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Unwinding Strings and T-duality of Kaluza-Klein and H-Monopoles

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Abstract

A fundamental string with non-zero winding number can unwind in the presence of a Kaluza-Klein monopole. We use this fact to deduce the presence of a zero mode for the Kaluza-Klein monopole corresponding to excitations carrying H-electric charge and we study the coupling of this zero mode to fundamental strings. We also describe a T-dual process in which the momentum of a fundamental string "unwinds" in the presence of an H-monopole. We use the coupling of string winding modes to the dyon collective coordinate of the Kaluza-Klein monopole to argue that there are stringy corrections to the Kaluza-Klein monopole which are in accordance with T-duality.

1 Introduction

Supersymmetric string theory compactified on T^6 contains Kaluza-Klein monopoles [1] carrying charge under a $U(1)$ arising from Kaluza-Klein reduction of the ten-dimensional metric, and H-monopoles [2] which carry charge under a $U(1)$ arising from reduction of the ten-dimensional Kalb-Ramond field B_{MN}.

Consider a Kaluza-Klein monopole associated with one of the compact directions, say x^5. At spatial infinity the spacetime has topology $S^3 \times T^5$. The factor T^5 is a compactifying torus. The factor S^3 has the following interpretation: S^3 is topologically the Hopf fibration, an S^1 fibration over S^2. In the Kaluza-Klein monopole S^3 is metrically a squashed three-sphere: at large spatial distances the radius of the Hopf fiber, parametrized locally by x^5, becomes constant and serves as a circle for Kaluza-Klein reduction. The base space S^2 is a metrically round S^2 and plays the role of the "spatial" points at fixed radius in $\mathbb{R}^3$. Consider now a string wound around the Hopf fiber. Although squashed, the three-sphere is nevertheless a nontrivial S^1 fiber bundle over S^2, and in particular the total space, which is topologically S^3, has $\pi_1 = 0$. Therefore, a string wound around a Hopf fiber at a large distance from the monopole can be unwound by motion that keeps it arbitrarily far away from the monopole. On the other hand, the winding number of the string is a conserved charge, the H-electric charge, and is coupled to a gauge field $B_{\mu 5}$. Since this charge is conserved and the charge of the string changes when the string unwinds, there must be another source of H-electric charge. The only plausible possibility is that the Kaluza-Klein monopole can also carry H-electric charge and that the H-electric charge of the string flows onto the monopole in the unwinding process. We will show below that this is in fact the case and identify the zero mode of the Kaluza-Klein monopole which carries this charge.

We can also consider a T-dual version of this process. T-duality takes the Kaluza-Klein monopole to an H-monopole and a string with non-zero winding to a string with non-zero momentum along the S^1. It must then be possible to "unwind" a string with momentum in the presence of an H-monopole, again without bringing the string near the monopole. This implies that H-monopoles can carry Kaluza-Klein electric charge, and we again identify the relevant zero mode.

Finally, we use the fact that strings can unwind in a Kaluza-Klein monopole background to show that there are stringy corrections to the Kaluza-Klein monopole solution and we argue that these resolve a number of puzzles regarding T-duality of H-monopoles and Kaluza-Klein monopoles. The interplay between T-duality, 5-branes, and ADE singularities has also been discussed in [3,4].

2 Unwinding Strings in a Kaluza-Klein Monopole Background

We begin by considering Type II string theory compactified on T^6. Many of our considerations also apply to heterotic strings, but there are some additional subtleties which will be discussed elsewhere. Type II theory has a Kaluza-Klein monopole solution [1] with the string metric given by

$$ds^2 = dt^2 - U(dx^5 + \frac{1}{2}R(1-\cos\theta)d\phi)^2 - U^{-1}(dr^2 + r^2 d\Omega_2^2)\ , \qquad (2.1)$$

with

$$U(r) = \left(1 + \frac{R}{2r}\right)^{-1} \qquad (2.2)$$

and with a flat metric on the remaining five compact coordinates. The metric is non-singular provided that x^5 is periodic with period $2\pi R$. In what follows it will be useful to have two coordinate patches to describe this solution. Note that as $\theta \to 0$ we can write the above metric in a non-singular fashion, but as $\theta \to \pi$, the metric appears singular. This is simply a coordinate singularity and can be removed by setting

$$\bar{x}^5 = x^5 + R\phi. \qquad (2.3)$$

Since ϕ has periodicity 2π, this is clearly a well defined transition function. The resulting t, r =constant surfaces are described by the Hopf fibration of S^1 over S^2 to give the total space S^3. Thus although *locally* for large r the spacetime has the appearance of being $\mathbb{R}^4 \times S^1 \times T^5$, it in fact has global topology $\mathbb{R}^5 \times T^5$.

Now consider a string which is wound once around the x^5 direction and is located at radial coordinate r_0 and at the North pole on the spatial S^2. Since $\pi_1(\mathbb{R}^5)$ is trivial, any closed loop is contractible and we should be able to unwind the string. One way this can happen is for the wound string to be pulled into the core of the monopole where it then unwinds. From 2.1 we see that the proper distance around x^5 decreases as we move in towards the center of the monopole. The wound string can thus decrease its winding energy by moving in towards the monopole. Another way of seeing this is to note that in the dimensional reduction of 2.1 to four dimensions, the wound string, which now appears to be a particle, picks up a factor of $\sqrt{U}$ in the usual action for a relativistic test particle. This introduces a modification to the geodesic equation which can be interpreted as a central attractive force. The string is therefore attracted to the core of the Kaluza-Klein monopole where it can unwind. This process was considered by Gibbons and Ruback for Nambu strings [5] and will play an important role in our analysis of T-duality in section 4.

However, since the spatial topology at infinity is S^3 it must also be possible to unwind this string while keeping it arbitrarily far from the monopole. This is accomplished by the following trajectory labelled by world-sheet coordinates σ, τ with $0 \le \sigma, \tau \le 1$:

$$
\begin{aligned}
x^5 &= 2\pi R\sigma \ , \\
\theta &= \pi\tau \ , \\
\phi &= -2\pi\sigma \ , \\
r &= r_0 \ .
\end{aligned} \tag{2.4}
$$

As $\tau \to 1$, $\theta \to \pi$, and we must use the barred coordinate patch in which $\bar{x}^5 = 0$ and the string is unwound. To understand where the winding charge has gone we need to analyze the problem in somewhat more detail.

The only fields relevant to the resolution of this puzzle can be packaged in terms of the dimensional reduction of a five-dimensional theory containing gravity and an antisymmetric tensor field. We also add a scalar field representing the dilaton. We thus for the time being ignore the five additional coordinates needed to embed the solution into string theory as well as the additional moduli of string theory.

Therefore, we consider the dimensional reduction of the action describing the above five-dimensional fields and their coupling to the string:

$$
\begin{aligned}
S_5 &= \frac{1}{2\kappa_5^2}\int d^5x\sqrt{\mathcal{G}}e^{-2\Phi}[-\mathcal{R} - 4(\partial_a\Phi)^2 + \frac{1}{12}\mathcal{H}^2_{abc}] \\
&- \frac{1}{4\pi\alpha'}\int d^5x\int d^2\sigma\delta^{(5)}\left(x^a - X^a(\sigma^A)\right) \\
&\times\left\{\sqrt{\gamma}\gamma^{AB}\frac{\partial X^a}{\partial\sigma^A}\frac{\partial X^b}{\partial\sigma^B}\mathcal{G}_{ab} + \epsilon^{AB}\frac{\partial X^a}{\partial\sigma^A}\frac{\partial X^b}{\partial\sigma^B}\mathcal{B}_{ab}\right\} \ .
\end{aligned} \tag{2.5}
$$

Our convention is that Roman indices run over 5 dimensions, Greek from 0 to 3, capitals from 0 to 1. The signature is mostly minus, and the calligraphic symbols denote five-dimensional objects which will ultimately be reduced to 4 dimensions. Note that the Kaluza-Klein monopole is still a solution within this theory with a constant dilaton and zero $\mathcal{B}$-field. We now dimensionally reduce the action 2.5 following [6, 7] setting

$$ds^2 = -e^{-4\sigma}[dx^5 + A^1_\mu dx^\mu]^2 + g_{\mu\nu}dx^\mu dx^\nu \ , \tag{2.6}$$

$$A^2_\mu = \mathcal{B}_{\mu 5} \ , \tag{2.7}$$

and

$$B_{\mu\nu} = \mathcal{B}_{\mu\nu} + \frac{1}{2}(A^1_\mu A^2_\nu - A^1_\nu A^2_\mu). \tag{2.8}$$

This theory has an $O(1,1)$ symmetry which we can make manifest by introducing the two by two matrices

$$L = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad M = \begin{pmatrix} e^{4\sigma} & 0 \\ 0 & e^{-4\sigma} \end{pmatrix}. \tag{2.9}$$

The field strengths are then defined by

$$F^{\hat{a}}_{\mu\nu} = \partial_\mu A^{\hat{a}}_\nu - \partial_\nu A^{\hat{a}}_\mu \ , \tag{2.10}$$

with $\hat{a} = 1, 2$ and

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} - \frac{1}{2} A^{\hat{a}}_\mu L_{\hat{a}\hat{b}} F^{\hat{b}}_{\nu\lambda} + \text{cyclic perm.} \tag{2.11}$$

Note that $B_{\mu\nu}$ now transforms under gauge transformations of A. In particular, under a gauge transformation $\delta A^1 = d\Lambda^1$, B transforms as $\delta B = \frac{1}{2} d\Lambda^1 \wedge A^2$. This means in particular that in the Kaluza-Klein monopole background both B and A^1 will be patch dependent with the relation between the fields in the two coordinate patches being given by

$$\begin{aligned} \bar{A}^1 &= A^1 - R d\phi \ , \\ \bar{B} &= B + \tfrac{1}{2} R d\phi \wedge A^2 \ . \end{aligned} \tag{2.12}$$

Continuing with the dimensional reduction, we see that the first part of the action becomes

$$S^1_4 = \frac{1}{2\kappa_4^2} \int d^4x e^{-2\phi} \sqrt{g} \times \left[-R - 4(\nabla\phi)^2 - \frac{1}{8} \mathrm{Tr} \nabla M L \nabla M L - \frac{1}{4} F^{\hat{a}} (LML)_{\hat{a}\hat{b}} F^{\hat{b}} + \frac{1}{12} H^2 \right] , \tag{2.13}$$

where $\phi = \Phi + \sigma$ is now the four-dimensional dilaton field and $\kappa_4^2 = \kappa_5^2 / 2\pi R$.

For the string part of the action, note that

$$\frac{\partial X^a}{\partial \sigma^A} \frac{\partial X^b}{\partial \sigma^B} \mathcal{G}_{ab} = h_{AB} - e^{-4\sigma} V_A V_B \ , \tag{2.14}$$

where

$$h_{AB} = \frac{\partial X^\mu}{\partial \sigma^A} \frac{\partial X^\nu}{\partial \sigma^B} g_{\mu\nu} \ , \qquad V_A = \frac{\partial X^5}{\partial \sigma^A} + A^1_A \ , \tag{2.15}$$

with the pullbacks of the gauge fields given by

$$A_A^{\hat{a}} = \frac{\partial X^\mu}{\partial \sigma^A} A_\mu^{\hat{a}} \, . \tag{2.16}$$

We also have

$$\begin{aligned} \epsilon^{AB} X^a_{,A} X^b_{,B} \mathcal{B}_{ab} &= \epsilon^{AB} X^\mu_{,A} X^\nu_{,B} \mathcal{B}_{\mu\nu} + 2\epsilon^{AB} X^5_{,A} X^\nu_{,B} \mathcal{B}_{5\nu} \\ &= -2\epsilon^{AB} V_A A_B^2 + \epsilon^{AB} A_A^1 A_B^2 + \epsilon^{AB} X^\mu_{,A} X^\nu_{,B} B_{\mu\nu} \, . \end{aligned} \tag{2.17}$$

So the string action reads

$$\begin{aligned} S_4^2 = &-\frac{1}{4\pi\alpha'} \int d^4x d^2\sigma \delta^{(4)} \left(x^\mu - X^\mu(\sigma^A)\right) \\ &\times \left\{ \sqrt{-\gamma}\gamma^{AB} [h_{AB} - e^{-4\sigma} V_A V_B] + \epsilon^{AB} \left[X^\mu_{,A} X^\nu_{,B} B_{\mu\nu} + A_A^1 A_B^2 - 2V_A A_B^2 \right] \right\}. \end{aligned} \tag{2.18}$$

The relevant equations of motion derived from this action are then

$$\nabla_\mu \left(e^{-2\phi} H^{\mu\nu\lambda} \right) = -\frac{\kappa_4^2}{\pi\alpha'} \int d^2\sigma \delta^{(4)} (x^\mu - X^\mu(\sigma,\tau)) \epsilon^{AB} X^\nu_{,A} X^\lambda_{,B} \, , \tag{2.19}$$

$$\begin{aligned} \nabla_\mu \left(e^{-2\phi} LMLF^{\mu\nu} \right)^{\hat{a}} &= j^{\hat{a}\nu} \\ &= \frac{e^{-2\phi}}{2} H^{\nu\mu\lambda} (LF_{\mu\lambda})^{\hat{a}} - \frac{\kappa_4^2}{\pi\alpha'} \int d^2\sigma \delta^{(4)} (x^\mu - X^\mu(\sigma,\tau)) \left[T^{\hat{a}AB} V_A X^\nu_{,B} \right] \, , \end{aligned} \tag{2.20}$$

where

$$T^{\hat{a}AB} = (e^{-4\sigma} \gamma^{AB}, \epsilon^{AB}) \, . \tag{2.21}$$

Note that the electric current $j^{\hat{a}}$ contains two terms, the first term depending on the spacetime fields while the second is localized on the string world-sheet. For a string wrapped exclusively around the fifth dimension, appearing as a particle from the four-dimensional point of view, the current associated to A^2 takes a canonical form:

$$j^{2\mu} = \frac{\kappa_5^2}{\pi\alpha'} \frac{dX^\mu}{dt} \delta^{(3)} (\vec{x} - \vec{X}(\tau)) \, , \tag{2.22}$$

i.e., the current appropriate to a charged particle (charge proportional to $\frac{\kappa_5^2}{\pi\alpha'}$) following a trajectory $X^\mu(\tau)$.

Now consider the unwinding trajectory 2.4. The Kaluza-Klein monopole gives rise to a magnetic field $F^1_{\theta\phi}$, and hence if $H_{\mu\theta\phi}$ becomes non-zero at any point current will be present. This is precisely what happens for the unwinding string. The string couples to $B_{\mu\nu}$ and its unwinding motion 2.4 gives rise to a non-zero $H_{r\theta\phi}$. These two effects combine to a give a radial component of the current j^2, that is, the charge flows in towards the center of the monopole. Meanwhile, the string also couples to A^2_μ and gives a θ-directional current on the worldsheet so that the total current is

$$j^{2\nu} = \frac{\kappa_5^2 U^{3/2}}{4\pi^2\alpha' r^2}\left[\frac{(1+\cos\theta)\delta(r-r_0)}{\sin\theta}\frac{dX^\nu}{dt} - \frac{dX^\theta}{dt}\Theta(r_0 - r)\delta^\nu_r\right]\delta(\theta - X^\theta(t)) \tag{2.23}$$

for the unwinding configuration 2.4.

Note how the contribution of winding-charge from the string disappears as θ moves from 0 to π, whereas there is a steady radial inflow of charge via j^r. The total inflow is obtained by integrating over the sphere at radius r_0, and gives, not surprisingly, $\frac{\kappa_5^2}{\pi\alpha'}$. This means that the Kaluza-Klein monopole must be able to carry the electric charge which couples to A^2.

We can also understand this directly as arising from the excitation of a collective coordinate of the Kaluza-Klein monopole. As usual, the bosonic collective coordinates of a magnetic monopole of charge one arise either from translations or from gauge transformations which do not vanish at infinity. The Kaluza-Klein monopole has 3 collective coordinates from translations in $\mathbb{R}^3$ but does not have a collective coordinate from translation of x^5 because this is an exact symmetry of the solution. Instead the remaining collective coordinate arises from a gauge transformation of the antisymmetric tensor field of the form $\delta\mathcal{B} = \alpha(t)d\Lambda$ with $d\mathcal{B} = 0$ for constant α [8, 9]. Here Λ is a one-form non-vanishing at infinity such that $d\Lambda$ is a harmonic two-form – thus guaranteeing that the deformation has zero energy for constant α. Explicitly we have

$$\Lambda = 2\pi RU(dx^5 + \frac{1}{2}R(1-\cos\theta)d\phi)\ . \tag{2.24}$$

We can derive an effective coupling between the string and the Kaluza-Klein monopole collective coordinate by substituting the above expression for $\mathcal{B}$ into the action 2.5. This leads to the coupling

$$-\frac{R}{\alpha'}\int d^2\sigma\alpha(t)\epsilon^{AB}\left\{UF^1_{\theta\phi}X^\theta_{,A}X^\phi_{,B} + U'X^r_{,A}V_B\right\}. \tag{2.25}$$

We can now see how the string excites the zero mode. For unwinding motions at constant r such as 2.4, the θ and ϕ variations of the string couple via the magnetic field of the Kaluza-Klein monopole to α. For radial infall of the string which remains wrapped around the S^1, the 'particle' couples to α via the geometry. In each case, suppose X^ϕ and X^5 are given by 2.4 (but X^θ, X^r are arbitrary); then we obtain the following Lagrangian for α:

$$\mathcal{L}_\alpha = \frac{1}{2}\dot{\alpha}^2 + \frac{\alpha\kappa_4^2}{4\pi^2\alpha' R}\left[U\sin\theta\frac{d\theta}{dt} - U'(1+\cos\theta)\frac{dr}{dt}\right] . \tag{2.26}$$

For a string sitting initially at $\theta = 0$ asymptotically far from the Kaluza-Klein monopole, with $\dot{\alpha}$ consequently zero at the start of the motion, we see that the solution for $\dot{\alpha}$ is therefore

$$\dot{\alpha}(t) = \frac{\kappa_4^2}{4\pi^2\alpha' R}\left[2 - U(1+\cos\theta)\right] . \tag{2.27}$$

Thus, either for the string which radially infalls, or for the string unwinding at infinity, the net result is that $\dot{\alpha}$ has increased by $\frac{\kappa_4^2}{2\pi^2\alpha' R}$ once the string has disappeared.

3 The T-Dual Process

We can also consider a T-dual version of this process by performing a T-duality transformation along the x^5 direction, thus working in a background where x^5 has periodicity $\tilde{R} = \alpha'/R$. This transformation maps the Kaluza-Klein monopole into an H-monopole [2] and a string with winding around x^5 into a string with momentum along x^5 [10]. The T-duality between the Kaluza-Klein and H-monopole solutions will be discussed in more detail in the following section.

The H-monopole solution takes the five-dimensional form

$$\begin{aligned} \mathcal{H}_{\alpha\beta\gamma} &= 2\epsilon_{\alpha\beta\gamma}{}^{\delta}\nabla_\delta\Phi , \\ \mathcal{G}_{\alpha\beta} &= e^{2\Phi}\delta_{\alpha\beta} , \end{aligned} \tag{3.1}$$

where $\alpha, \beta, \cdots$ now run over $1,2,3,5$ and Φ is given by

$$e^{2\Phi}(x^5, \vec{x}) = e^{2\Phi_0} + \frac{\alpha'}{2\tilde{R}r}\frac{\sinh(r/\tilde{R})}{\cosh(r/\tilde{R}) - \cos\frac{x^5 - x_0^5}{\tilde{R}}} , \tag{3.2}$$

where $r = |\vec{x}|$ and x_0^5 is the location of the H-monopole along the S^1.

Note in particular that the H-monopole is *not* invariant under x^5 translations, unlike the Kaluza-Klein monopole. As a result its bosonic collective coordinates are given by translations in $\mathbb{R}^3$ as well as translation along the S^1. This implies that there is a zero mode carrying Kaluza-Klein electric charge, as expected from T-duality.

Since translation in x^5 is no longer a symmetry, there is no conservation law which forbids the decay of a string with momentum along x^5. However at large distance from the monopole this symmetry is approximately valid and thus we might expect a string with momentum at large distances from the monopole to be stable. We can, however, by analogy to the previous discussion write down a string trajectory which does remove the momentum of the string while keeping it infinitely far from from the monopole. The T-dual of the previous string trajectory is

$$\begin{aligned} x^5 &= 2\pi(\alpha'/\tilde{R})\tau \ , \\ \theta &= \pi\tau \ , \\ \phi &= -2\pi\sigma \ , \\ r &= r_0 \ , \end{aligned} \tag{3.3}$$

To an observer looking at the string near the north pole it appears that this configuration carries one unit of momentum in the x^5 direction. However the full expression for the momentum p_5 in the fifth direction is

$$p_5 = \int d\sigma \left[g_{5a}\dot{X}^a + B_{5a}X'^a \right] \ . \tag{3.4}$$

Far from the monopole the metric is approximately flat, but we must remember that it is necessary to use two different patches to describe the B field which is the vector potential of a Dirac monopole. Thus to evaluate p_5 in the Southern hemisphere we must use the appropriate form of B, which leads to

$$p_5 = 2\pi(\alpha'/\tilde{R})(1 + \cos\theta) \ , \tag{3.5}$$

which does indeed vanish at the south pole.

One way to think of this is as follows. In string theory we should distinguish between left and right-moving coordinates on S^1. In the Kaluza-Klein monopole background a string wound around the geometrical coordinate $x^5_- = x^5_L - x^5_R$ can unwind because the S^1 is fibered non-trivially over the S^2 at infinity. The T-dual version has a string with momentum along the geometrical coordinate x^5_+ which is equivalent to a string with winding around the dual coordinate $x^5_- = x^5_L - x^5_R$. The patching of the B field in the H-monopole is equivalent to saying that the global topology at infinity has S^1 with coordinate x^5_- fibered over the S^2 at infinity. This illustrates the fact that *both* string coordinates $x^5_\pm$ play a non-trivial role in the full solution.

An analysis similar to the previous one shows that Kaluza-Klein electric charge is conserved in this process by an inflow of current from the string into the H-monopole and excitation of the zero mode carrying Kaluza-Klein electric charge.

4 T-Duality of H-Monopoles and Kaluza-Klein Monopoles

T-duality of string theory is an exact discrete gauge symmetry of non-perturbative string theory. We may regard both the H-monopole and the Kaluza-Klein monopole as particular excitations of the type II string vacua $\mathbb{R}^4 \times S^1_{\tilde{R}} \times T^5$ and $\mathbb{R}^4 \times S^1_R \times T^5$, respectively. Regarded as states in string field theory these two solutions must be identical. There are, however, a number of puzzling points about the relation between the two solutions as given here.

Let us call the geometrical coordinate of $S^1_{\tilde{R}}$ x^5_+ and that of S^1_R in the T-dual background x^5_-. The first puzzle is that the Kaluza-Klein monopole appears to have an isometry while the H-monopole does not: The H-monopole solution is known to correspond to an exact conformal field theory and is thus an exact solution to string theory without any α' corrections. As noted in the previous section it is not invariant under translations in x^5_+. Thus it is not clear how to apply the standard rules for T-duality transformations in x^5_+ since these require the existence of a Killing vector field. On the other hand, the T-dual Kaluza-Klein monopole does have a Killing vector given by translation in x^5_- and is also supposed to be an exact solution of string theory since the metric is hyperkahler. The second puzzle is that the H-monopole exhibits a "throat" metric at small values of $\vec{x}$ and x^5_+. This is a reflection of the standard throat metric transverse to a solitonic fivebrane. On the other hand the Kaluza-Klein monopole metric has no sign of such behavior. How can these facts be reconciled with T-duality?

In this section we will argue that these puzzles are resolved by modifications to the Kaluza-Klein monopole solution in string theory due to massive string winding modes. In particular, we will argue that the Kaluza-Klein monopole has a sort of throat behavior which would be probed by scattering of string winding modes in the same way that the H-monopole throat would be probed by the scattering of string momentum states.

There are several ways to understand why these modes are important in the Kaluza-Klein monopole background. First, at the core of the monopole the radius of x^5_- approaches zero. Thus near the the core of the monopole the string winding modes become arbitrarily light and must be included in the dynamics. Second, from the T-dual point of view, we can Fourier decompose the H-monopole solution by writing

$$e^{2\Phi}(x^5_+, \vec{x}) = \sum_{n=-\infty}^{+\infty} e^{inx^5_+/\tilde{R}} \Psi_n(\vec{x}) \ , \tag{4.1}$$

with

$$\Psi_0 = e^{2\phi_0} + \frac{\alpha'}{2r\tilde{R}} \tag{4.2}$$

and

$$\Psi_n = \frac{\alpha'}{2r\tilde{R}} e^{-|n|r/\tilde{R}} e^{-inx^5_{+,0}/\tilde{R}} , \tag{4.3}$$

for $n \neq 0$. The Kaluza-Klein monopole solution arises from applying the standard T-duality transformation laws to the metric and $\mathcal{B}$ field determined from Ψ_0. The Ψ_n can be thought of as the classical values of the Kaluza-Klein momentum states in the H-monopole background. Note that they transform by a phase under a shift of $x^5_{+,0}$ which is the zero mode collective coordinate for translation in the x^5_+ position of the 5-brane. Under T-duality, we would expect that there would be classical values for the string winding fields in the dual background. Thus under T-duality we expect that in a background with x^5_- having radius $R = \alpha'/\tilde{R}$ and the x^5_+ independent fields being those of the Kaluza-Klein monopole, that there should be classical values for the fields for a string of winding n given by

$$\tilde{\Psi}_n = \frac{R}{2r} e^{-|n|rR/\alpha'} e^{-inx^5_{+,0}R/\alpha'} , \tag{4.4}$$

where $x^5_{+,0}$ is the collective coordinate for translations in the coordinate x^5_+.

The values 4.4 of the classical winding fields can be understood, at least qualitatively, in three ways. Two ways are string-theoretic, the third involves an effective field theory. The first way involves string field theory. We regard the Kaluza-Klein monopole as a particular state in the type II string theory built on the vacuum $\mathbb{R}^4 \times S^1_R \times T^5$. As such it must be described by a vector in the space of states $\mathcal{H}_0$ of the conformal field theory associated to this vacuum. Thus, the Kaluza-Klein monopole must be described by a string wavefunction $\Psi_{KK}[X(\sigma)] \in \mathcal{H}_0$. Concretely, the values of the wavefunction are defined by evaluating the worldsheet path integral on the disk using the Kaluza-Klein background σ-model and imposing boundary conditions $X(\sigma, r=1) = X(\sigma)$. On the other hand, since it is a state in $\mathcal{H}_0$ we can expand in a basis of states $\{\mathcal{O}\}$ for $\mathcal{H}_0$: $\Psi_{KK}[X(\sigma)] = \sum_{\mathcal{O}} c_{\mathcal{O}} \Psi_{\mathcal{O}}$. Moreover, we can choose an orthonormal basis with respect to the natural bilinear pairing defined by the 2-point function on the sphere. The values of the coefficient $c_{\mathcal{O}}$ correspond to the values of the spacetime fields associated to the vertex operator $\mathcal{O}$. In particular, the value of the coefficient $c_{\mathcal{O}}$ for vertex operators creating winding strings in $\mathbb{R}^4 \times S^1_R \times T^5$ is given by sewing into the disk amplitude defining $\Psi_{KK}[X(\sigma)]$ the disk amplitude with $\mathcal{O}$ at its center. In the semiclassical theory in α' the effect of the vertex operator is to impose winding boundary conditions on the Kaluza-Klein path integral. The classical action then gives the worldsheet disk instanton factor in 4.4.

A second way to understand 4.4 (pointed out by E. Witten) is that in the standard argument for T-duality [10, 11] obtained by gauging an isometry with a $U(1)$ gauge field A and then integrating out the gauge field, instantons in the gauge field A will spoil the translation symmetry which ordinarily forbids classical field values for $\tilde{\Psi}_n$.

A third way to understand 4.4 uses a low energy effective field theory description. For simplicity consider the following reduction of the problem. We will work in a regime where we can represent the dyon degree of freedom of the monopole as being localized at the origin. Now of all the many fields describing excitations of the string (as in string field theory) we keep only the fields $\tilde{\Psi}_n$ which create strings with winding n around x^5_- far from the monopole where, at least locally, the metric makes spacetime look like $\mathbb{R}^3 \times S^1$. We now want to write an effective Lagrangian describing these winding fields and their coupling to the dyon degree of freedom, $\alpha(t)$.

As a first step note that for a *classical* winding mode, $x^5_- = 2\pi nR$, the effective action can be seen to be (c.f. 2.25 and 2.26)

$$\frac{(2\pi R)^3}{2\kappa_4^2}\int \frac{1}{2}\dot{\alpha}^2 dt - \frac{|n|R}{\alpha'}\int \left(\frac{1}{2}e^{-2\sigma}|\dot{X}^\mu| + 2\pi R\alpha U'\dot{X}^\tau\right) d\tau \ . \tag{4.5}$$

The second part of this is the action for a charged relativistic particle of mass $m_n = |n|R/\alpha'$, charge proportional to $n\alpha$, in the conformal frame $\hat{g}_{\mu\nu} = e^{-4\sigma}g_{\mu\nu}$. Therefore, when describing the winding mode as a quantum field, we must remember to include the correct conformal factor in front of the mass term, as well as including the additional factor of $e^{-2\phi} = e^{-2\sigma}$ resulting from the dimensional reduction.

$$\begin{aligned} S_{\text{eff}} &= \frac{1}{2\kappa_4^2}\int d^3x dt \left[(D_\mu\tilde{\Psi}_n)^*(D^\mu\tilde{\Psi}_n) + e^{-4\sigma}m_n^2\tilde{\Psi}_n^*\Psi_n\right]\sqrt{-g}e^{-2\sigma} \\ &\quad + \delta^3(x)\left[(2\pi R)^3(\partial_0\alpha + A_0)^2 + (2\pi R)b(e^{in\alpha}\tilde{\Psi}_n^* + e^{-in\alpha}\tilde{\Psi}_n)\right] . \end{aligned} \tag{4.6}$$

Since the string winding states can decay we know that there is a mixing between α and the $\tilde{\Psi}_n$, the form of which is dictated by gauge invariance. The dimensionless constant b represents the ambiguity in replacing the A^2_μ field by a point source at the origin; however, note that the magnitude of this coupling is independent of n, as it should be since the geodesic motion for string winding states is independent of n [5].

We can now solve for the static configuration of the $\tilde{\Psi}_n$ in a background with $A^2_\mu = 0$

$$(-\nabla^2 + m_n^2)\tilde{\Psi}_n = 2\pi R b e^{in\alpha}\delta^3(x) \ , \tag{4.7}$$

(where ∇^2 is now the flat space laplacian), with solution

$$\tilde{\Psi}_n = \frac{bR}{2r} e^{in\alpha} e^{-m_n r} \equiv \frac{R}{2r} e^{in\alpha} e^{-|n| r R/\alpha'} \ . \tag{4.8}$$

This agrees with what we obtained by T-duality if we identify α with the dual collective coordinate for translations in x^5_+: $\alpha = -x^5_{+,0} R/\alpha'$, and set $b = 1$. Of course it is not clear that the correct T-duality transformation on massive fields does not include some additional scaling, but the argument given here should give an accurate description of the asymptotic value of the string winding modes.

The above results suggest that the Kaluza-Klein monopole as a solution to string theory also exhibits the same kind of throat behavior as the H-monopole solution. This throat could be probed by scattering string winding states of the full Kaluza-Klein monopole solution in the same way that the H-monopole throat can be probed by scattering string momentum states. Unfortunately it is difficult to give a precise description of the throat behavior purely in field theoretic terms. The reason is that the collective coordinate α can also be thought of as a translational zero mode for the dual stringy coordinate x^5_+ and it is difficult to find a field theoretic formalism which incorporates both x^5_- as well as x^5_+ as geometrical coordinates.

5 Conclusions and Outlook

The construction we have described is clearly quite general. The global topology of the Kaluza-Klein and H-monopoles ensures that a string with winding or momentum can be unwound at infinity. Since the winding number is a conserved charge, there must therefore be an excitation of the monopole which can carry this charge. We deduce this purely from the physics at large distances from the monopole but can then verify that this is in fact consistent with the exact structure of the solution. This is in some ways reminiscent of the anomaly inflow argument of [12] which also allows one to deduce zero mode structure on various defects from an analysis of couplings at large distances from the defect.

One possible application of this mechanism is to the study of black hole entropy. Since the analysis involves only the large distance physics, it clearly applies quite generally to black holes or D-brane configuration which carry Kaluza-Klein or H magnetic charge and provides a way to change the charge of the black hole by moving strings around at infinity. Both charged fundamental strings and charged black holes can carry non-zero entropy; it would be interesting to see if this mechanism can be used to transfer entropy from a black hole to a fundamental string at infinity. If so this might provide a concrete way to identify the microstate of a black hole outside the D-brane regime.

We have also resolved – at least qualitatively – the puzzles associated with T-duality between Kaluza-Klein and H-monopoles. In particular, the zero mode structure is precisely what one would expect from T-duality. In string theory we should distinguish between x^5_L and x^5_R. The H-monopole solution is not invariant under shifts of $x^5_+ = x^5_L + x^5_R$ due to the excited Kaluza-Klein momentum states but is invariant under shifts of $x^5_- = x^5_L - x^5_R$. The same holds for the T-dual Kaluza-Klein monopole with the crucial difference that the role of the stringy coordinate x^5_- and the geometrical coordinate used to write the metric x^5_+ are interchanged: In the Kaluza-Klein monopole x^5_- is the geometrical coordinate, while x^5_+ is stringy. Note that the coordinate x^5_+ is associated with a topologically trivial fibration over space, while the coordinate x^5_- is only locally defined and is a fiber coordinate in a nontrivial S^1 fibration. This holds true in *both* the Kaluza-Klein and H-monopole backgrounds, as required by T-duality. From this understanding of T-duality we are forced to conclude that the classical values of the string winding fields in the Kaluza-Klein monopole background generate a throat which cannot however be described purely in field theoretic terms and must be probed by scattering with winding strings.

Finally, we note that this sort of effect is likely to lead to stringy modifications to other solutions of string theory which are also related by T-duality. The throat metric and its T-dual also appear in a number of other contexts, for example in describing the dynamics of D1-branes in the background of D5-branes and in the compactifications of this much-studied system. The string corrections to the Kaluza-Klein monopole solution described here (i.e. to the Taub-Nut metric) are likely to have application to these systems as well.

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