Adv. Theor. Math. Phys. 1 (1997) 271–282

New String Theories in Six Dimensions via Branes at Orbifold Singularities

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Abstract

We present several classes of new 6 dimensional string theories which arise via branes at orbifold singularities. They have compact moduli spaces, associated with tensor multiplets, given by Weyl alcoves of non-Abelian groups. We discuss T-duality and Matrix model applications upon compactification.

1 Introduction

It was recently pointed out in [1] that new 6d theories, which include stringy excitations but without gravity, can be obtained in the world-volume of five-branes by taking $g_s \to 0$ with M_s held fixed. Four different classes were obtained in [1]:

[1] new address

(iia) Theories with $\mathcal{N} = (1,1)$ supersymmetry, which are obtained in type IIB five-branes or, alternatively [2], via type IIA with a $\mathbb{C}^2/\Gamma_G$ ALE singularity [2].

(iib) Theories with $\mathcal{N} = (2,0)$ supersymmetry, which are obtained in the world-volume of type IIA (or M-theory) five-branes or, alternatively [2], via type IIB with a $\mathbb{C}^2/\Gamma_G$ singularity.

(o) Theories with $\mathcal{N} = (1,0)$ supersymmetry in the world-volume of $SO(32)$ heterotic small instantons or type I five-branes.

(e) Theories with $\mathcal{N} = (1,0)$ supersymmetry in the world-volume of E_8 small instantons.

The (o)-theory has a global $SO(32)$ symmetry and the (e)-theory has a global $E_8 \times E_8$ symmetry.

The infrared limit of these theories, with energies small compared to M_s, appear to be local quantum field theories. In the (iib) and (e) cases these are non-trivial, interacting, RG fixed points, while the (iia) and (o) cases are IR free. Despite their different IR behavior, upon compactification to five dimensions on a circle, T-duality exchanges the (iia) $\leftrightarrow$ (iib) and (o) $\leftrightarrow$ (e) theories. Thus the full theories are *not* local quantum field theories [1].

In this paper, we discuss new 6d $\mathcal{N} = (1,0)$ theories associated with type II or heterotic five-branes at orbifold singularities in the orthogonal four dimensions. As in [1], we take $g_s \to 0$ with M_s fixed. The fact that new theories could be thus obtained was also mentioned during the course of this work in a footnote in [3]. It was there pointed out that one could have a general, Ricci-flat, non-compact manifold $\mathcal{M}_4$ in the remaining four directions, giving theories which, in principle, could depend on the uncountably infinite parameters needed to specify $\mathcal{M}_4$. However, as in [4], we expect most of these parameters are irrelevant in the $g_s \to 0$ limit and that only the $\mathbb{C}^2/\Gamma_G$ singularity type matters. Clearly the singularity itself cannot be ignored; indeed, it breaks the supersymmetry of the (iia)- or (iib)-theories to $\mathcal{N} = (1,0)$ supersymmetry.

The 6d string theories which we present have compact "Coulomb branches", associated with expectation values of the scalar components of 6d $\mathcal{N} = (1,0)$ tensor multiplets, which are the "Coxeter boxes" (also referred to as the "Weyl alcove") of non-Abelian groups. For any group $\mathcal{G}$ of rank r, the Coxeter box is a compact subspace of $\mathbb{R}^r$ given by all $\vec{\Phi} \in \mathbb{R}^r$ which satisfy

$$\vec{\alpha}_\mu \cdot \vec{\Phi} + M_s^2 \delta_{\mu 0} \geq 0, \qquad \mu = 0, \ldots, r , \tag{1.1}$$

[2] We label $\Gamma_G \subset SU(2)$ using the well-known correspondence with the simply-laced groups $G = A_r,\ D_r,\ E_{6,7,8}$.

where $\vec{\alpha}_\mu$ are the simple roots, including the extended root $\mu = 0$, with $\sum_{\mu=0}^{r} n_\mu \vec{\alpha}_\mu = 0$ (n_μ are the Dynkin indices). The $\mu \neq 0$ conditions in 1.1 give the non-compact Weyl chamber $\mathbb{R}^r/\mathcal{W}_G$, where $\mathcal{W}_G$ is the Weyl group. Including the $\mu = 0$ condition gives the Coxeter box $\mathbb{R}^r/\mathcal{C}_\mathcal{G} \cong (S^1)^r/\mathcal{W}_\mathcal{G}$, where the Coxeter group $\mathcal{C}_\mathcal{G}$ includes translations in the root lattice of $\mathcal{G}$. Compact Coxeter box moduli spaces, of size R^{-1}, also arise via Wilson loops upon reducing a $\mathcal{G}$ gauge theory on a circle of radius R. We have written the size of the Coxeter box 1.1 as M_s^2 because here this is what it will be.

Coxeter boxes already appear in the theories (iib) and (e) mentioned above. Part of the moduli space of the (iib)-theory obtained from K parallel five-branes is the $U(K)$ Coxeter box of size M_s^2. The (iib)-theory obtained from type IIB string theory on a $\mathbb{C}^2/\Gamma_G$ ALE singularity has, as part of its moduli space, the Coxeter box of size M_s^2 of the corresponding ADE group G. The (e)-theory obtained from K small $E_8 \times E_8$ instanton five-branes has the Coxeter box, again of size M_s^2, for $Sp(K)$ as its Coulomb branch.

We will simply note some basic features of the new 6d string theories, saving a more detailed analysis for further study. In the next section, we discuss theories associated with type IIB NS five-branes at orbifold singularities. The tensor multiplet moduli live on the Coxeter box of the simply laced group G associated with the singularity. In sect. 3 we discuss theories associated with $SO(32)$ heterotic or type I branes at orbifold singularities. In these examples, the tensor multiplet moduli can live in the Coxeter box of a non-simply-laced subgroup of G. In sect. 4, we discuss theories associated with $E_8 \times E_8$ branes at orbifold singularities. In sect. 5, we discuss T-duality upon compactification. Finally, in sect. 6, we discuss applications of the theories to providing a definition of M-theory on (ALE) $\times T^5 \times \mathbb{R}^{1,1}$ and M-theory on (ALE) $\times (T^5/\mathbb{Z}_2) \times \mathbb{R}^{1,1}$.

2 Type IIB Branes at a $\mathbb{C}^2/\Gamma_G$ Orbifold Singularity

For our first class of examples, consider K parallel type IIB NS five-branes at a $\mathbb{C}^2/\Gamma_G$ orbifold singularity in the transverse directions. Having five-branes but no ALE singularity would lead to a (iia)-theory of [1]. Having the ALE singularity but no five-branes would lead to a (iib)-theory of [1]. Putting the two situations together leads to new $\mathcal{N} = (1,0)$ string theories, whose field theory infrared limit was discussed in [5].

As discussed in [5], the $\mathcal{N} = (1,0)$ theory has gauge group

$$\prod_{\mu=0}^{r} U(Kn_\mu), \tag{2.1}$$

with matter multiplets in the representations $\frac{1}{2} \oplus_{\mu\nu} a_{\mu\nu} (\square_\mu, \overline{\square}_\nu)$. In addition, there are $r \equiv \operatorname{rank} G$ hypermultiplets and tensor multiplets (which would give r $\mathcal{N} = (2,0)$ matter multiplets for the theory with no five-branes). r of the $U(1)$ factors in 2.1 have charged matter and are thus anomalous in 6d. As in [6, 7], this means that these $U(1)$ factors are spontaneously broken; they pair with the r hypermultiplets mentioned above to get a mass. The massless, unbroken gauge group is thus

$$U(1) \times \prod_{\mu=0}^{r} SU(Kn_\mu), \tag{2.2}$$

with the $U(1)$ factor decoupled, with no charged matter. Although the $U(1)$ factors in 2.1 are massive, their D term equations still constrain the moduli space. Supersymmetry implies that the expectation values of the r hypermultiplets involved in the $U(1)$ anomaly cancelation appear as Fayet-Iliopoulos terms in these constraints [6]; these are the ALE blowing-up modes, which enter as background parameters in the 6d theory.

Taking $g_s \to 0$ with M_s fixed, the tensor multiplet moduli space is the Coxeter box 1.1 of the corresponding ADE group G. This can be seen starting from the (iib)-theory associated with the ALE space and no branes.

Using results found in [5, 8] via anomalies, the effective gauge coupling of the $r+1$ gauge groups on the Coulomb branch can be written as

$$g_\mu^{-2}(\vec{\Phi}) = \vec{\alpha}_\mu \cdot \vec{\Phi} + M_s^2 \delta_{\mu 0} \ , \tag{2.3}$$

where, as in 1.1, the $\vec{\alpha}_\mu$ are the simple and extended roots of the ADE group G associated with the singularity. Using $\vec{\alpha}_\mu \cdot \vec{\alpha}_\mu = \widetilde{C}_{\mu\nu}$, the extended Cartan matrix of G, the couplings in 2.3 cancel the reducible $\widetilde{C}_{\mu\nu}\mathrm{tr}F_\mu^2\mathrm{tr}F_\nu^2$ anomaly terms found in [5, 8]. We see that, as required, all $g_\mu^{-2} \geq 0$ over the entire Coulomb box 1.1, with the various $g_\mu^{-2} = 0$ along the boundaries of the Coulomb box. The "Landau pole" mentioned in [5, 8] has been eliminated by the compactness of the Coulomb branch for finite M_s.

There is a Higgs mode of the theory corresponding to moving the K five-branes away from the $X_G \cong \mathbb{C}^2/\Gamma_G$ ALE space. This Higgs branch moduli space is $\mathcal{M}_H \cong (X_G)^K/S_K$, as expected, with 2.2 broken to the diagonal $U(K)_D$ away from the origin (or with non-zero Fayet-Iliopoulos parameters). This $U(K)_D$ theory is the (iia)-theory of the branes away from the singularity, with gauge coupling $g_D^{-2} = \sum_{\mu=0}^{r} n_\mu g_\mu^{-2} = M_s^2$ as expected. The low energy theory has an enhanced, accidental $\mathcal{N} = (1,1)$ supersymmetry which is not respected by the massive field theory and stringy modes.

There are also interesting new 6d theories associated with type IIA NS 5-branes at orbifold singularities, which require further understanding. For the case of K-branes at a $\mathbb{C}^2/\mathbb{Z}_M$ singularity, the 6d theory could be the same theory as that of M type IIB branes at a $\mathbb{C}^2/\mathbb{Z}_K$ singularity (up to a decoupled tensor multiplet in the former and vector multiplet in the latter).

3 New Theories from $SO(32)$ Branes at ALE Singularities

Our next class of new 6d string theories with $\mathcal{N} = (1,0)$ supersymmetry arise from $SO(32)$ heterotic or type I 5-branes at $\mathbb{C}^2/\Gamma_G$ orbifold singularities. The low energy limit of these theories was discussed in [5,8,9] and also, via F-theory, in [10,11]. The gauge group is

$$\prod_{\mu\in\mathcal{R}} Sp(v_\mu) \times \prod_{\mu\in\mathcal{P}} SO(v_\mu) \times \prod_{\mu\in\mathcal{C}} U(v_\mu), \tag{3.1}$$

where the nodes of the extended G Dynkin diagram have been grouped into the sets $\mathcal{R}$, $\mathcal{P}$, $\mathcal{C}$, $\overline{\mathcal{C}}$ discussed in detail in [5]. As in the discussion following 2.1, the overall $U(1)$ factor in each $U(v_\mu)$ is anomalous and thus pairs with a hypermultiplet to get a mass.

The tensor multiplet structure is related to the Coxeter box of the corresponding simply-laced group G, but modded out by a $\mathbb{Z}_2$ action $*$ which takes $\mathcal{C} \leftrightarrow \overline{\mathcal{C}}$. From the analysis in [5,9], the result is that the tensor multiplets for a $\mathbb{C}^2/\Gamma_G$ singularity live in the Coxeter box of $H \subset G$ with $G \to H$ as

$$\begin{array}{ccc} SU(2P) & \to & Sp(P)\ , \\ SO(4P+2) & \to & SO(4P+1)\ , \\ SO(4P) & \to & SO(4P)\ , \\ E_6 & \to & F_4\ , \\ E_7 & \to & E_7\ , \\ E_8 & \to & E_8\ . \end{array} \tag{3.2}$$

The operation in 3.2 is the same modding out which appeared in the description of [12,13] for obtaining composite gauge invariance with non-simply-laced gauge groups. Although it is outside of the focus of this work, we note that the hyper-Kähler quotient construction of [5,8] for the moduli space of $SO(N)$ instantons on ALE spaces suggests an interesting analog of the results of Nakajima. Briefly put, Nakajima [14] showed that $\widehat{G}_N$ affine Lie algebras arise in analyzing the moduli space of $U(N)$ instantons on $\mathbb{C}^2/\Gamma_G$. Similarly, we expect $\widehat{H}_N$ affine Lie algebras to arise in analyzing the moduli space of $SO(N)$ instantons on $\mathbb{C}^2/\Gamma_G$, with $G \to H$ as in 3.2. The results of [14] find physical application, for example in [15], in showing that simply-laced composite gauge invariance is properly represented on massive modes. The conjectured appearance of $\widehat{H}_N$ affine Lie algebras could find similar application in compactifications with non-simply-laced composite gauge invariance.

Other 6d theories can be obtained by making use of the fact, as in [7], that the gauge group of the heterotic or type I theory is actually $Spin(32)/\mathbb{Z}_2$. The low-energy limit of these string theories in the case of $\mathbb{C}^2/\mathbb{Z}_{2P}$ singularities was discussed in [8], where it was (sloppily) referred to as the case without vector structure. The result is a theory based on the "type I5 quiver diagrams" of [6], with gauge group

$$\prod_{i=1}^{P} SU(v_\mu) \tag{3.3}$$

and tensor multiplets which live in the Coxeter box, of size M_s^2, of $Sp(P-1)$. For the simplest example, $\mathbb{C}^2/\mathbb{Z}_2$, the low energy theory is $SU(2K)$ with two matter fields in the $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$ and sixteen in the $\Box$ and no tensor multiplet.

4 New Theories from $E_8 \times E_8$ Branes at Orbifold Singularities

Our next class of new 6d string theories with $\mathcal{N} = (1,0)$ supersymmetry arise via $E_8 \times E_8$ 5-branes at orbifold singularities in the $g_s \to 0$ with M_s fixed limit. The gauge group and number of tensor multiplets associated with point-like E_8 instantons at ADE orbifold singularities was obtained via F-theory in [11]. We take this opportunity to briefly spell out the massless matter content of these theories, which we determine from the results of [11] combined with anomaly considerations, as it was not presented in [11]. First, the irreducible $\mathrm{tr}F^4$ gauge anomalies must vanish; the remaining reducible anomalies must then be canceled by coupling to the tensor multiplets. In addition, as discussed in [16], a π_6 anomaly restricts $SU(2)$ to have $n_2 \equiv 4 \pmod 6$, $SU(3)$ to have $n_3 \equiv 0 \pmod 6$, and G_2 to have $n_7 \equiv 1 \pmod 3$. A further general condition is

$$n_H - n_V + 29 n_T = 30K + r, \tag{4.1}$$

where n_H is the total number of hypermultiplets, n_V is the total number of vector multiplets, n_T is the number of tensor multiplets, K is the number of small instantons or five-branes, and $r \equiv \mathrm{rank}\ G$ is the number of ALE blowing-up modes. The condition 4.1 is a 6d analog of a 't Hooft anomaly matching condition for the gravitational anomaly.

The theory (e) for K $E_8 \times E_8$ five-branes and no singularity has a Coulomb branch with $n_T = K$ tensor multiplets and no vector multiplet gauge group. Putting the K 5-branes at a $\mathbb{C}^2/\mathbb{Z}_M$ singularity, with $K \geq 2M$, the result of [11] is that there is a Coulomb branch, again with $n_T = K$ tensor multiplets, but with new gauge fields, with gauge group

$$\begin{aligned} &SU(2) \otimes SU(3) \otimes \cdots \otimes SU(M-1) \otimes SU(M)^{\otimes(K-2M+1)} \otimes \\ &\quad SU(M-1) \otimes \cdots \otimes SU(2). \end{aligned} \tag{4.2}$$

The massless matter content consists of bi-fundamentals charged under each neighboring pair of gauge groups in 4.2 as well as an extra fundamental flavor for each of the two $SU(2)$s at the ends and for each of the two $SU(M)$s at the end of the string of $SU(M)s$. As remarked in [11], the gauge group in 4.2 agrees (up to replacing the $SU(n)$ with $U(n)$) with that of [17, 18] which is mirror dual in three dimensions to $U(M)$ gauge theory with K flavors; the above hypermultiplet content also agrees with that of [17, 18]. The theory with this gauge group and matter content is properly free of gauge anomalies (making use of couplings to $K-3$ of the tensor multiplets to cancel the reducible gauge anomalies).

The theory with the above gauge group and matter content properly has a $(K+M-1)$-dimensional Higgs branch, with the gauge group generically completely broken. $M-1$ of the Higgs branch moduli correspond to the blowing-up modes of the $\mathbb{C}^2/\mathbb{Z}_M$ orbifold. The remaining K dimensions is the K-fold symmetric product of the ALE space with those $M-1$ moduli, corresponding to the locations of the K identical, point-like instantons on the ALE space. For generic values of these moduli, the 5-branes are away from any singularity and there are no vector multiplets; the gauge symmetry 4.2 is unHiggsed when the moduli are tuned, corresponding to putting the 5-branes on the singularity.

For $K=6$ five-branes at a $G=D_4$ singularity, the result of [11] is that the gauge group is $SU(2)\times G_2\times SU(2)$ with $n_T=6$ tensor multiplets. The matter content is determined by anomaly considerations to be $\frac{1}{2}(\mathbf{2},\mathbf{1},\mathbf{1})\oplus\frac{1}{2}(\mathbf{2},\mathbf{7},\mathbf{1})\oplus\frac{1}{2}(\mathbf{1},\mathbf{7},\mathbf{2})\oplus\frac{1}{2}(\mathbf{1},\mathbf{1},\mathbf{2})\oplus 2(\mathbf{1},\mathbf{7},\mathbf{1})$. This theory has a 10-dimensional Higgs branch, with the gauge group generically completely broken, corresponding to the location of the six point-like instantons on the ALE space and its four blowing-up modes. Giving an expectation value to a matter fields in the $(\mathbf{1},\mathbf{7},\mathbf{1})$ corresponds to smoothing the D_4 singularity to an A_2 singularity.

For $K\geq 7$ five-branes at a $G=D_4$ singularity, the result of [11] is gauge group $SU(2)\times G_2\times SO(8)^{K-7}\otimes G_2\otimes SU(2)$ with $n_T=2K-6$. The matter content is determined by anomaly considerations to be $\frac{1}{2}(\mathbf{2},\mathbf{1})\oplus\frac{1}{2}(\mathbf{2},\mathbf{7})$ for each $SU(2)\times G_2$ pair and no other matter fields.

For E_6, the result of [11] is $n_T=4K-22$, with gauge group $SU(2)\times G_2\times F_4\times G_2\times SU(2)$ for $K=8$ and gauge group $SU(2)\times G_2\times F_4\times SU(3)\times(E_6\times SU(3))^{K-9}\times F_4\times G_2\times SU(2)$ for $K>8$. The matter content is determined by anomaly considerations to consist, as above, of the minimal $SU(2)\times G_2$ matter $\frac{1}{2}(\mathbf{2},\mathbf{1})\oplus\frac{1}{2}(\mathbf{2},\mathbf{7})$ in each pair of $SU(2)\times G_2$. For $K=8$

the F_4 has a single matter field in the **26** (giving it an expectation value breaks $F_4 \to SO(9) \to SO(8)$, corresponding to smoothing the singularity from $E_6 \to D_5 \to D_4$). For $K > 8$ each $SU(2) \times G_2$ pair has the same minimal matter content as above, and there is no other matter.

For $K \geq 10$ five-branes at an E_7 singularity, the result of [11] is $(SU(2) \times G_2)^4 \times F_4^2 \times E_7 \times (SU(2) \times SO(7) \times SU(2) \times E_7)^{K-10}$ with $n_T = 6K - 40$. Each $SU(2) \times G_2$ factor has the minimal matter appearing above. Each $SU(2) \times SO(7) \times SU(2) \times E_7$ factor has matter $\frac{1}{2}(\mathbf{2},\mathbf{8},\mathbf{1},\mathbf{1}) \oplus \frac{1}{2}(\mathbf{1},\mathbf{8},\mathbf{2},\mathbf{1})$. There is no other matter.

For $K \geq 10$ five-branes at an E_8 singularity, the result of [11] is gauge group $E_8^{(K-9)} \times F_4^{(K-8)} \times (SU(2) \times G_2)^{2K-16}$ with $n_T = 12K - 96$. Each $SU(2) \times G_2$ factor has the minimal matter content appearing above and there is no other matter.

The result of [11] for $K = 2m + 6$ five-branes at a D_{m+4} singularity is $n_T = 2K-6$ and gauge group $SU(2) \times G_2 \times SO(9) \times SO(3) \times SO(11) \times SO(5) \times \cdots \times SO(2m+5) \times SO(2m-1) \times SO(2m+7) \times SO(2m-1) \times \cdots \times SO(9) \times G_2 \times SU(2)$. For $K > 2m + 6$ five-branes at a D_{m+4} singularity, [11], again find $n_T = 2K - 6$ and, in addition to the gauge group factors for $K = 2m+6$, $Sp(m) \times (SO(2m + 8) \times Sp(m))^{(K-2m-7)} \times SO(2m + 7)$. For $m > 1$, we were not able to find a solution for matter content which is compatible with anomaly considerations and these gauge groups, though perhaps one does exist[3].

5 Compactification and T-Duality

It is natural to expect that, upon compactification on a circle, the new theories associated with five-branes at singularities are related by T-duality,

[3]**Note added** (in revised version, 9/3/97): There is a slight modification of the above gauge groups for which there is a matter content which *is* nicely compatible with all of the anomaly considerations. For $K = 2m + 6$ five-branes at a D_{m+4} singularity, with $n_T = 2K-6$ as in [11], the modified gauge group is $SU(2) \times G_2 \times SO(9) \times Sp(1) \times SO(11) \times Sp(2) \times \cdots \times SO(2m+5) \times Sp(m-1) \times SO(2m+7) \times Sp(m-1) \times \cdots \times SO(9) \times G_2 \times SU(2)$. The matter content which satisfies all of the anomaly equations is given by the minimal $\frac{1}{2}((\mathbf{2},\mathbf{1}) \oplus (\mathbf{2},\mathbf{7}))$ in each $SU(2) \times G_2$ factor and a half-hypermultiplet bi-fundamental charged under each neighboring SO and Sp, i.e. a $\frac{1}{2}(\mathbf{2k+7},\mathbf{2k})$ under each neighboring $SO(2k+7) \times Sp(k)$ and a $\frac{1}{2}(\mathbf{2k},\mathbf{2k+9})$ under each neighboring $Sp(k) \times SO(2k+9)$. In addition, the middle $SO(2m+7)$ gauge group has a hypermultiplet in the $\mathbf{2m+7}$ which is uncharged under the other gauge groups. For the cases $m = 2,3$, where the gauge group agrees with that of [11] (as $Sp(1) \cong SO(3)$ and $Sp(2) \cong SO(5)$), this matter content was first worked out by G. Rajesh. I am very grateful for his correspondence on the $m = 2,3$ cases, which helped to inspire the above modified gauge groups and matter content for $m > 3$. I also thank P.S. Aspinwall and D.R. Morrison for helpful correspondence on these issues. A similar modification of the gauge group and matter content applies for $K > 2m + 6$.

generalizing that of [1] between (iia) $\leftrightarrow$ (iib) and (o) $\leftrightarrow$ (e). As in [1], this can be put to a simple test.

Upon compactifying on a circle, both the Cartan of the 6d gauge group and the 6d tensor multiplets lead to 5d $U(1)$ gauge fields with scalar moduli. The number of 5d scalar moduli is thus r_V+n_T, where r_V is the rank of the 6d vector multiplet gauge group and n_T is the number of 6d tensor multiplets. Two 6d theories related by T-duality must thus have $r_V + n_T = \widetilde{r}_V + \widetilde{n}_T$. More precisely, tensor multiplets in 6d have a compact "Coulomb branch," with the scalar moduli living on a box of size M_s^2. Upon reducing to 5d and rescaling the modulus to have dimension one, it lives on a box of size M_s^2R. On the other hand, reducing a 6d vector multiplet to 5d leads to a scalar modulus which lives on a box of size R^{-1}. Because T-duality relates a theory compactified on a circle of radius R to another theory compactified on a circle of radius $\widetilde{R} \equiv (M_s^2R)^{-1}$, it exchanges 5d moduli associated with 6d tensor multiplets with those associated with 6d vector multiplets. Thus T-dual theories must satisfy the stronger conditions $\widetilde{r}_V = n_T$ and $\widetilde{n}_T = r_V$.

This can be thought of as a reason why, as we have seen, the Coulomb branch of 6d tensor multiplets is the Coxeter box of a non-Abelian group. Compactifying on a circle, there should be a T-dual theory where these moduli *do* arise from a gauge theory with that gauge group.

For example, the vector multiplets of the (iia)-theory compactified on a circle of radius R and the tensor multiplets of the (iib)-theory compactified on a circle of radius $\widetilde{R} \equiv (M_s^2R)^{-1}$ both lead to moduli living on a Coxeter box of size R^{-1}, compatible with their equivalence [1]. Similarly, both the $SO(32)$ theory (o), compactified on a circle of radius R, with Wilson lines which break it to $SO(16) \times SO(16)$, and the theory (e) on a circle of radius $\widetilde{R} \equiv (M_s^2R)^{-1}$ lead to a 5d moduli space which is the Coxeter box of $Sp(K)$, of size R^{-1}.

The theories associated with $SO(32)$ and $E_8 \times E_8$ branes at $\mathbb{C}^2/\Gamma_G$ singularities do satisfy the condition $r_V + n_T = \widetilde{r}_V + \widetilde{n}_T$. Indeed, as also noted in [19], in both cases, $r_V + n_T = C_2(G)K - |G|$, where $C_2(G)$ is the dual Coxeter number of the ADE group G and $|G|$ is its dimension[4]. On the other hand, the two theories do not satisfy the stronger conditions $\widetilde{r}_V = n_T$ and $\widetilde{n}_T = r_V$. It is not presently known how this failure should be interpreted or resolved.

[4] As also noted in [19], this agrees with the dimension (in hypermultiplet units) of the moduli space of K G-instantons on $K3$. Duality between the heterotic theory on T^3 and M-theory on $K3$ suggests that the quantum-corrected Coulomb branch for the theory compactified to 3d on a T^3 *actually is* the moduli space of K G-instantons on $K3$. Similarly, compactifying the theory of sect. 2 associated with type II branes at orbifold singularities, the dimension of the Coulomb branch is $C_2(G)K$. Duality between type II on a T^3 and M-theory on T^4 suggests that the quantum-corrected Coulomb branch for the theory compactified to 3d on a T^3 is the moduli space of K G-instantons on T^4.

6 Matrix Model Applications of the Theories.

Following [20], it was suggested in [21] that a M(atrix) description of M-theory on $X_G \times \mathbb{R}^{6,1}$, where X_G is an ALE space asymptotic to $\mathbb{C}^2/\Gamma_G$, is given by quantum mechanics with 8 supersymmetries and gauge group $\prod_{\mu=0}^{r} U(v_\mu)$ with matter $\frac{1}{2}\oplus_{\mu\nu=0}^{r} a_{\mu\nu}(\Box_\mu,\overline{\Box}_\nu)$. The (classical) moduli space of vacua of this theory for $v_\mu = Kn_\mu$ is [5] $(X_G \times \mathbb{R}^5)^K/S_K$, corresponding to the location of K identical zero-branes in the light-cone $X_G \times \mathbb{R}^5$. We propose a slight variant of this conjecture.

Now consider M-theory on $X_G \times T^5 \times R^{1,1}$. Following [1], it is expected[6] that a definition of this theory is given by compactifying the new 6d theory of sect. 2 on a $\widehat{T}^5$. As in [1], there are 25 compactification parameters living in $SO(5,5,\mathbb{Z})\backslash SO(5,5)/(SO(5)\times SO(5))$. Taking a rectangular torus with no B field, $\widehat{T}^5$ is related to T^5, as in [1], by:

$$\begin{aligned}\widehat{L}_i &= \frac{l_p^3}{RL_i},\\ M_s^2 &= \frac{R^2L_1L_2L_3L_4L_5}{l_p^9},\end{aligned}\tag{6.1}$$

where R is the radius of the longitudinal direction and l_p is the eleven-dimensional Planck length. Indeed, this gives the correct light-cone $X_G\times T^5$ space-time from the moduli space of vacua (subject to the same discussion about the situation at the quantum level as in [1,22]).

In the limit of large T^5, this reduces to a slight variant of the suggestion of [21] outlined above. The massless gauge group of the $6d$ theory is given by 2.2 rather than $\prod_{\mu=0}^{r} U(v_\mu)$; in addition, there are the $n_T = r$ tensor multiplets. Upon compactification, the tensor multiplets yield $U(1)^r$ gauge fields, the same number which became massive because of the anomaly. It is thus tempting to conclude that, upon compactification, the tensor multiplets simply give back the same $U(1)$ factors which became massive in 6d because of the anomaly, giving back the original $\prod_\mu U(Kn_\mu)$ theory in lower dimensions. However, this does not seem to be the case. The difference is that the matter fields $\frac{1}{2}\oplus_{\mu\nu=0}^{r} a_{\mu\nu}(\Box_\mu,\overline{\Box}_\nu)$ were charged under the $U(1)^r$ which became massive because of the 6d anomaly. On the other hand, these matter fields are neutral under the $U(1)^r$ which the tensor multiplets give back upon compactification; the new $U(1)^r$ has no charged matter. Taking the limit of large T^5 in 6.1 thus yields a slight variant of the gauge theory of [21].

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[5] This is the moduli space for generic Higgs expectation values. There is a larger Coulomb branch, of dimension $5KC_2(G)$, at the origin.

[6] I thank N. Seiberg for suggesting this.

Following [1], we similarly expect that the 6d string theory from $SO(32)$ or $E_8 \times E_8$ heterotic five-branes at a X_G singularity, when compactified on $\widehat{T}^5$ (which depends on the 105 parameters in $SO(21,5,\mathbb{Z})\backslash SO(21,5)/(SO(21)\times SO(5))$), gives a definition of M-theory on $X_G \times (T^5/\mathbb{Z}_2) \times \mathbb{R}^{1,1}$.

Acknowledgments

I would like to thank J. Blum, D. R. Morrison, S. J. Rey, S. Sethi, E. Witten, and especially N. Seiberg for useful discussions. This work was supported by NSF PHY-9513835, the W. M. Keck Foundation, an Alfred Sloan Foundation Fellowship, and the generosity of Martin and Helen Chooljian. The final stage of this work was also supported by UCSD grant DOE-FG03-97ER40506.

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