TX)^2 - T^2 M^{12} M^{21})^{-1} \begin{pmatrix} cc1 - TX & TM^{12} \\ TM^{21} & 1 - TX \end{pmatrix}.$$

Finally, we record that

$$\det(1 - \Sigma \mathcal{V}) = \det(1 - 2TX + T^2 X), \tag{2.32}$$

where use was made the first equation in (2.19) reading $M^{12} M^{21} = X^2 - X$.

It is now a simple matter to substitute (2.31) into (2.29) and expand out eliminating all reference of M^{12} and M^{21} in favor of X by use of eqs.(2.19). The result is the condition:

$$(T - 1)(XT^2 - (1 + X)T + X) = 0. \tag{2.33}$$

This gives the solution for S:[10]

$$S = CT, \qquad T = \frac{1}{2X}(1 + X - \sqrt{(1 + 3X)(1 - X)}). \tag{2.34}$$

We can now verify that S obtained this way satisfies equations (2.21) and (2.27). Indeed, since CS is a function of X, and since $X (\equiv CV^{11})$ commutes with CV^{rs}, CS also commutes with CV^{rs}. Furthermore, since V^{11} is twist invariant, so is X. It then follows that the inverse of X and any polynomial

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[10]Of the two other solutions, $T = 1$ gives the identity state $|\mathcal{I}_m\rangle$, whereas the third solution has diverging eigenvalues and hence is badly behaved.

in X are twist invariant. Therefore T is twist invariant, and, as desired, S is twist invariant.

Demanding that the normalization factors in $|\Psi_m\rangle$ and $|\Psi_m * \Psi_m\rangle$ match gives

$$\mathcal{N} = \det(1 - \Sigma\mathcal{V})^{1/2} = (\det(1 - X)\det(1 + T))^{1/2}, \qquad (2.35)$$

where we have used eqn.(2.32) and simplified it further using (2.33). Thus the solution is given by

$$|\Psi_m\rangle = \{\det(1 - X)^{1/2}\det(1 + T)^{1/2}\}^{26} \exp(-\frac{1}{2}\eta_{\mu\nu}\sum_{m,n\geq 1} S_{mn} a_m^{\mu\dagger} a_n^{\nu\dagger})|0\rangle. \tag{2.36}$$

This is the matter part of the state found in ref.[16] (referred to as $|\mathcal{T}_m\rangle$ in the introduction) after suitable correction to the normalization factor. From eq.(2.7) we see that the value of the action associated with this solution has the form:

$$\mathcal{S}|_\Psi = K\,\mathcal{N}^{52} \langle 0| \exp(-\frac{1}{2}\eta_{\mu'\nu'}\sum_{m',n'\geq 1} S_{m'n'} a_{m'}^{\mu'} a_{n'}^{\nu'}) \exp(-\frac{1}{2}\eta_{\mu\nu}\sum_{m,n\geq 1} S_{mn} a_m^{\mu\dagger} a_n^{\nu\dagger})|0\rangle. \tag{2.37}$$

By evaluating the matrix element using eq.(2.25), and using the normalization:

$$\langle 0|0\rangle = \delta^{(26)}(0) = \frac{V^{(26)}}{(2\pi)^{26}}, \qquad (2.38)$$

where $V^{(26)}$ is the volume of the 26-dimensional space-time, we get the value of the action to be

$$\begin{aligned}
\mathcal{S}|_\Psi &= K\,\frac{V^{(26)}}{(2\pi)^{26}}\,\mathcal{N}^{52}\{\det(1 - S^2)^{-1/2}\}^{26} \\
&= K\,\frac{V^{(26)}}{(2\pi)^{26}}\,\{\det(1 - X)^{3/4}\det(1 + 3X)^{1/4}\}^{26}. \qquad (2.39)
\end{aligned}$$

In arriving at the right hand side of eq.(2.39) we have made use of eqs.(2.34) and (2.35). Thus the tension of the D25-brane is given by

$$\mathcal{T}_{25} = K\,\frac{1}{(2\pi)^{26}}\,\{\det(1 - X)^{3/4}\det(1 + 3X)^{1/4}\}^{26}. \qquad (2.40)$$

2.3   Identification with the sliver state

In [10] a family of surface states was constructed corresponding to once
punctured disks with a special kind of local coordinates. They were called
wedge states because the half-disk representing the local coordinates could
be viewed as a wedge of the full unit disk. The puncture was on the boundary
and the wedge has an angle $360°/n$ at the origin, where n is an integer. A
complete description of the state $|n\rangle$ is provided by the fact that for any
state $|\phi\rangle$,[11]

$$\langle n|\phi\rangle = \langle f_n \circ \phi(0)\rangle\,, \tag{2.41}$$

where $f_n \circ \phi(z)$ denotes the conformal transform of $\phi(z)$ by the map

$$f_n(z) = \frac{n}{2}\tan\left(\frac{2}{n}\tan^{-1}(z)\right). \tag{2.42}$$

In the $n \to \infty$ limit this reduces to

$$f(z) \equiv f_\infty(z) = \tan^{-1}(z)\,. \tag{2.43}$$

It was found in [10] that the states $|n\rangle$ can be written in terms of the full
Virasoro operators as:

$$\begin{aligned}
\left|n\right\rangle = {}&\exp\Big(-\frac{n^2-4}{3n^2}L_{-2} + \frac{n^4-16}{30n^4}L_{-4} \\
&-\frac{(n^2-4)(176+128n^2+11n^4)}{1890n^6}L_{-6} + \cdots\Big)|0\rangle\,.
\end{aligned} \tag{2.44}$$

For $n = 1$ the state reduces to the identity string field: $|n = 1\rangle = |\mathcal{I}\rangle$. For
$n = 2$ we get the vacuum: $|n = 2\rangle = |0\rangle$. For $n \to \infty$, which corresponds
to a vanishingly thin wedge state, and will be called the sliver state $|\Xi\rangle$, we
find a smooth limit

$$|\Xi\rangle \equiv |\infty\rangle = \exp\Big(-\frac{1}{3}L_{-2} + \frac{1}{30}L_{-4} - \frac{11}{1890}L_{-6} + \frac{34}{467775}L_{-8} + \cdots\Big)|0\rangle\,. \tag{2.45}$$

It was also shown in [10] that

$$|n\rangle * |m\rangle = |n + m - 1\rangle\,. \tag{2.46}$$

Thus the state $|\Xi\rangle$ has the property that $|n\rangle * |\Xi\rangle = |\Xi\rangle$ for any $n \geq 1$.
In particular, $|\Xi\rangle$ squares to itself. Some properties of this state have been
discussed in appendix D.

---

[11] For brevity, we have modified the notation of [10]. The states $|\frac{360°}{n}\rangle$ are now simply
called $|n\rangle$. Also we have included an extra scaling by $n/2$ in the definition (2.42) of the
conformal map f_n compared to ref.[10]. This does not affect the definition of $|n\rangle$ due to
SL(2,R) invariance of the correlation functions.

L	S_{11}	S_{22}	S_{13}	S_{33}	S_{24}	S_{44}
20	0.2888	−0.0627	−0.1263	0.0706	0.0440	−0.0347
40	0.2970	−0.0638	−0.1316	0.0740	0.0451	−0.0358
80	0.3033	−0.0647	−0.1356	0.0766	0.0459	−0.0365
160	0.3081	−0.0652	−0.1387	0.0785	0.0465	−0.0371
∞	0.3419	−0.0665	−0.1588	0.0905	0.0476	−0.0382

Table 1: Numerical results for the elements of the matrix S. We compute S by restricting the indices m, n of V_{mn}^{11} and C_{mn} to be $\leq L$ so that V^{11} and C are $L \times L$ matrices, and then using eq.(2.34). The last row shows the interpolation of the various results to $L = \infty$, obtained via a fitting function of the form $a_0 + a_1/\ln(L) + a_2/(\ln(L))^2 + a_3/(\ln(L))^3$.

Given the split $L = L^m + L^g$ of the Virasoro operators into commuting Virasoro subalgebras, the state $|\Xi\rangle$ can be written in factorized form: an exponential of matter Virasoros, and an identical exponential of ghost Virasoros:

$$|\Xi\rangle = |\Xi_m\rangle \otimes |\Xi_g\rangle. \tag{2.47}$$

In particular, it follows from (2.45) that

$$|\Xi_m\rangle = \widehat{\mathcal{N}}^{26} \exp\left(-\frac{1}{3}L_{-2}^m + \frac{1}{30}L_{-4}^m - \frac{11}{1890}L_{-6}^m + \cdots\right)|0\rangle, \tag{2.48}$$

where $\widehat{\mathcal{N}}$ is a normalization factor to be fixed shortly. The property $|\Xi\rangle *$ $|\Xi\rangle = |\Xi\rangle$ implies that

$$|\Xi_m\rangle *^m |\Xi_m\rangle = \lambda \widehat{\mathcal{N}}^{52}|\Xi_m\rangle, \tag{2.49}$$

where the constant λ could possibly be vanishingly small or infinite. We shall choose $\widehat{\mathcal{N}}$ such that $\lambda\widehat{\mathcal{N}}^{52} = 1$, so that $|\Xi_m\rangle$ squares to itself.

In order to show that this sliver state is the matter state identified in the previous subsection for the D25-brane, we must compare (2.48) with

$$|\Psi_m\rangle = \mathcal{N}^{26} \exp\left(-\frac{1}{2}\eta_{\mu\nu}a^{\mu\dagger} \cdot S \cdot a^{\nu\dagger}\right)|0\rangle, \tag{2.50}$$

where S is the matrix calculated in the previous subsection. Since the Virasoro operators contain both positively and negatively moded oscillators

a comparison requires expansion. While we have done this as a check, the techniques of ref.[28] enable one to use the local coordinate (2.43) to give a direct oscillator construction of the sliver state

$$|\Xi_m\rangle = \widehat{\mathcal{N}}^{26} \exp(-\frac{1}{2}\eta_{\mu\nu}a^{\mu\dagger} \cdot \widehat{S} \cdot a^{\nu\dagger})\,, \qquad (2.51)$$

where,

$$\widehat{S}_{mn} = -\frac{1}{\sqrt{mn}} \oint \frac{dw}{2\pi i} \oint \frac{dz}{2\pi i} \frac{1}{z^n w^m (1+z^2)(1+w^2)(\tan^{-1}(z) - \tan^{-1}(w))^2}\,. \qquad (2.52)$$

$\oint$ denotes a contour integration around the origin. As required by twist invariance, $\widehat{S}_{mn}$ vanishes when $m+n$ is odd. Explicit computations give:

$$\widehat{S}_{11} = \frac{1}{3} \simeq .3333\,, \qquad \widehat{S}_{22} = -\frac{1}{15} \simeq -.0667\,, \qquad \widehat{S}_{13} = -\frac{4}{15\sqrt{3}} \simeq -.1540\,,$$

$$\widehat{S}_{33} = \frac{83}{945} \simeq .0878\,, \qquad \widehat{S}_{24} = \frac{32\sqrt{2}}{945} \simeq .0479\,, \qquad \widehat{S}_{44} = -\frac{109}{2835} \simeq -.0384\,. \qquad (2.53)$$

On the other hand a level expansion computation for S_{mn}, together with a fit, has been shown in table 1. The data shows rather remarkable agreement between S_{mn} and $\widehat{S}_{mn}$. The errors are of the order of 3%. Once S_{mn} and $\widehat{S}_{mn}$ agree, the normalization factors must agree as well, since both states square to themselves under $*^m$-product. This is convincing evidence that the matter part of the state representing the D25-brane solution is identical to the matter part $|\Xi_m\rangle$ of the sliver state up to an overall normalization.

3   Construction of the lump solutions

In this section we shall discuss the construction of lump solutions of eq.(2.5) representing lower dimensional D-branes. We are able to give these solutions in closed form and to express the ratio of tensions of branes of different dimensions in terms of determinants of infinite dimensional matrices. Numerical calculation of these ratios in the level expansion gives remarkable agreement with the expected values.

3.1   Lump solutions and their tensions

We begin by noting that the solution (2.36) representing the D25 brane has the form of a product over 26 factors, each involving the oscillators associated

with a given direction. This suggests that in order to construct a solution of codimension k representing a D-$(25-k)$-brane, we need to replace k of the factors associated with directions transverse to the D-brane by a different set of solutions, but the factors associated with directions tangential to the D-brane remains the same. (This is precisely what happens in the case of p-adic string theory [29, 30, 31], background independent open string field theory [32, 33, 34, 35, 36, 37, 38, 39, 40], as well as non-commutative solitons [41, 42, 43].) A procedure for constructing such space(-time) dependent solutions was given in ref.[16]. Suppose we are interested in a D-$(25-k)$ brane solution. Let us denote by $x^{\bar{\mu}}$ ($0 \leq \bar{\mu} \leq (25-k)$) the directions tangential to the brane and by x^{α} ($(26-k) \leq \alpha \leq 25$) the directions transverse to the brane. We now use the representation of the vertex in the zero mode oscillator basis for the directions x^{α}, as given in appendix B. For this we define, for each string,

$$a_0^{\alpha} = \frac{1}{2}\sqrt{b}\,\hat{p}^{\alpha} - \frac{1}{\sqrt{b}}i\hat{x}^{\alpha}, \qquad a_0^{\alpha\dagger} = \frac{1}{2}\sqrt{b}\,\hat{p}^{\alpha} + \frac{1}{\sqrt{b}}i\hat{x}^{\alpha}, \qquad (3.1)$$

where b is an arbitrary constant, and $\hat{x}^{\alpha}$ and $\hat{p}^{\alpha}$ are the zero mode coordinate and momentum operators associated with the direction x^{α}. We also denote by $|\Omega_b\rangle$ the normalized state which is annihilated by all the annihilation operators a_0^{α}, and by $|\Omega_b\rangle_{123}$ the direct product of the vacuum $|\Omega_b\rangle$ for each of the three strings. As shown in appendix B (eq.(B.6)), the vertex $|V_3\rangle$ defined in eq.(2.10) can be rewritten in this new basis as:

$$
\begin{aligned}
|V_3\rangle \;=\; & \int d^{26-k}p_{(1)}d^{26-k}p_{(2)}d^{26-k}p_{(3)}\delta^{(26-k)}(p_{(1)}+p_{(2)}+p_{(3)}) \\
& \exp\Big(-\frac{1}{2}\sum_{\substack{r,s \\ m,n\geq 1}}\eta_{\bar{\mu}\bar{\nu}}a_m^{(r)\bar{\mu}\dagger}V_{mn}^{rs}a_n^{(s)\bar{\nu}\dagger} - \sum_{\substack{r,s \\ n\geq 1}}\eta_{\bar{\mu}\bar{\nu}}p_{(r)}^{\bar{\mu}}V_{0n}^{rs}a_n^{(s)\bar{\nu}\dagger} \qquad (3.2) \\
& \qquad -\frac{1}{2}\sum_{r}\eta_{\bar{\mu}\bar{\nu}}p_{(r)}^{\bar{\mu}}V_{00}^{rr}p_{(r)}^{\bar{\nu}}\Big)|0,p\rangle_{123} \\
& \otimes\Big(\frac{\sqrt{3}}{(2\pi b^3)^{1/4}}(V_{00}^{rr}+\frac{b}{2})\Big)^{-k}\exp\Big(-\frac{1}{2}\sum_{\substack{r,s \\ m,n\geq 0}}a_m^{(r)\alpha\dagger}V_{mn}^{\prime rs}a_n^{(s)\alpha\dagger}\Big)|\Omega_b\rangle_{123}\,.
\end{aligned}
$$

In this expression the sums over $\bar{\mu},\bar{\nu}$ run from 0 to $(25-k)$, and sum over α runs from $(26-k)$ to 25. Note that in the last line the sums over m, n run over 0, 1, 2 .... The coefficients $V_{mn}^{\prime rs}$ have been given in terms of V_{mn}^{rs} in eq.(B.7).

In Appendix B it is shown that $V^{\prime rs}$, regarded as matrices with indices running from 0 to ∞, satisfy (see (B.19) and (B.21))

$$V^{\prime rs} = \frac{1}{3}(C' + \omega^{s-r}U' + \omega^{r-s}\bar{U}'), \qquad (3.3)$$

where we have dropped the explicit b dependence from the notation, $C'_{mn} = (-1)^m \delta_{mn}$ with indices m, n now running from 0 to ∞, and U', $\bar{U}' \equiv U'^*$ viewed as matrices with $m, n \geq 0$ satisfy the relations:

$$\bar{U}' = C'U'C', \qquad U'^2 = \bar{U}'^2 = 1, \qquad U'^\dagger = U'. \tag{3.4}$$

We note now the complete analogy with equations (2.15) and (2.17) [16]. It follows also that the V' matrices, together with C' will satisfy equations exactly analogous to (2.18), (2.19). Thus we can construct a solution of the equations of motion (2.5) in an identical manner with the unprimed quantities replaced by the primed quantities. Taking into account the extra normalization factor appearing in the last line of eq.(3.2), we get the following form of the solution of eq.(2.5):

$$|\Psi'_m\rangle = \{\det(1-X)^{1/2} \det(1+T)^{1/2}\}^{26-k} \exp(-\frac{1}{2}\eta_{\bar\mu\bar\nu} \sum_{m,n\geq 1} S_{mn} a_m^{\bar\mu\dagger} a_n^{\bar\nu\dagger})|0\rangle$$

$$\otimes \left(\frac{\sqrt{3}}{(2\pi b^3)^{1/4}}(V_{00}^{rr} + \frac{b}{2})\right)^k \{\det(1-X')^{1/2} \det(1+T')^{1/2}\}^k$$

$$\exp\left(-\frac{1}{2}\sum_{m,n\geq 0} S'_{mn} a_m^{\alpha\dagger} a_n^{\alpha\dagger}\right)|\Omega_b\rangle, \tag{3.5}$$

where

$$S' = C'T', \qquad T' = \frac{1}{2X'}(1 + X' - \sqrt{(1+3X')(1-X')}), \tag{3.6}$$

$$X' = C'V'^{11}. \tag{3.7}$$

Using eq.(2.7) we can calculate the value of the action associated with this solution. It is given by an equation analogous to (2.39):

$$\mathcal{S}_{\Psi'} = K \frac{V^{(26-k)}}{(2\pi)^{26-k}} \{\det(1-X)^{3/4} \det(1+3X)^{1/4}\}^{26-k} \tag{3.8}$$

$$\times \left(\frac{3}{(2\pi b^3)^{1/2}}(V_{00}^{rr} + \frac{b}{2})^2\right)^k \{\det(1-X')^{3/4} \det(1+3X')^{1/4}\}^k,$$

where $V^{(26-k)}$ is the D-$(25-k)$-brane world-volume. This gives the tension of the D-$(25-k)$-brane to be

$$\mathcal{T}_{25-k} = K \frac{1}{(2\pi)^{26-k}} \{\det(1-X)^{3/4} \det(1+3X)^{1/4}\}^{26-k} \tag{3.9}$$

$$\times \left(\frac{3}{(2\pi b^3)^{1/2}}(V_{00}^{rr} + \frac{b}{2})^2\right)^k \{\det(1-X')^{3/4} \det(1+3X')^{1/4}\}^k.$$

L	$\frac{\mathcal{T}_k}{2\pi\mathcal{T}_{k+1}}$ for $b = .5$	$\frac{\mathcal{T}_k}{2\pi\mathcal{T}_{k+1}}$ for $b = 1$	$\frac{\mathcal{T}_k}{2\pi\mathcal{T}_{k+1}}$ for $b = 2$	$\frac{\mathcal{T}_k}{2\pi\mathcal{T}_{k+1}}$ for $b = 4$	$\frac{\mathcal{T}_k}{2\pi\mathcal{T}_{k+1}}$ for $b = 8$	$(2\pi) \cdot \left(\frac{\mathcal{T}_{25}}{K}\right)^{\frac{1}{26}}$
0	1.38117	1.08677	.96195	.94748	1.00992	1
50	.94968	.95490	.96518	.98485	1.02101	.69705
100	.95488	.95959	.96884	.98659	1.01946	.64875
200	.95906	.96333	.97172	.98789	1.01802	.60244
400	.96250	.96640	.97408	.98891	1.01672	.55839
800	.96539	.96898	.97606	.98976	1.01558	.51718
1600	.96785	.97118	.97775	.99048	1.01456	.48082
3200	.97007	.97316	.97927	.99113	1.01361	.44664
∞	1.00042	1.00061	1.00063	1.00031	.99939	-.12638

Table 2: Numerical results for the ratio $\frac{\mathcal{T}_k}{2\pi\mathcal{T}_{k+1}}$. The first column shows the level up to which we calculate the matrices V_{mn}^{rs} and $V_{mn}'^{rs}$. The second to sixth column shows the ratio $\frac{\mathcal{T}_k}{2\pi\mathcal{T}_{k+1}}$ for different values of the parameter b. The last column gives $(2\pi)^{26}\mathcal{T}_{25}/K$. The last row shows the interpolation of the various results to $L = \infty$, obtained via a fitting function of the form $a_0 + a_1/\ln(L) + a_2/(\ln(L))^2 + a_3/(\ln(L))^3$.

Clearly for $k = 0$ this agrees with (2.40). From eq.(3.9) we get

$$\frac{\mathcal{T}_{24-k}}{2\pi\mathcal{T}_{25-k}} = \frac{3}{\sqrt{2\pi b^3}}\left(V_{00}^{rr} + \frac{b}{2}\right)^2 \frac{\{\det(1 - X')^{3/4}\det(1 + 3X')^{1/4}\}}{\{\det(1 - X)^{3/4}\det(1 + 3X)^{1/4}\}}. \quad (3.10)$$

This ratio can be calculated if we restrict m, n to be below a given level L, so that $X = CV^{11}$ is an $L \times L$ matrix and $X' = C'V'^{11}$ is an $(L+1) \times (L+1)$ matrix. The values of V^{11} and V'^{11} can be found from eqs.(A.3) and (B.7). In particular for $L = 0$ only the matrix X' contributes. From eq.(3.7), (B.7) and (A.3) we get

$$V_{00}^{11} = \ln\frac{27}{16}, \qquad X_{00}' = V_{00}'^{11} = 1 - \frac{2}{3}\frac{b}{\ln(27/16) + \frac{b}{2}}. \quad (3.11)$$

Thus in the $L = 0$ approximation, the ratio (3.10) is given by

$$\frac{3}{\sqrt{2\pi}}\left(\frac{2}{3}\right)^{3/4}(4\ln(27/16))^{1/4}\left(\ln(27/16) + \frac{b}{2}\right)b^{-3/4}. \qquad (3.12)$$

For larger values of L the ratio is calculated numerically. The results of the numerical analysis are given in table 2. As seen from this table, the ratio $\mathcal{T}_k/(2\pi\mathcal{T}_{k-1})$ approaches 1 as $L \to \infty$ for all b. This is exactly what is expected if the lump solutions discussed here describe lower dimensional D-branes. It is also seen from the table that $\mathcal{T}_{25}/K$ extrapolated to $L = \infty$ gives a negative number. We take this as an evidence that it approaches 0 as $L \to \infty$. This indicates that the matter component $|\Psi_m\rangle$ of the string field has zero norm. We expect that this will be compensated by the ghost sector contribution K, so that the contribution to the action from the full string field approaches a finite limit as we take the level of approximation L to ∞.

As in the previous section (footnote 10), one can construct two other solutions to eq.(2.5). For one of them T' is the inverse of the solution for T' given in eq.(3.6). For this solution the eigenvalues of T' diverge and so the state is not well behaved. The other solution corresponds to $T'_{mn} = \delta_{mn}$, *i.e.* $S'_{mn} = C'_{mn}$. Using eq.(B.5) for $p^\alpha = 0$ to go from oscillator basis to the momentum basis, one can easily verify that this again is the identity string state $|\mathcal{I}_m\rangle$. Thus we do not get a new solution.

3.2   *b*-dependence of the solution

The analysis of the last section generates a one-parameter family of lump solutions characterized by the parameter b.[12] Thus we are now faced with an embarrassment of riches, − for these solutions to have the interpretation as D-branes there should be a unique solution (up the possibility of translating the solution in the transverse direction) and not a family of solutions. There are two possibilities that come to mind.

1. Although in the oscillator basis the solution seems to depend on b, the relationship between the oscillator and the momentum basis is b-dependent, and when we rewrite the solution in the momentum basis it is actually b-independent.

2. Even after rewriting the solution in the momentum basis it is b-dependent,

---

[12]Actually for a codimension k lump we have a k parameter family of solutions since we can choose different parameters b corresponding to different directions.

L	S'_{00} for $b=1$	S'_{00} for $b=2$	S'_{00} for $b=4$	S'_{00} for $b=6$	S'_{00} for $b=8$
20	.25980	.08001	$-.10718$	$-.21475$	$-.28836$
40	.25796	.07692	$-.11183$	$-.22034$	$-.29458$
60	.25711	.07548	$-.11403$	$-.22302$	$-.29758$
80	.25659	.07460	$-.11539$	$-.22469$	$-.29947$
100	.25623	.07398	$-.11636$	$-.22588$	$-.30081$
120	.25596	.07352	$-.11709$	$-.22678$	$-.30184$
∞	.2497	.0619	$-.1372$	$-.2534$	$-.3333$

Table 3: Numerical results for S'_{00}. The first column shows the level up to which we calculate the matrices V^{rs}_{mn} and V'^{rs}_{mn}. The second to sixth column shows S'_{00} for different values of the parameter b. The last row shows the interpolation of the various results to $L = \infty$, obtained via a fitting function of the form $a_0 + a_1/\ln(L) + a_2/(\ln(L))^2 + a_3/(\ln(L))^3$.

but the solutions for different values of b are related to each other by gauge transformation.

We shall begin by exploring the first possibility. In order to get basis independent information about the lump solution, we can calculate its inner product with states in the momentum basis. Let us, for example, consider the inner product $\langle \{p^\alpha\} | \Psi'_m \rangle$. Using eqs.(B.5), (3.5) and (2.25) we get

$$\langle \{p^\alpha\} | \Psi'_m \rangle \propto \exp\left(-\frac{p^2}{2}\left(\frac{b}{2} + b\frac{S'_{00}}{1 - S'_{00}}\right) \right), \qquad p^2 = p^\alpha p^\alpha. \tag{3.13}$$

The numerical results for the values of S'_{00} for different values of b are shown in table 3, and the values of $b/2 + bS'_{00}/(1 - S'_{00})$ have been shown in table 4. From this we see clearly that $\langle \{p^\alpha\} | \Psi'_m \rangle$ is not independent of b.

This brings us to the second possibility: could the different solutions be related by gauge transformation? Since the different solutions have the same ghost component, such a gauge transformation must be of a special kind that changes the matter part but not the ghost part. So the question

b	1	2	4	6	8
$b/2 + bS'_{00}/(1 - S'_{00})$	.833	1.132	1.517	1.787	2.000

Table 4: Numerical results for $b/2 + bS'_{00}/(1 - S'_{00})$ for different values of b. We use the results for S'_{00} given in the last row of table 3.

is: are such gauge transformations possible? If we choose the gauge transformation parameter $|\Lambda\rangle$ to be of the form $|\Lambda_g\rangle \otimes |\Lambda_m\rangle$, then, under a gauge transformation

$$\delta(|\Psi_g\rangle \otimes |\Psi_m\rangle) \qquad (3.14)$$
$$= \mathcal{Q}|\Lambda_g\rangle \otimes |\Lambda_m\rangle + |\Psi_g *^g \Lambda_g\rangle \otimes |\Psi_m *^m \Lambda_m\rangle - |\Lambda_g *^g \Psi_g\rangle \otimes |\Lambda_m *^m \Psi_m\rangle.$$

Now suppose $|\Lambda_g\rangle$ is such that

$$\mathcal{Q}|\Lambda_g\rangle = 0, \qquad |\Lambda_g *^g \Psi_g\rangle = |\Psi_g *^g \Lambda_g\rangle = |\Psi_g\rangle. \qquad (3.15)$$

In that case (3.14) can be written as

$$\delta(|\Psi_g\rangle \otimes |\Psi_m\rangle) = |\Psi_g\rangle \otimes (|\Psi_m *^m \Lambda_m\rangle - |\Lambda_m *^m \Psi_m\rangle). \qquad (3.16)$$

Thus effectively the gauge transformation induces a transformation on the matter part of the solution without any transformation on the ghost sector. It is our guess that solutions with different values of b are related by gauge transformations of this kind. Although we do not have a complete proof of this, a partial analysis of this problem has been carried out in appendix C. If this is indeed true, then this will imply that the width of the solution in the position space, given by $\sqrt{b/2 + bS'_{00}/(1 - S'_{00})}$, is a gauge dependent quantity.

Without having detailed knowledge of the operator $\mathcal{Q}$ we cannot know whether there is some ghost number zero state $|\Lambda_g\rangle$ in the ghost sector satisfying eq.(3.15). Note however that if $\mathcal{Q}$ annihilates the identity $|\mathcal{I}\rangle$ of the $*$ product then taking $|\Lambda_g\rangle = |\mathcal{I}_g\rangle$, where $|\mathcal{I}_g\rangle$ denotes the component of $|\mathcal{I}\rangle$ in the ghost sector, automatically satisfies eq.(3.15). On the other hand since eq.(3.15) needs to be satisfied only for a special $|\Psi_g\rangle$ which represents D-brane solutions, there may be other $|\Lambda_g\rangle$ satisfying these equation.

Note that even if we did not discover the existence of multiple solutions labeled by different values of b, we would still have an embarrassment of riches if there were no $|\Lambda_g\rangle$ satisfying eq.(3.15). This is due to the fact that

given any solution of eq.(2.5), we can generate other solutions by deforming $|\Psi_m\rangle$ as follows:

$$\delta|\Psi_m\rangle = |\Psi_m *^m \Lambda_m\rangle - |\Lambda_m *^m \Psi_m\rangle. \qquad (3.17)$$

In order to make sense of these solutions, we must show that when we combine them with $|\Psi_g\rangle$ to construct solutions of the full string field theory equations of motion, they are related by gauge transformations. The postulate of existence of a $|\Lambda_g\rangle$ satisfying eq.(3.15) makes this possible.

4   Open questions

Clearly many questions remain unanswered. In this concluding section we shall try to make a list of questions which we hope will be answered in the near future.

1. The most pressing question at this time seems to be understanding the ghost sector of the solution. We expect that a proper analysis of the ghost sector will not only lead to the solution, but will also fix uniquely (up to the field redefinition ambiguity) the form of the kinetic operator $\mathcal{Q}$. Since the matter part of the D25-brane solution is given by the matter part Ξ_m of the sliver state, our guess is that the full solution is given by a ghost number one operator built purely out of ghosts acting on the full sliver state Ξ.

2. For the D-branes of dimension < 25, we have found families of candidate solutions labeled by the parameter b. Since a physical D-brane does not admit continuous deformations other than the translational motion transverse to the brane, we need to show that these additional deformations are gauge artifacts. We have given some arguments to this effect in appendix C, but a complete proof is lacking.

3. Although we have shown that the ratios of tensions of our solutions agree very well with the expected answer, in order to establish conclusively that these solutions describe D-branes, we need to analyze the fluctuations of the string field around these solutions and show that the spectrum and interaction of these fluctuations agree with those of conventional open strings living on the D-brane. Clearly, the knowledge of $\mathcal{Q}$ is crucial for this study.

4. Although the matter parts of our solutions are given in analytic form, calculation of the ratios of tensions of these solutions, involving computation of determinants of infinite dimensional matrices, was done

numerically. It will be nice to have an analytic expression for this ratio.

5. If our solution really describes D-branes, then we expect that there should be static multiple lump solutions representing multiple D-branes. One should be able to construct such solutions in our string field theory. It is natural to assume that these multi-lump solutions will also have factorized form, with the ghost part being described by the same universal state as the single lump solutions. Thus this analysis can be carried out without a detailed knowledge of $\mathcal{Q}$.

6. The procedure that we have followed to construct lower dimensional branes from the D25 brane solution bears a suggestive formal similarity with the solution generating techniques which have appeared recently in studies of non-commutative solitons and have been conjectured to be relevant to string field theory [41, 44, 45]. In that context, new space-time dependent solutions are obtained by acting on a translational invariant solution with "non-unitary isometries", like the "shift operator" in an infinite dimensional Hilbert space. The construction of the matrix T' from T is quite reminiscent of some sort of shift operation. It would be interesting to investigate this connection precisely.

A   The coefficients V_{mn}^{rs}

In this appendix we give the coefficients V_{mn}^{rs} introduced in the text. These results are taken from refs.[25, 26]. First we define the coefficients A_n and B_n for $n \geq 0$ through the relations:

$$\left(\frac{1+ix}{1-ix}\right)^{1/3} = \sum_{n\,even} A_n x^n + i \sum_{n\,odd} A_n x^n\,, \quad \left(\frac{1+ix}{1-ix}\right)^{2/3} = \sum_{n\,even} B_n x^n + i \sum_{n\,odd} B_n x^n\,.$$

$$(\mathrm{A.1})$$

In terms of A_n and B_n we define the coefficients $N_{mn}^{r,\pm s}$ as follows:

$$N_{nm}^{r,\pm r} = \frac{1}{3(n \pm m)} (-1)^n (A_n B_m \pm B_n A_m) \quad \text{for} \quad m + n \text{ even}, \ m \neq n,$$

$$= 0 \qquad\qquad \text{for} \quad m + n \text{ odd},$$

$$N_{nm}^{r,\pm(r+1)} = \frac{1}{6(n \pm m)} (-1)^{n+1} (A_n B_m \pm B_n A_m) \quad \text{for} \quad m + n \text{ even}, \ m \neq n,$$

$$= \frac{1}{6(n \pm m)} \sqrt{3} (A_n B_m \mp B_n A_m) \quad \text{for} \quad m + n \text{ odd},$$

$$N_{nm}^{r,\pm(r-1)} = \frac{1}{6(n \mp m)} (-1)^{n+1} (A_n B_m \mp B_n A_m) \quad \text{for} \quad m + n \text{ even}, \ m \neq n,$$

$$= -\frac{1}{6(n \mp m)} \sqrt{3} (A_n B_m \pm B_n A_m) \quad \text{for} \quad m + n \text{ odd}. \qquad (A.2)$$

The coefficients V_{mn}^{rs} are then given by

$$V_{nm}^{rs} = -\sqrt{mn} (N_{nm}^{r,s} + N_{nm}^{r,-s}) \quad \text{for} \quad m \neq n, \ m, n \neq 0,$$

$$V_{nn}^{rr} = -\frac{1}{3}[2 \sum_{k=0}^{n} (-1)^{n-k} A_k^2 - (-1)^n - A_n^2], \quad \text{for} \quad n \neq 0,$$

$$V_{nn}^{r(r+1)} = V_{nn}^{r(r+2)} = \frac{1}{2}[(-1)^n - V_{nn}^{rr}] \quad \text{for} \quad n \neq 0,$$

$$V_{0n}^{rs} = -\sqrt{2n} (N_{0n}^{r,s} + N_{0n}^{r,-s}) \quad \text{for} \quad n \neq 0,$$

$$V_{00}^{rr} = \ln(27/16). \qquad (A.3)$$

The value of V_{nn}^{rr} quoted above corrects the result for $N_{nn}^{rr}(\equiv -V_{nn}^{rr}/n)$ quoted in eqn.(1.18) of [26]. In writing down the expressions for V_{0n}^{rs} and V_{00}^{rr} we have taken into account the fact that we are using $\alpha' = 1$ convention, as opposed to the $\alpha' = 1/2$ convention used in refs.[25, 26].

Finally we would like to point out that our convention for $|A *^m B\rangle$, defined through eq.(2.9), differs from that in refs.[25, 26]. In particular, with the values of V_{mn}^{rs} give in eq.(A.3), our $|A *^m B\rangle$ would correspond to $|B *^m A\rangle$ in ref.[25]. Since the string field equation of motion involves $|\Psi * \Psi\rangle$, it is not affected by this difference in convention. However, if we want our convention for $|A *^m B\rangle$ to agree with that of ref.[25], we should replace V^{rs} by V^{sr} everywhere in eqs.(A.3).

B   Conversion from momentum to oscillator basis

We start with the three string vertex in the matter sector as given in section (2.2):

$$|V_3\rangle = \int d^{26}p_{(1)}d^{26}p_{(2)}d^{26}p_{(3)}\delta^{(26)}(p_{(1)} + p_{(2)} + p_{(3)}) \exp(-E)|0, p\rangle_{123} \tag{B.1}$$

where

$$E = \frac{1}{2} \sum_{\substack{r,s \\ m,n \geq 1}} \eta_{\mu\nu} a_m^{(r)\mu\dagger} V_{mn}^{rs} a_n^{(s)\nu\dagger} + \sum_{\substack{r,s \\ n \geq 1}} \eta_{\mu\nu} p_{(r)}^{\mu} V_{0n}^{rs} a_n^{(s)\nu\dagger} + \frac{1}{2} \sum_r \eta_{\mu\nu} p_{(r)}^{\mu} V_{00}^{rr} p_{(r)}^{\nu}. \tag{B.2}$$

Note that using the freedom of redefining V_{00}^{rs} using momentum conservation, we have chosen V_{00}^{rs} to be zero for $r \neq s$. Due to the same reason, a redefinition $V_{0n}^{rs} \to V_{0n}^{rs} + A_n^s$ by some r independent constant A_n^s leaves the vertex unchanged. We shall use this freedom to choose:

$$\sum_r V_{0n}^{rs} = 0. \tag{B.3}$$

It can be easily verified that V_{0n}^{rs} given in eq.(A.3) satisfy these conditions.

We now pass to the oscillator basis for a subset of the space-time coordinates x^α ($(26 - k) \leq \alpha \leq 25$), by relating the zero mode operators $\hat{x}^\alpha$ and $\hat{p}^\alpha$ to oscillators a_0^α and $a_0^{\alpha\dagger}$. For this one writes:

$$a_0^\alpha = \frac{1}{2}\sqrt{b}\,\hat{p}^\alpha - \frac{1}{\sqrt{b}}i\hat{x}^\alpha, \qquad a_0^{\alpha\dagger} = \frac{1}{2}\sqrt{b}\,\hat{p}^\alpha + \frac{1}{\sqrt{b}}i\hat{x}^\alpha, \tag{B.4}$$

where b is an arbitrary constant. Then a_0^α, $a_0^{\alpha\dagger}$ satisfy the usual commutation rule $[a_0^\alpha, a_0^{\beta\dagger}] = \delta^{\alpha\beta}$ (we are assuming that the directions x^α are space-like; otherwise we shall need $\eta^{\alpha\beta}$), and we can define a new vacuum state $|\Omega_b\rangle$ such that $a_0^\alpha|\Omega_b\rangle = 0$. The relation between the momentum basis and the new oscillator basis is given by (for each string)

$$|\{p^\alpha\}\rangle = (2\pi/b)^{-k/4} \exp\left[-\frac{b}{4}p^\alpha p^\alpha + \sqrt{b}a_0^{\alpha\dagger}p^\alpha - \frac{1}{2}a_0^{\alpha\dagger}a_0^{\alpha\dagger}\right]|\Omega_b\rangle. \tag{B.5}$$

In the above equation $\{p^\alpha\}$ label momentum eigenvalues. Substituting eq.(B.5) into eq.(B.1), and integrating over $p_{(i)}^\alpha$, we can express the three string vertex

as

$$
|V_3\rangle = \int d^{26-k}p_{(1)} d^{26-k}p_{(2)} d^{26-k}p_{(3)} \delta^{(26-k)}(p_{(1)}+p_{(2)}+p_{(3)})
$$

$$
\exp\Big(-\frac{1}{2}\sum_{\substack{r,s \\ m,n\geq 1}} \eta_{\bar{\mu}\bar{\nu}} a_m^{(r)\bar{\mu}\dagger} V_{mn}^{rs} a_n^{(s)\bar{\nu}\dagger} - \sum_{\substack{r,s \\ n\geq 1}} \eta_{\bar{\mu}\bar{\nu}} p_{(r)}^{\bar{\mu}} V_{0n}^{rs} a_n^{(s)\bar{\nu}\dagger} \qquad (B.6)
$$

$$
-\frac{1}{2}\sum_r \eta_{\bar{\mu}\bar{\nu}} p_{(r)}^{\bar{\mu}} V_{00}^{rr} p_{(r)}^{\bar{\nu}}\Big)|0,p\rangle_{123}
$$

$$
\otimes\left(\frac{\sqrt{3}}{(2\pi b^3)^{1/4}}(V_{00}^{rr}+\frac{b}{2})\right)^{-k} \exp\Big(-\frac{1}{2}\sum_{\substack{r,s \\ m,n\geq 0}} a_m^{(r)\alpha\dagger} V_{mn}^{\prime rs} a_n^{(s)\alpha\dagger}\Big)|\Omega_b\rangle_{123}.
$$

In this expression the sums over $\bar{\mu}, \bar{\nu}$ run from 0 to $(25 - k)$, and the sum over α runs from $(26 - k)$ to 25. Note that in the last line the sums over m, n run over 0, 1, 2 .... The new b-dependent V' coefficients are given in terms of the V coefficients by

$$
V_{mn}^{\prime rs}(b) = V_{mn}^{rs} - \frac{1}{V_{00}^{rr}+\frac{b}{2}} \sum_{t=1}^{3} V_{0m}^{tr} V_{0n}^{ts}, \qquad m,n\geq 1,
$$

$$
V_{0n}^{\prime rs}(b) = V_{n0}^{\prime sr} = \frac{1}{V_{00}^{rr}+\frac{b}{2}} \sqrt{b}\, V_{0n}^{rs}, \qquad n\geq 1,
$$

$$
V_{00}^{\prime rs}(b) = \frac{1}{3}\frac{b}{V_{00}^{rr}+\frac{b}{2}}, \qquad r\neq s,
$$

$$
V_{00}^{\prime rr}(b) = 1 - \frac{2}{3}\frac{b}{V_{00}^{rr}+\frac{b}{2}}. \qquad (B.7)
$$

In deriving the above relations we have used eq.(B.3). These relations can be readily inverted to find

$$
V_{mn}^{rs} = V_{mn}^{\prime rs}(b) + \frac{2}{3}\frac{1}{1-V_{00}^{\prime rr}(b)} \sum_{t=1}^{3} V_{m0}^{\prime rt}(b) V_{0n}^{\prime ts}(b), \qquad m,n\geq 1,
$$

$$
V_{0n}^{rs} = \frac{2}{3}\frac{1}{1-V_{00}^{\prime rr}(b)} \sqrt{b}\, V_{0n}^{\prime rs}(b), \qquad n\geq 1,
$$

$$
V_{00}^{rr} = \frac{b}{6}\frac{1+3V_{00}^{\prime rr}(b)}{1-V_{00}^{\prime rr}(b)}. \qquad (B.8)
$$

We shall now describe how our variables V_{mn}^{rs} and $V_{mn}^{\prime rs}$ are related to the variables introduced in ref.[25]. For this we begin by comparing the variables in the oscillator representation. Since ref.[25] uses the $\alpha' = 1/2$ convention rather than the $\alpha' = 1$ convention used here, every factor of $p\,(x)$ in [25]

should be multiplied (divided) by $\sqrt{2\alpha'}$, and then α' should be set equal to one in order to compare with our equations. With this prescription eqs.(2.5b) of [25] giving $a_0 = \frac{1}{2}\hat{p} - i\hat{x}$ becomes $a_0 = \frac{1}{\sqrt{2}}\hat{p} - \frac{i}{\sqrt{2}}\hat{x}$, which corresponds to our (B.4) for $b = 2$. Thus, we can directly compare our variables with those of [25] for the case $b = 2$.

Ref.[25] introduced a matrix U which appears, for example, in their eq.(2.47). We shall denote this matrix by U^{gj}. This matrix appears in the construction of the vertex in the oscillator basis ([25], eqn.(2.52) and (2.53)). This implies that the V' coefficients for $b = 2$ can be expressed in terms of U^{gj} using their results. In particular, defining V'^{rs} to be the matrices V'^{rs}_{mn} with m, n now running from 0 to ∞, we have (see [25], eqn.(2.53)):[13]

$$V'^{rs}(2) = \frac{1}{3}(C' + \omega^{s-r}U^{gj} + \omega^{r-s}\bar{U}^{gj}), \qquad (B.9)$$

where $\omega = \exp(2\pi i/3)$, $C'_{mn} = (-1)^m \delta_{mn}$ with $m, n \geq 0$, and the matrix U^{gj} satisfies the relations (eq.(2.51) of [25]):

$$U^{gj\,\dagger} = U^{gj}, \quad \bar{U}^{gj} \equiv (U^{gj})^* = C'U^{gj}C', \quad U^{gj}U^{gj} = 1. \qquad (B.10)$$

Eq.(B.9) gives us, $V'^{rr}_{00}(2) = \frac{1}{3}(1 + 2U^{gj}_{00})$. With this result, the last equation in (B.8) can be used with $b = 2$ to find

$$V^{rr}_{00} = \frac{1 + U^{gj}_{00}}{1 - U^{gj}_{00}}. \qquad (B.11)$$

Similarly, the second equation in (B.8) gives:

$$V^{rs}_{0n} = \frac{1}{1 - U^{gj}_{00}}\sqrt{2}\,V'^{rs}_{0n}(2), \qquad \text{for} \quad n \geq 1. \qquad (B.12)$$

Making use of (B.9) and $\bar{U}^{gj}_{0n} = (U^{gj}_{0n})^*$ we find that we can write, for $n \geq 1$:

$$V^{rs}_{0n} = \frac{1}{3}(\omega^{s-r}W_n + \omega^{r-s}W^*_n), \qquad (B.13)$$

where

$$W_n = \frac{\sqrt{2}U^{gj}_{0n}}{1 - U^{gj}_{00}}. \qquad (B.14)$$

---

[13]As explained at the end of appendix A, U^{gj} should really be identified with $\bar{U}$ of ref.[25].

The first equation in (B.8) together with (B.9) gives us [16]

$$V^{rs} = \frac{1}{3}(C + \omega^{s-r}U + \omega^{r-s}\bar{U}),\tag{B.15}$$

where V^{rs}, U and C are regarded as matrices with indices running over $m, n \geq 1$, $C_{mn} = (-1)^m \delta_{mn}$ and U is given as

$$U_{mn} = U_{mn}^{gj} + \frac{U_{m0}^{gj} U_{0n}^{gj}}{1 - U_{00}^{gj}}.\tag{B.16}$$

By virtue of this relation, and the identities in (B.10) we have that the matrix U satisfies

$$\bar{U} \equiv U^* = CUC, \qquad U^2 = \bar{U}^2 = 1, \qquad U^\dagger = U, \quad \bar{U}^\dagger = \bar{U}.\tag{B.17}$$

It follows from (B.10) and (B.14) that W_n satisfies the relations:

$$W_n^* = (-1)^n W_n, \qquad \sum_{n \geq 1} W_n U_{np} = W_p, \qquad \sum_{m \geq 1} W_m^* W_m = 2V_{00}^{rr}.\tag{B.18}$$

Finally, using (B.7), (B.13), (B.15), the coefficients $V'(b)$ for arbitrary b can be made into matrices with $m, n \geq 0$, and, just as for the case $b = 2$ in (B.9), can be written as

$$V'^{rs}(b) = \frac{1}{3}(C' + \omega^{s-r}U' + \omega^{r-s}\bar{U}'),\tag{B.19}$$

where

$$
\begin{aligned}
U'_{00} &= 1 - \frac{b}{V_{00}^{rr} + \frac{b}{2}}, \\
U'_{0n} &= (U'_{n0})^* = \frac{\sqrt{b}}{V_{00}^{rr} + \frac{b}{2}} W_n, \quad n \geq 1, \\
U'_{mn} &= U_{mn} - \frac{W_m^* W_n}{V_{00}^{rr} + \frac{b}{2}} \quad m, n \geq 1.
\end{aligned}
\tag{B.20}
$$

Using eqns.(B.17) and (B.18) one can show that U', $\bar{U}' \equiv U'^*$ viewed as a matrix with $m, n \geq 0$ satisfies the relations:

$$\bar{U}' = C'U'C', \qquad U'^2 = \bar{U}'^2 = 1, \qquad U'^\dagger = U'.\tag{B.21}$$

C   On the b-dependence of the lump solution

In this appendix we shall address the question as to whether the apparent b-dependence of the lump solution given in eq.(3.5) could be a gauge artifact. In order to avoid cluttering up the formulæ we shall focus on the matter part associated with a single direction transverse to the lump:

$$|\Psi'\rangle \equiv \mathcal{N}' \exp(-\frac{1}{2}\sum_{m,n\geq 0} S'_{mn} a^\dagger_m a^\dagger_n)|\Omega_b\rangle\,, \qquad (C.1)$$

where

$$\mathcal{N}' = \left(\frac{\sqrt{3}}{(2\pi b^3)^{1/4}}(V^{rr}_{00}+\frac{b}{2})\right)\{\det(1-X')^{1/2}\det(1+T')^{1/2}\}\,. \qquad (C.2)$$

Since all states under discussion are in the matter sector associated with this single direction, we shall refrain from adding the subscript m to various states and the $*$ operation. The b dependence of the state given above comes from four sources:

1. b dependence of $\mathcal{N}'$ (including implicit b dependence of X' and T' through eqs.(B.7), (3.6), (3.7)).

2. b dependence of S'_{mn} through eqs.(B.7), (3.6), (3.7).

3. b dependence of $a^\dagger_0$ through eq.(3.1). Under an infinitesimal change in b, eq.(3.1) gives

$$\delta a^\dagger_0 = \frac{\delta b}{2b}\,a_0, \qquad \delta a_0 = \frac{\delta b}{2b}\,a^\dagger_0\,. \qquad (C.3)$$

4. b dependence of $|\Omega_b\rangle$ due to the change in the definitions of a_0, $a^\dagger_0$. Requiring that $(a_0 + \delta a_0)$ annihilates $|\Omega_b\rangle + \delta|\Omega_b\rangle$ gives

$$\delta|\Omega_b\rangle = -\frac{\delta b}{4b}(a^\dagger_0)^2|\Omega_b\rangle\,. \qquad (C.4)$$

A straightforward calculation (involving expansion of the exponential to first order in δb using the Baker-Campbell-Hausdorff formula) gives:

$$\delta|\Psi'\rangle = \left(\delta\ln\mathcal{N}' - \frac{1}{4}\frac{\delta b}{b}S'_{00}\right)|\Psi'\rangle - \frac{1}{2}\left\{\delta S'_{mn} + \frac{\delta b}{2b}(-S'_{0m}S'_{0n} + \delta_{0m}\delta_{0n})\right\}a^\dagger_m a^\dagger_n|\Psi'\rangle\,.$$
$$(C.5)$$

We would now like to ask if the expression for $\delta|\Psi'\rangle$ given above can be represented as a gauge transformation of the kind given in eq.(3.16)

$$\delta_{gauge}|\Psi'\rangle = |\Psi' *^m \Lambda\rangle - |\Lambda *^m \Psi'\rangle, \qquad (C.6)$$

for some state $|\Lambda\rangle$ in the matter sector. Eq.(C.5) suggests that we look for a $|\Lambda\rangle$ of the form:

$$|\Lambda\rangle = \Lambda_{mn} a_m^\dagger a_n^\dagger |\Psi'\rangle. \qquad (C.7)$$

(Note that we could have included a term in $|\Lambda\rangle$ proportional to $|\Psi'\rangle$, but this does not contribute to the gauge transformation of $|\Psi'\rangle$.) Using eq.(2.9) and the general formula (2.25) together with the identity

$$\langle 0| \exp\left(\lambda_i a_i - \frac{1}{2} P_{ij} a_i a_j\right) a_p a_q \exp\left(\mu_i a_i^\dagger - \frac{1}{2} Q_{ij} a_i^\dagger a_j^\dagger\right)|0\rangle \qquad (C.8)$$

$$= \frac{\partial^2}{\partial\lambda_p \partial\lambda_q}\left(\langle 0| \exp\left(\lambda_i a_i - \frac{1}{2} P_{ij} a_i a_j\right) \exp\left(\mu_i a_i^\dagger - \frac{1}{2} Q_{ij} a_i^\dagger a_j^\dagger\right)|0\rangle\right),$$

we can show that

$$\delta_{gauge}|\Psi'\rangle = -Tr(\mathcal{B}\mathcal{V}'(1 - \Sigma'\mathcal{V}')^{-1})|\Psi'\rangle \qquad (C.9)$$

$$+ \left\{ (V'^{31}, V'^{32})(1 - \Sigma'\mathcal{V}')^{-1}\mathcal{B}(1 - \mathcal{V}'\Sigma')^{-1}\begin{pmatrix} V'^{13} \\ V'^{23} \end{pmatrix}\right\}_{mn} a_m^\dagger a_n^\dagger |\Psi'\rangle,$$

where

$$\mathcal{B} = \begin{pmatrix} -C'\Lambda C' & 0 \\ 0 & C'\Lambda C' \end{pmatrix}, \quad \Sigma' = \begin{pmatrix} S' & 0 \\ 0 & S' \end{pmatrix}, \quad \mathcal{V}' = \begin{pmatrix} V'^{11} & V'^{12} \\ V'^{21} & V'^{22} \end{pmatrix}. \qquad (C.10)$$

Thus requiring that $\delta_{gauge}|\Psi'\rangle$ is equal to $\delta|\Psi'\rangle$ given in eq.(C.5) gives,

$$Tr(\mathcal{B}\mathcal{V}'(1 - \Sigma'\mathcal{V}')^{-1}) = -\left(\delta \ln\mathcal{N}' - \frac{1}{4}\frac{\delta b}{b} S'_{00}\right); \qquad (C.11)$$

and

$$\left\{ (V'^{31}, V'^{32})(1 - \Sigma'\mathcal{V}')^{-1}\mathcal{B}(1 - \mathcal{V}'\Sigma')^{-1}\begin{pmatrix} V'^{13} \\ V'^{23} \end{pmatrix}\right\}_{mn}$$

$$= -\frac{1}{2}\left\{\delta S'_{mn} + \frac{\delta b}{2b}(-S'_{0m}S'_{0n} + \delta_{0m}\delta_{0n})\right\}. \qquad (C.12)$$

These give a set of linear equations for Λ_{mn}. In order to show that the change in b corresponds to a gauge transformation, we need to show the existence of Λ_{mn} satisfying these equations. Although we do not have a proof of this, the fact that $\delta|\Psi'\rangle$ given in eq.(C.5) satisfies the various consistency relations (*e.g.* $|\Psi' * \delta\Psi'\rangle + |\delta\Psi' * \Psi'\rangle = 0$, which follows from the fact that $|\Psi'\rangle + |\delta\Psi'\rangle$ satisfies eq.(2.5)) gives us hope that the solutions to these equations do exist.

D   Conservation laws for the sliver

In this section we derive conservation laws obeyed by the sliver state, following the methods of [10]. The surface state $\langle \Xi |$ is defined by the conformal map

$$\tilde{z} = \tan^{-1}(z) \quad \rightarrow \quad \frac{d\tilde{z}}{dz} = \frac{1}{1+z^2}, \quad S(\tilde{z}, z) = -\frac{2}{(1+z^2)^2}. \qquad (D.1)$$

Here $\tilde{z}$ is the global coordinate on the once punctured upper half plane Im $(\tilde{z}) > 0$ with the punture at $\tilde{z} = 0$, and z is the local coordinate around the puncture. $S(\tilde{z}, z)$ denotes the standard Schwartzian derivative.

To obtain Virasoro conservations laws, we consider a globally defined vector field $\tilde{v}(\tilde{z})$, holomorphic everywhere except for a possible pole at the puncture $\tilde{z} = 0$. The standard contour deformation argument then gives [10]

$$\langle \Xi | \oint dz \, v(z) \left(T(z) - \frac{c}{12} S(\tilde{z}, z) \right) = 0, \qquad (D.2)$$

where contour integration is around the origin, and $v(z) = \tilde{v}(\tilde{z}) \left(\frac{d\tilde{z}}{dz} \right)^{-1}$. Taking $\tilde{v}(\tilde{z}) = \tilde{z}^p$ (with $p \le 2$ so that there is no pole at $\tilde{z} = \infty$), we get

$$v(z) = [\tan^{-1}(z)]^p (1 + z^2). \qquad (D.3)$$

We can write explicitly the first few conservation laws:

$$\langle \Xi | \left(L_1 + \frac{1}{3} L_3 - \frac{7}{45} L_5 + \cdots \right) = 0 \qquad (D.4)$$

$$\langle \Xi | \left(L_0 + \frac{2}{3} L_2 - \frac{2}{15} L_4 + \frac{2}{35} L_6 + \cdots \right) = 0 \qquad (D.5)$$

$$\langle \Xi | \left(L_{-1} + L_1 \right) = 0 \qquad (D.6)$$

$$\langle \Xi | \left(L_{-2} + \frac{c}{6} - \frac{29}{45} L_2 + \frac{128}{945} L_4 - \frac{848}{14175} L_6 + \cdots \right) = 0 \qquad (D.7)$$

$$\langle \Xi | \left(L_{-3} + \frac{61}{189} L_3 - \frac{2176}{14175} L_5 + \cdots \right) = 0 \qquad (D.8)$$

$$\langle \Xi | \left(L_{-4} - \frac{c}{3} + \frac{608}{945} L_2 - \frac{629}{4725} L_4 + \frac{1312}{22275} L_6 + \cdots \right) = 0. \qquad (D.9)$$

Here L_m could stand for either matter or ghost Virasoro operators (or even the Virasoro operators associated with a subsector of the matter conformal field theory) and c is the corresponding central charge. The perfect conservation of $K_1 = L_1 + L_{-1}$ holds more generally for any "wedge state" $\langle n |$, defined by the conformal map (see (2.42))

$$\tilde{z} = \frac{n}{2} \tan \left(\frac{2}{n} \tan^{-1} z \right). \qquad (D.10)$$

Indeed, with $\tilde{v}(\tilde{z}) = \tilde{z}^2 + 1$ we get $v(z) = (z^2 + 1)$ and deduce that for all n,

$$\langle n|K_1 = 0 \,. \tag{D.11}$$

The operator K_1 is a derivation of the $*$ product, and the observation that it annihilates all wedge states fits nicely with the fact that the wedge states form a subalgebra under $*$ multiplication.

Conservation laws for the antighost b are identical to Virasoro conservations with vanishing central charge, since b is a true conformal primary of dimension two.

Conservations laws involving c_n's can be derived analogously. Here we need to consider holomorphic quadratic differentials $\tilde{\varphi}(\tilde{z})$. We can take $\tilde{\varphi} = 1/\tilde{z}^p$, with $p \geq 4$ for the quadratic differential to be regular at $\tilde{z} \to \infty$. Back in z coordinates one has

$$\varphi(z) = \left(\frac{d\tilde{z}}{dz} \right)^2 \tilde{\varphi}(\tilde{z}) = \frac{1}{[\tan^{-1}(z)]^p (1 + z^2)^2} \,. \tag{D.12}$$

Contour deformation argument now gives

$$\langle \Xi | \oint dz\, \varphi(z) c(z) = 0 \,. \tag{D.13}$$

The first few conservations read

$$\langle \Xi | \left(c_{-2} - \frac{2c_0}{3} + \frac{29c_2}{45} - \frac{608c_4}{945} + \cdots \right) = 0 \tag{D.14}$$

$$\langle \Xi | \left(c_{-3} - \frac{c_{-1}}{3} + \frac{c_1}{3} - \frac{61c_3}{189} + \cdots \right) = 0 \tag{D.15}$$

$$\langle \Xi | \left(c_{-4} + \frac{2c_0}{15} - \frac{128c_2}{945} + \frac{629c_4}{4725} + \cdots \right) = 0 \tag{D.16}$$

$$\langle \Xi | \left(c_{-5} + \frac{7c_{-1}}{45} - \frac{7c_1}{45} + \frac{2176c_3}{14175} + \cdots \right) = 0 \tag{D.17}$$

$$\langle \Xi | \left(c_{-6} - \frac{2c_0}{35} + \frac{848c_2}{14175} - \frac{1312c_4}{22275} + \cdots \right) = 0 \,. \tag{D.18}$$

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