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Erratum
of section (2.2) of 'Prepotential, Mirror Map and F-Theory on K3'

W. Lerche and S. Stieberger

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This erratum repeats the computation of the $\mathrm{Tr}F_\alpha^2\mathrm{Tr}F_\beta^2$ couplings (for $\alpha \neq \beta$), given in eqs. (2.23) and (2.24) of section 2.2 of [1]. All other couplings of this subsection remain unchanged[1]. Moreover, the conclusions do not change much, as the couplings can still be written in the form of eq. (2.20) of [2]. For the correction Δ_{12}, eq. (2.18) is modified:

$$\Delta_{12} = \int \frac{d^2\tau}{\tau_2} \Bigg\{ 2Z_{(2,2)}(q,\bar{q},\widetilde{T},U) + \sum_{i=1,2,3} Z_{(2,2)_i}(q,\bar{q},\widetilde{T},U) \left(\frac{1}{4}\widehat{\mathcal{B}}_i - 4\right) - 6Z_{(2,2)}(q,\bar{q},\widetilde{T},U) + 4 \sum_{i=1,2,3} Z^*_{(2,2)_i}(q,\bar{q},\widetilde{T},U) \Bigg\} . \quad (1)$$

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[1] For simplicity, we worked in the $SO(32)$ heterotic string and thus an exchange in T and U has to be performed in the final results. E.g.: $\Delta_{\mathcal{R}^4} = 4\Delta_{(\mathcal{R}^2)^2} = -360 \ln T_2U_2|\eta(2U)|^4|\eta(T)|^4 + \text{const.}$.

An additional[2] $\mathbb{Z}_2$ lattice partition function $Z^*_{(2,2)_i}$ with $A_1^* = \{m_1 \in 2\mathbb{Z}, m_2, n^j \in \mathbb{Z}\}$ and volume factor $\nu_i = vol(\mathcal{N}_{2,2_i}) = \{1, \frac{1}{2}, \frac{1}{2}\}$ appears (see the $\mathbb{Z}_8$–orbifold example of [26] for further details). Using results of [26] and thanks to the relation (2.21), the integral (1) evaluates[3] to

$$\Delta_{12} = 4\ln|\eta(T)|^4 - 4\ln|\eta(2T)|^4 + \text{const.} \tag{2}$$

Similary, we get for the other couplings:

$$\begin{aligned}\Delta_{13} &= 4\ln|\eta(T)|^4 - 4\ln|\eta\Big(\frac{T}{2}\Big)|^4 + \text{const.}\,, \\ \Delta_{14} &= -8\ln|\eta(T)|^4 + 4\ln|\eta(2T)|^4 + 4\ln|\eta\Big(\frac{T}{2}\Big)|^4 + \text{const.}\end{aligned} \tag{3}$$

It can easily be checked that these couplings can be written in the form (2.20) of ref. [2], by appropriately choosing the coefficients α_i. As discussed in ref. [2], the extra corrections μ_i do not appear in this model, unlike as it is for the models with exceptional gauge symmetries. This is because the "Atkin-Lehner" cancellations (2.21) appear only for the presently discussed model with $SO(8)^4$ gauge symmetry.

References

[1] W. Lerche and S. Stieberger, *Prepotential, mirror map and F-theory on K3,* Adv. Theor. Math. Phys., **2** (1998), 1105-1140.

[2] W. Lerche, S. Stieberger, and N. Warner, *Quartic gauge couplings from K3 geometry,* `hep-th/9811228`.

[2]This additional contribution had been overlooked in [1]. We thank B. Pioline for bringing this fact to our attention.

[3]In fact, with $E_8 \times E_8$ and the Wilson lines $a_1^I = \frac{1}{2}(0,0,0,0,1,1,1,1;$ $0,0,0,0,1,1,1,1)$, $a_2^I = \frac{1}{2}(-2,0,0,0,0,0,0,0;-2,0,0,0,0,0,0,0)$ we also calculated: $\Delta_{12} = 4\ln|\eta(2T)|^4 - 4\ln|\eta(T)|^4 + \text{const.}$