On The Construction of

Gauge Theories From

Non Critical Type 0 Strings

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Abstract

We investigate Polyakov's proposal of constructing Yang-Mills theories by using non critical type 0 strings. We break conformal invariance by putting the system at finite temperature and find that the entropy of the cosmological solutions for these theories matches that of a gas of weakly interacting Yang-Mills bosons, up to a numerical constant. The computation of the entropy using the effective action approach presents some novelties in that the whole contribution comes from the RR fields. We also find an area law and a mass gap in the theory and show that such behavior persists for $p > 4$. We comment on the possible physical meaning of this result.

1   Introduction

The recent progress in string theory has unearthed deep connections between gravity and gauge theories. The conjecture [1, 2, 3] that $\mathcal{N} = 4$ super Yang-Mills theory is dual to type IIB supergravity in $AdS_5 \times S^5$ has led to the computation of many correlation functions of local observables and Wilson loops for this theory. An even more exciting possibility is the applicability of these techniques to non supersymmetric gauge theories. In this context, Witten [4] has proposed to compactify the $4 + 1$ dim. theory of a D4-brane on a circle with supersymmetry breaking boundary conditions, leaving a $3 + 1$ dimensional pure Yang-Mills theory with an effective cut-off given by the radius of the circle. This proposal has enjoyed many successes, yielding qualitatively reasonable results for bare quantities such as an area law for the spatial Wilson loop and a mass gap [5, 6, 7, 8, 9, 10, 11, 12]. A very difficult unsolved problem in this approach is how to take the continuum limit, which corresponds to taking the curvature of the background to infinity.

A different proposal, due to Polyakov [13], is to consider a string theory with RR fields and a diagonal GSO projection that removes all fermions from the spectrum while preserving modular invariance at one loop. Such theories are referred to as type 0A or 0B, depending on the way one implements the projection [14, 15]. They have open string descendants that are of interest for the D-brane construction [16, 17, 18, 19, 20]. The bulk fields of these theories are the tachyon, the usual massless fields of the respective type II theories (with a doubling of the RR part of the spectrum) and an infinite tower of bosonic massive modes. If D-branes are included, the GSO projection in the open string sector removes the world-volume tachyon and all the fermionic partners of the gauge bosons, leaving, in the critical case, only a bosonic Yang-Mills-Higgs theory obtained by dimensional reduction of pure Yang-Mills in $9 + 1$ dimensions to $p + 1$ dimensions. For $p = 3$ this theory is asymptotically free (recall that the additional presence of the fermions would make the theory exactly finite).

Klebanov and Tseytlin [21] have recently made the important observation that a large background RR field provides a shift in the tachyon potential that effectively induces the tachyon to condense at a value of order one. The tachyon condensate has three effects: first it makes the theory well defined in spite of the naive instability, second it induces an effective central charge proportional to $\langle T \rangle^2$ even in $d = 10$, and third, it provides a mechanism for breaking conformal invariance in the world volume theory.

Assuming that the tachyon condenses, it pays off to consider the theory off criticality, that is at arbitrary values of d, with an effective central charge

given by

$$c_{eff.} = 10 - d + \frac{d-2}{16}\langle T \rangle^2.$$ (1)

This has the advantage of eliminating some or all of the Higgs fields from the Yang-Mills theory under study, leaving in principle only the pure gauge theory if $d = p + 2$. Also, if $d/2 - 2 < p \le d - 2$ and $d \ne p + 3$ there exist solutions to the Einstein equations having a constant dilaton, suggesting the existence of a conformal fixed point [13]. Compactifying the conformal theory provides an alternative way to break the conformal symmetry introducing a deformation parameter proportional to the size of the compactified manifold.

In this paper we study the simplest example of such mechanism — the compactification on an Euclidean time circle of period inversely proportional to Hawking's temperature. In section two we write down the tree level gravity equations under the assumption that the tachyon condenses and that there is only one RR field present. We solve these equations and show that it is possible to have arbitrary temperature while keeping the dilaton constant.

In section three we compute the entropy for this solution and show that it agrees with what is expected from a gas of weakly interacting YM particles. We compute the entropy in two different ways, by evaluating the free energy (effective action) and the horizon area, finding the same results. In the effective action approach all the contribution comes from the RR fields.

In section four we compute the spatial Wilson loop in the two limiting cases and show that it evolves from the behavior expected from a conformal field theory ($1/L$ potential) to that of a confining theory (area law). However, compared with the standard construction, the bare string tension scales differently with the bare coupling[1]. The fact that in this context one has one less free parameter than in the standard type II/M-theory construction presents a potentially serious problem in taking the continuum limit. We also present the equation for the mass gap as a straightforward generalization of the previous analysis [4]. It is puzzling that the area law and mass gap persist for $p > 4$. We comment on the possible physical meaning of such result.

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[1] For a discussion of the Wilson loop in relation to zig-zag symmetry see [22].

2   The gravity equations

Assuming that the tachyon condenses, the one loop σ-model β-functions (tree level gravity equations) for the metric g, dilaton Φ and RR $p+1$ gauge potential C can be derived from the action[2]

$$I = \int dx^d \sqrt{|g|} \left\{ e^{-2\Phi} \left(\frac{c_{eff.}}{2} + R + 4(\partial\Phi)^2 \right) - \frac{1}{2(p+2)!} G^2 \right\}, \quad (2)$$

where $c_{eff.}$ is given in (1), $G = dC$ is the RR $p+2$ field strength and d includes the Liouville direction r. For us r is just another coordinate representing the radial direction.

The equations of motion are

$$R_{\mu\nu} = -2D_\mu D_\nu \Phi + e^{2\Phi} T_{\mu\nu} \tag{3}$$

$$c_{eff.}/2 + R = 4D_\mu \Phi D^\mu \Phi - 4D^\mu D_\mu \Phi \tag{4}$$

$$D^\mu G_{\mu\nu_1\cdots\nu_{p+1}} = 0, \tag{5}$$

where

$$T_{\mu\nu} = \frac{1}{(p+1)!} \left(G_{\mu\nu_1\cdots\nu_{p+1}} G_\nu{}^{\nu_1\cdots\nu_{p+1}} - \frac{1}{2(p+2)} g_{\mu\nu} G_{\nu_1\cdots\nu_{p+2}} G^{\nu_1\cdots\nu_{p+2}} \right) \tag{6}$$

is the stress energy tensor of the RR field. Note that (6) is traceless only if $d = 2p + 4$.

We want to deform the solution found in [13] to allow for a non zero temperature. We thus consider the following ansatz:

$$g_{rr} = 1; \quad g_{00} = \gamma^2(r); \quad g_{ij} = a^2(r)\delta_{ij}; \quad g_{ab} = b^2(r)\hat{g}_{ab}, \tag{7}$$

where r is the Liouville direction, 0 is the Euclidean time, $i,j = 1\cdots p$ the spatial coordinates and $a,b = 1\cdots d-p-2$ the (possible) internal coordinates of a sphere. The Greek indices in (3,4,5,6) are meant to run over the whole range $\mu = (r, 0, i, a)$ The metric $\hat{g}$ is normalized to have

$$\hat{R}_{abcd} = (\hat{g}_{ac}\hat{g}_{bd} - \hat{g}_{ad}\hat{g}_{bc}). \tag{8}$$

With the further ansatz that

$$\Phi \equiv \Phi(r) \quad \text{and} \quad C_{0i_1\cdots i_p} = c(r)\epsilon_{i_1\cdots i_p}, \tag{9}$$

the equation of motion (5) gives the standard solution

$$c' = N \frac{\gamma a^p}{b^{d-p-2}} \tag{10}$$

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[2]Throughout this paper we set $\alpha' = 1$.

that can be used to eliminate c from the other equations. The remaining equations (3, 4) yield

$$\frac{\gamma''}{\gamma} + p\frac{a''}{a} + (d-p-2)\frac{b''}{b} = 2\Phi'' + \frac{N^2}{b^{2d-2p-4}}e^{2\Phi}$$

$$\frac{\gamma''}{\gamma} + p\frac{\gamma'}{\gamma}\frac{a'}{a} + (d-p-2)\frac{\gamma'}{\gamma}\frac{b'}{b} = 2\frac{\gamma'}{\gamma}\Phi' + \frac{N^2}{b^{2d-2p-4}}e^{2\Phi}$$

$$\frac{a''}{a} + \frac{\gamma'}{\gamma}\frac{a'}{a} + (p-1)\frac{a'^2}{a^2} + (d-p-2)\frac{a'}{a}\frac{b'}{b} = 2\frac{a'}{a}\Phi' + \frac{N^2}{b^{2d-2p-4}}e^{2\Phi} \quad (11)$$

$$\frac{b''}{b} + \frac{\gamma'}{\gamma}\frac{b'}{b} + p\frac{a'}{a}\frac{b'}{b} + (d-p-3)\frac{b'^2}{b^2} - \frac{d-p-3}{b^2} = 2\frac{b'}{b}\Phi' - \frac{N^2}{b^{2d-2p-4}}e^{2\Phi}$$

$$4\Phi'^2 - 2\Phi'' - 2\left(\frac{\gamma'}{\gamma} + p\frac{a'}{a} + (d-p-2)\frac{b'}{b}\right)\Phi' = \frac{c_{eff.}}{2} - \frac{(2p+4-d)N^2}{b^{2d-2p-4}}e^{2\Phi}.$$

The advantage of working in a non critical theory is that (11) admit solutions with a constant dilaton even for $p \neq 3$. Such solutions are those of interest to us, so let us specialize to this case by setting $\lambda_{p+1} = Ne^\Phi =$ const. (λ_{p+1} is the 't Hooft coupling in the appropriate units of α'). We also search for solutions with constant b, so that the equations reduce to

$$\frac{\gamma''}{\gamma} + p\frac{a''}{a} = \frac{\lambda_{p+1}^2}{b^{2d-2p-4}}$$

$$\frac{\gamma''}{\gamma} + p\frac{\gamma'}{\gamma}\frac{a'}{a} = \frac{\lambda_{p+1}^2}{b^{2d-2p-4}}$$

$$\frac{a''}{a} + \frac{\gamma'}{\gamma}\frac{a'}{a} + (p-1)\frac{a'^2}{a^2} = \frac{\lambda_{p+1}^2}{b^{2d-2p-4}} \quad (12)$$

$$\frac{d-p-3}{b^2} = \frac{\lambda_{p+1}^2}{b^{2d-2p-4}}$$

$$\frac{c_{eff.}}{2} - \frac{(2p+4-d)\lambda_{p+1}^2}{b^{2d-2p-4}} = 0.$$

The equations don't allow for $d = p+3$, that is, for the compact dimension to be a circle. When $d > p + 3$, from the last two equations one can solve for the 't Hooft coupling and for b, which are fixed to be (in units of α')

$$\lambda_{p+1}^2 = (d-p-3)\left(\frac{2(2p+4-d)(d-p-3)}{c_{eff.}}\right)^{d-p-3}$$

$$b^2 = \frac{2(2p+4-d)(d-p-3)}{c_{eff.}}. \quad (13)$$

The zero temperature solution is that of [13] and is found by setting $a = \gamma = \exp(r/R)$, where R is the radius of curvature. Eqs. (12) and (13) then give

$$R^2 = \frac{2(p+1)(2p+4-d)}{c_{eff.}} \tag{14}$$

and the relation between the 't Hooft coupling and the radius of curvature reads

$$\lambda_{p+1}^2 \sim \left(R^2\right)^{d-p-3}. \tag{15}$$

Note that for $d = 10$, $p = 3$ we obtain Maldacena' s scaling. We emphasize that it is only in the case of critical type II theory that one is truly free to vary the parameters in (15) although it is tempting to hope that a better control of the tachyon field will allow to give a precise physical meaning to the type 0 construction as well.

The case with no compact dimensions $(d = p+2)$ should describe a theory without any scalar. In this case the quantity b drops out of the equations (11), whose solution is then

$$\lambda_{p+1}^2 = \frac{p+1}{R^2} = \frac{c_{eff.}}{2(p+2)}. \tag{16}$$

Notice the peculiar behavior of R that scales in a way inversely proportional to the 't Hooft coupling, contrary to the standard situation. Again, this dependence should be interpreted with a grain of salt because at this stage both values are fixed in terms of $c_{eff.}$ and, barring a novel mechanism that allows to vary the effective central charge, we cannot take the limit $\lambda_{p+1} \to 0$.

The thermal deformation of this solution is more easily obtained by going to the gauge

$$ds^2 = \frac{\rho^2 f(\rho)}{R^2} dt^2 + \frac{\rho^2}{R^2} dx_i^2 + \frac{R^2}{\rho^2 f(\rho)} d\rho^2 + b^2 d\Omega^2. \tag{17}$$

Substituting into (12) yields

$$(p+1)f(\rho) + \rho \frac{df}{d\rho} = p+1, \tag{18}$$

whose solution is

$$f(\rho) = 1 - \frac{\rho_T^{p+1}}{\rho^{p+1}}. \tag{19}$$

Notice that in the extremal limit there exist solutions with $AdS_{p+2} \times S^{d-p-2}$ geometry for generic values of p and d. The Hawking temperature for this solution is easily computed to be

$$T_H = \frac{p+1}{4\pi} \frac{\rho_T}{R^2}. \tag{20}$$

3   Thermodynamics of non-critical p-branes

In this section we investigate the thermodynamic properties of the system and find evidence in favor of the conjectured string theory/ gauge theory correspondence. We compute the entropy [23, 4] of the non-critical, non-extremal p-brane solution using two different methods and give an interpretation of the results in terms of the light degrees of freedom living on the brane, namely Yang-Mills theory [24]. A similar analysis is performed in [10] for the type II D-branes.

For Yang-Mills theory in the weak coupling limit, one can neglect interactions between gluons and compute microscopically thermodynamic quantities using a free Bose gas approximation. In the case of $SU(N)$ gauge theory in $p + 1$ dimensions the energy and entropy per unit volume read

$$\frac{E}{V} \sim N^2 T^{p+1} \qquad\qquad \frac{S}{V} \sim N^2 T^p \tag{21}$$

which are, up to a numerical coefficient, dictated just by dimensional arguments, N^2 being the number of degrees of freedom.

Let us follow [4] and [23] and identify the free energy F of the black-brane as the (subtracted) Euclidean action (c.f.r. (2)) times the Hawking temperature ($\beta = \frac{1}{T_H}$),

$$\beta F = I_E[g_{\mu\nu}, \Phi, G; T_H] - I_E[g_{\mu\nu}, \Phi, G; 0] \tag{22}$$

where as usual we subtract the zero temperature action to get a finite result. Notice that by virtue of equation of motion (4) the Einstein and cosmological terms drop out for constant dilaton and the action gets contribution only from the RR field. In fact,

$$I_E = \int dx^d \sqrt{|g|} \frac{1}{2(p+2)!} G^2 = \frac{1}{2} \frac{N^2}{R^{2d-2p-4}} \int dx^d \sqrt{|g|} \tag{23}$$

After putting a cutoff in the "radial" integration, one has to evaluate two invariant volumes, where in the black-brane configuration the integration is to be performed in the physical region outside the horizon,

$$V(\rho_\infty) = \int_0^\beta dt \int_{\rho_T}^{\rho_\infty} d\rho \int d^p x \frac{\rho^p}{R^p} \int R^{d-p-2} d\Omega_{d-p-2} \tag{24}$$

and

$$V_0(\rho_\infty) = \int_0^{\beta'} dt \int_0^{\rho_\infty} d\rho \int d^p x \frac{\rho^p}{R^p} \int R^{d-p-2} d\Omega_{d-p-2} \tag{25}$$

and let $\rho_\infty \to \infty$ after subtracting them. Here the radius of compactification β' has to be matched with β for the hyper-spheres in the two geometries to be comparable, i.e.

$$\beta' \frac{\rho_\infty}{R} = \beta \frac{\rho_\infty}{R} \sqrt{1 - \frac{\rho_T^{p+1}}{\rho_\infty^{p+1}}}. \tag{26}$$

The result is

$$\beta F = \frac{N^2}{2R^{2d-2p-4}} \lim_{\rho_\infty \to \infty} (V - V_0) \sim -R^{4+2p-d}\Omega_{d-p-2}N^2 V_p \frac{1}{\beta^p} \tag{27}$$

where Ω_{d-p-2} is the volume of the $(d - p - 2)$-sphere and V_p is the total volume of the p-space. Now recall from (14) that $R \sim 1$ so that we finally get the energy

$$E = \frac{\partial}{\partial \beta}(\beta F) \sim N^2 V_p T_H^{p+1} \tag{28}$$

and the entropy

$$S = \beta(E - F) \sim N^2 V_p T_H^p. \tag{29}$$

One can also compute the Bekenstein-Hawking entropy. In fact, going to the Einstein frame[3]

$$ds_E^2 = e^{-\frac{4}{d-2}\Phi} ds^2 \tag{30}$$

the area of the horizon is easily found to be (recall that $b \sim R$)

$$A \sim \Omega_{d-p-2} R^{4+2p-d} N^2 V_p T_H^p, \tag{31}$$

also in agreement with (29). So we find that the Hawking relation is reproduced and the entropy has the ideal gas scaling behavior.

This result is in agreement with the conclusions of [25] for critical black p-branes, where it is pointed out that constant dilaton is a sufficient condition for such a scaling. Nevertheless, off criticality allows for more general values of p.

Thus, we argue that there should be a correspondence between the gravity approximation of type 0 string theory and a non supersymmetric Yang-Mills theory of N^2 degrees of freedom, finding support for Polyakov's conjecture.

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[3]This is the same as doing the calculation in the string frame and remembering that now the Newton constant depends on the dilaton and behaves like λ_{p+1}^2/N^2.

4   The spatial Wilson loop and the mass gap

We are now in the position of computing the Wilson loop for the theory described by the metric in section two using the techniques exposed in [26, 27] and further developed in [11, 10].

Let us parameterize the world sheet of the string as $x^1 = \sigma$, $x^2 = \tilde{\sigma}$, $\rho = \rho(\sigma)$ where $-L/2 < \sigma < L/2$, $-\tilde{L}/2 < \tilde{\sigma} < \tilde{L}/2$ and $L << \tilde{L}$. The action to be minimized is

$$S = \frac{\tilde{L}}{2\pi} \int_{-L/2}^{L/2} d\sigma \sqrt{\frac{\rho'^2}{1 - \rho_T^{p+1}/\rho^{p+1}} + \frac{\rho^4}{R^4}}. \tag{32}$$

The conserved quantity derived from this action is

$$\frac{1}{\rho^4} \sqrt{\frac{\rho'^2}{1 - \rho_T^{p+1}/\rho^{p+1}} + \frac{\rho^4}{R^4}} = \frac{1}{\rho_0^2 R^2}, \tag{33}$$

where $\rho(0) = \rho_0$ and $\rho'(0) = 0$ for symmetry reasons. ρ_0 measures how close the world sheet approaches the horizon at ρ_T and the behavior of the Wilson loop is governed by the ratio $\epsilon = \rho_T/\rho_0$. For $\epsilon \to 0$ we recover the conformal fixed point, whereas for $\epsilon \to 1$ we should approach the p dimensional theory.

The minimum action is given by the integral (after subtracting the infinite energy of the string)

$$S_{\min} = \frac{\tilde{L}\rho_0}{\pi} \left\{ \epsilon - 1 + \int_1^\infty dy \left[\frac{y^{(p+5)/2}}{\sqrt{(y^4 - 1)(y^{p+1} - \epsilon^{p+1})}} - 1 \right] \right\}, \tag{34}$$

where ρ_0 is expressed in terms of R, L and ρ_T by the implicit function

$$\frac{L}{2} = \frac{R^2}{\rho_0} \int_1^\infty dy \frac{y^{(p-3)/2}}{\sqrt{(y^4 - 1)(y^{p+1} - \epsilon^{p+1})}}. \tag{35}$$

In the regime $\epsilon \to 0$ we obtain the results of [26, 27]: $S_{min} \sim R^2 \times (\tilde{L}/L)$. Note that this is the same behavior as in [26, 27] only if expressed in terms of R. The relation between R and the 't Hooft coupling being different (c.f.r. (15)) if not in the critical dimension.

The interesting regime is when $\epsilon \to 1$. In this case, both integrals in (34), (35) scale like $|\log(1 - \epsilon)|$ and we must eliminate the divergence by taking the ratio of the two quantities. This leaves a dependence on ρ_0 but this is easily fixed by realizing that as $\epsilon \to 1$, $\rho_0 \to \rho_T$, yielding

$$S_{\min} = \frac{\rho_T^2}{2\pi R^2} \times \tilde{L}L. \tag{36}$$

Eq. (36) represents the area law for the p dimensional gauge theory, from which one can read off the bare string tension (always in units of α')

$$\mathcal{T}_{YM} = \frac{\rho_T^2}{R^2} \sim T_H^2. \tag{37}$$

We immediately see a potentially serious problem with this construction. In the most optimistic scenario, one would like to compute the renormalized string tension by taking the limit $T_H \to \infty$ while the p dimensional coupling $\hat{\lambda}_p$ goes to zero as $1/\log(T_H/\Lambda_{QCD})$. So far, this computation has been out of reach even for the standard construction. At least in that case, however, one has two truly independent bare parameters to vary, namely T_H and $\hat{\lambda}_p$. Here instead, the relation $\hat{\lambda}_p = \lambda_{p+1} T_H$ and the fact that λ_{p+1} is fixed to be of order one by the equations of motion forces $\hat{\lambda}_p \sim T_H$. We are thus led back to the issue raised in section two on whether it is possible to relax eqs. (13). This issue remains open.

Finally, we address the question of whether a mass gap will emerge in the p dimensional theory (at zero temperature), consistently with the area law found above.

As explained in [3, 4] it will be sufficient to study the equation of motion of a quantum field propagating in the background given by (17) and determine its spectrum in the p-dimensional sense. So, let us consider the dilaton equation of motion (4). In spite of the presence of the cosmological constant, the constant dilaton background renders the fluctuation field effectively massless (in d dimensions), so that we must still solve for

$$\partial_\mu(\sqrt{|g|}g^{\mu\nu}\partial_\nu\delta\Phi) = 0 \tag{38}$$

and we search for solution of the form $\delta\Phi = \chi(\rho)e^{ikx}$, $x \in R^p$.

After defining $y = \frac{\rho}{\rho_T}$ the equation of motion for χ following from (38) is

$$\partial_y\left[(y^{p+2} - y)\partial_y\chi\right] + \rho_T^{-2}R^4M^2y^{p-2}\chi = 0, \tag{39}$$

$M^2 = -k^2$ being the mass squared of the glueball. $M \sim T_H$ as it should, since the bare mass scales with the UV cutoff, the Hawking temperature in this case.

Thus, it is straightforward to repeat the arguments of section 3.3 in [4] and conclude that the eigenvalue problem (39) has normalizable solutions only for discrete and strictly positive values of M^2. In fact (39) actually reduces to the equation appearing in [4] for $p = 3$, while for $p = 4$ we also

obtain a mass gap for four dimensional gauge theory, in accordance with the area law.

It is puzzling that we find a mass gap and an area law even for $p > 4$. This could be an artifact of the approximation but could also have a field theoretical explanation. Yang-Mills theory in more than four dimensions is perturbatively non-renormalizable — however, seen from the point of view of the ϵ-expansion, the $4 + \epsilon$ theory has a phase transition at a finite value of the bare coupling constant. In the strong coupling phase, the theory behaves, at low energies, in a way similar to four dimensional Yang-Mills. It may happen that this is the phase relevant to the string theory description.

5   Acknowledgments

References

[1] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231, `hep-th/9711200`.

[2] S.S. Gubser, I.R. Klebanov, and A.M. Polyakov Phys. Lett. **B 428** (1998) 105, `hep-th/9802109`.

[3] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253, `hep-th/9802150`.

[4] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 505, `hep-th/9803131`.

[5] D.J. Gross and H. Ooguri, Phys. Rev. **D 58** (1998) 106002, `hep-th/9805129`.

[6] C. Csaki, H. Ooguri, Y. Oz and J. Terning, `hep-th/9806021`.

[7] R. De Mello Koch, A. Jevicki, M. Mihailescu and J. Nunes, `hep-th/9806125`.

[8] A. Hashimoto and Y. Oz, `hep-th/9809106`.

[9] C. Csaki, Y. Oz, J. Russo and J. Terning, `hep-th/9810186`.

[10] A. Brandhuber, N. Itzhaki, J. Sonnenschein, S. Yankielowicz J. High Energy Phys. **06** (1998) 001, `hep-th/9803263`.

[11] S.J. Rey, S. Theisen and J. Yee, Nucl. Phys. **B 527** (1998) 171, hep-th/9803135.

[12] J.A. Minahan, hep-th/9811156.

[13] A.M. Polyakov, hep-th/9809057.

[14] L. Dixon and J. Harvey, Nucl. Phys. **B 274** (1986) 93.

[15] N. Seiberg and E. Witten, Nucl. Phys. **B 276** (1986) 272.

[16] M. Bianchi and A. Sagnotti, Phys. Lett. **B 247** (1990) 517.

[17] A. Sagnotti, hep-th/9509080.

[18] A. Sagnotti, Nucl. Phys. Proc. Suppl. **B 56** (1997) 332, hep-th/9702093.

[19] C. Angelantonj, hep-th/9810214.

[20] O. Bergman and M. Gaberdiel, Nucl. Phys. **B 499** (1997) 183, hep-th/9701137.

[21] I.R. Klebanov and A.A. Tseytlin, hep-th/9811035.

[22] E. Alvarez, C. Gomez and T. Ortin, hep-th/9806075.

[23] S. Hawking and D. Page, Commun. Math. Phys. **87** (1983) 577.

[24] S.S. Gubser, I.R. Klebanov and A.W. Peet, Phys. Rev. **D 54** (1996) 3915, hep-th/9602135.

[25] I.R. Klebanov and A.A. Tseytlin, Nucl. Phys. **B 475** (1996) 164, hep-th/9604089.

[26] S.J. Rey, J. Yee, hep-th/9803001.

[27] J. Maldacena, Phys. Rev. Lett. **80** (1998) 4859, hep-th/9803002.