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Comments on Neveu-Schwarz Five-Branes

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Abstract

We study the theory of NS five-branes in string theory with a smooth non-trivial transverse space. We show that, in the limit that the bulk physics decouples, these theories become equivalent to theories with a flat and non-compact transverse space. We present a matrix model description of the type IIA theory on $\mathbb{R}^9 \times S^1$ with NS five-branes located at points on the circle. Consequently, we obtain a description of the dual configuration of Kaluza-Klein monopoles in the type IIB theory.

1 Introduction

The five-branes of M-theory and string theory are extremely interesting objects [1]. The theory on k coincident five-branes in M-theory is an interacting field theory at a non-trivial fixed point of the renormalization group. This theory was first found in [2, 3]; for a review, see e.g. [4]. We will refer to this theory as the $(2,0)$ field theory. To obtain this field theory, we have to consider the limit where the eleven dimensional Planck scale M_{pl} goes to infinity. In this limit, all interactions with the modes in the bulk of spacetime, including the interactions with gravity, decouple and we are left with a *complete* theory on the five-branes. The moduli space of vacua for this theory is

$$\mathcal{M} = \frac{(\mathbb{R}^5)^k}{\mathbf{S}_k}. \tag{1.1}$$

These theories are naturally associated with the A_{k-1} groups. Extensions to other groups and in particular to D_k and $E_{6,7,8}$ were discussed in [2, 4]. For other groups, the quotient by the permutation group is replaced by the appropriate Weyl group.

A generalization of this theory was found in [5]. There, M theory on $\mathbb{R}^{10} \times S^1$ was studied with k five-branes at points on the circle. To find a complete theory, we should again make sure that the modes on the five-branes decouple from the modes in the bulk of spacetime. Again, this is achieved by considering the limit $M_{pl} \to \infty$. However, unlike the previous case, we now have another parameter – the radius of the circle L. Therefore, we can find a family of new theories which depend on this parameter. More specifically, by taking

$$\begin{aligned} &M_{pl} \to \infty, \\ &L \to 0, \end{aligned} \tag{1.2}$$

while holding fixed

$$M_s^2 = LM_{pl}^3, \tag{1.3}$$

we find a new theory which depends on M_s. Equivalently, by starting with the type IIA theory, rather than with M-theory, we can define this theory by taking the string coupling g_s^A to zero, while holding fixed the string scale M_s. We will refer to this theory as the $(2,0)$ string theory, since it includes string-like excitations with tension M_s^2. The moduli space of vacua is now

$$\mathcal{M} = \frac{(\mathbb{R}^4 \times S^1)^k}{\mathrm{S}_k}, \tag{1.4}$$

where the radius of the S^1 factor is

$$P = LM_{pl}^3 = M_s^2. \tag{1.5}$$

This follows since P is clearly proportional to L and the factor of M_{pl}^3 appears on dimensional grounds, since M_{pl} is the only scale in the problem. In the "zero slope limit," $M_s \to \infty$, this theory reduces to the $(2,0)$ field theory. As a check, note that in this limit (1.4) becomes the same as (1.1). These theories are naturally associated with the A_{k-1} groups. Extensions to other groups, and in particular to D_k and $E_{6,7,8}$, are straightforward.

Another "non-critical string theory" with $(1,1)$ supersymmetry is similarly obtained by starting with k NS five-branes in type IIB string theory, in the limit where the string coupling g_s^B vanishes with the string scale held fixed [5]. After compactification on a longitudinal circle of radius R, these $(1,1)$ string theories are the same as the $(2,0)$ string theories compactified on a circle of radius $\frac{1}{RM_s^2}$. This fact has led to the conclusion that these theories are *not* local quantum field theories [5]. This non-locality distinguishes them from the $(2,0)$ field theories which also have string-like excitations, but appear to be ordinary local quantum field theories.

It is natural to ask whether we can find more theories by compactifying more transverse directions, or by considering five-branes on more general geometries than flat non-compact transverse spaces. In section two, we study the NS five-branes in IIA and IIB with a transverse circle. This circle compactifies some of the directions in the moduli space of vacua of these theories. However, the size of these directions has a factor of $\frac{1}{g_s}$ (g_s is the type IIA or type IIB string coupling) relative to the naive expectation. This factor has significant consequences. First, to decouple the physics in the bulk in order to find a complete theory on the brane, we have to set the string coupling to zero. This leads to the decompactification of these directions. Therefore, the brane theories with vanishing string coupling in these cases are the same as those in [5]. Second, it shows that unlike D-branes [6], which can serve as probes [7, 8], the NS five-branes do not probe the background transverse space at zero string coupling.

In section three, we focus on the NS five-branes in type IIA string theory with a transverse circle of radius R_A at arbitrary string coupling. We consider the matrix model [9] description of this configuration. It is given by a 2+1 dimensional field theory similar to that of [10]. In the limit $R_A \to \infty$, the theory becomes a 1+1 dimensional theory describing the NS five branes in type IIA theory. For $R_A \to 0$ another 1+1 dimensional theory appears, which describes an A_{k-1} singularity in IIB theory. Our 2+1 dimensional theory interpolates between these two limits. For $g_s^A = 0$, its Higgs branch decouples from its Coulomb branch and gives the matrix model description of the $(2,0)$ string theory [11, 12].

2 String Theory NS Five-Branes on $\mathbb{R}^9 \times S^1$

2.1 Type IIA Five-Branes on $\mathbb{R}^9 \times S^1$

We start by considering M-theory on $\mathbb{R}^9 \times T^2$ with k five-branes at points on the torus T^2. The T^2 is defined by its complex structure τ and its volume. For simplicity, let us take the torus to be rectangular with radii R_1 and R_2. This choice will not affect any of the following discussion in a significant way. The moduli space for the five-branes is then

$$\mathcal{M} = \frac{(\mathbb{R}^3 \times T^2)^k}{\mathbf{S}_k}. \tag{2.1}$$

Using the argument leading to (1.5) we find that the radii of the T^2 factors are

$$P_1 = M_{pl}^3 R_1 \ , \qquad P_2 = M_{pl}^3 R_2 \ . \tag{2.2}$$

M-theory on a two-torus is equivalent to the type IIB string [13] on a circle of radius

$$R_B = \frac{1}{M_{pl}^3 R_1 R_2} = \frac{1}{M_s^2 R_2}, \tag{2.3}$$

with string coupling

$$g_s^B = \frac{R_1}{R_2}. \tag{2.4}$$

This result can be obtained by going from M-theory to type IIA on R_1. Then $\mathbb{T}$ duality on R_2 maps us to the IIB theory on a circle with radius (2.3) and coupling (2.4). The NS five-branes in the IIA theory, which are located at points on the transverse circle R_2, are mapped under the T duality into Kaluza-Klein monopoles [14]. Therefore, our theory of k M-theory five-branes on T^2 is a theory of k Kaluza-Klein monopoles in type IIB string theory.

Let us review some basic facts about Kaluza-Klein monopoles. The monopole solution is constructed by taking flat space tensored with the four dimensional multi-Taub-NUT metric [15]. In the case of string theory, this construction gives a five-brane, while in M theory, we obtain a six-brane [16]. The non-trivial metric on $\mathbb{R}^3 \times S^1$ is

$$ds^2 = V(x)\, d\vec{x}^2 + V(x)^{-1}(d\theta + \vec{A}\cdot d\vec{x})^2, \tag{2.5}$$

where $\vec{x}$ is three dimensional and $\vec{A}$ is related to V by

$$\nabla V = \nabla \times \vec{A}. \tag{2.6}$$

The scalar function V depends on a single free parameter r:

$$V = 1 + r \sum_{i=1}^{k} \frac{1}{|\vec{x} - \vec{x}^i|}. \tag{2.7}$$

The positions of the k branes are specified by the $\vec{x}^i$ and the angular variable θ has a period proportional to r. The parameter r sets the scale of the solution, and can be rescaled by rescaling $\vec{x}$ and θ. It corresponds to the size of the circle S^1 in the limit $|x| \to \infty$. In our problem $r = R_B$. When all the branes are separated the space is smooth. For $k > 1$ coalescing branes the multi-Taub-NUT has an A_{k-1} singularity at the position of the branes. In the limit $r \to \infty$ the circle which is coordinatized by θ decompactifies everywhere except at the positions of the branes and the space becomes $\mathbb{R}^4/Z_k$.

Finally, we should mention that the multi-Taub-NUT has a number of non-trivial two-cycles. Some of these cycles collapse when the $\vec{x}^i$ coalesce; note that there is a non-trivial two-cycle even for $k = 1$ [17].

For recent discussions of Kaluza-Klein monopoles in M theory and string theory see [18–23].

It is useful to re-express the relations (2.2) in terms of the string scale and string couplings

$$\begin{aligned} P_1 &= M_s^2 , \\ P_2 &= \frac{M_s^2}{g_s^B} = \frac{M_s^3 R_2}{g_s^A} . \end{aligned} \tag{2.8}$$

The key feature is that P_2 always contains a factor of $1/g_s$ whether expressed in terms of the type IIA or type IIB string coupling.

There is a simple reason for these factors of $\frac{1}{g_s}$. The collective coordinates of each NS five-brane in IIA, or Kaluza-Klein monopole in IIB, are a two-form and five scalars (for a recent discussion, see [18]). The two-form and one of the scalars Φ_1 arise from the RR sector – for the type IIB Kaluza-Klein monopole they arise from the RR four-form and the RR two-form reduced on the non-trivial two-cycle. The other four scalars are NS-NS fields – they correspond to the three deformations of the metric, and a compact deformation Φ_2 of the NS-NS two-form. The natural normalization of these fields is with a factor of $\frac{1}{g_s^2}$ in front of the kinetic terms for the NS-NS fields, but not in front of the kinetic terms for the RR fields. In order to keep the $(2,0)$ supersymmetry on the five-brane manifest, we rescale the NS-NS scalars to have no $\frac{1}{g_s^2}$ in their kinetic terms. This leads to the crucial factor of $\frac{1}{g_s}$ in P_2.

This situation should be contrasted with that of D-branes. There, all the collective coordinates appear from open strings. Both the gauge fields and the scalars have the same normalization, $\frac{1}{g_s}$, in their kinetic terms and therefore no rescaling is necessary. Therefore, these scalars "see" the underlying geometry, and D-branes can be used as probes. On the other hand, the NS five-branes and the Kaluza-Klein monopoles are not good probes. In particular, for finite R_2 the value of P_2 diverges as the string coupling goes to zero.

We now want to decouple the bulk physics to obtain a complete theory. This can be accomplished only if $g_s^A = g_s^B = 0$. It is clear from (2.8) that in this case $P_2 = \infty$. A more careful analysis immediately shows that this conclusion cannot be avoided by taking various limits of R_1 and/or R_2. For example, if we take $g_s^A, R_2 \to 0$, while holding $\frac{R_2}{g_s^A}$ fixed, P_2 is finite. However, since $R_2 \to 0$ this theory is better thought of as the type IIB theory in $\mathbb{R}^{10}$ with a finite coupling, and the bulk physics no longer decouples.

The spacetime geometry in this limit depends on R_2. When $R_2 \to 0$

the type IIB Kaluza-Klein monopoles are the better description, while for $R_2 \to \infty$ the type IIA NS five-branes are the right description. However, the decoupled physics on the brane is actually independent of R_2 since P_2 has gone to infinity. We will find further evidence favoring the uniqueness of this decoupling limit from the M(atrix)-theory description of this configuration, discussed in the following section.

Our analysis leads us to a description of the decoupled physics on A_{k-1} singularities in free type IIB string theory. It is given by the same "non-critical string theory" as the k NS five-branes in type IIA theory. This fact is known for the low energy $(2,0)$ field theories, and here we recover it for the $(2,0)$ string theory. The $(2,0)$ string theory has two kinds of strings: those which exist in the $(2,0)$ field theory, whose tension vanishes at the singularities in the moduli space, and other strings with tension M_s^2. In the IIA description both kinds of strings are membranes stretching between five-branes and wrapping the compact direction. In the type IIB A_{k-1} theory, strings of the first kind are associated with IIB three-branes which wrap collapsing two-cycles. Strings of the second kind are bound states at threshold of strings from the bulk with the Kaluza-Klein monopoles.

Essentially the same scaling analysis applies when more transverse circles are present. The extra factor of $\frac{1}{g_s}$ rescales the metric, and decompactifies the transverse space in the $g_s \to 0$ limit. Furthermore, we can consider type IIA five-branes with an arbitrary smooth transverse metric. It seems that a similar factor of $\frac{1}{g_s}$ would make the general target space geometry as "probed" by the five-branes flat and non-compact in the decoupling limit. The case of five-branes at a singularity will be discussed in [24].

2.2 Type IIB Five-Branes on $\mathbb{R}^9 \times S^1$

We now consider the case of k type IIB five-branes whose world volume theory has $(1,1)$ supersymmetry. We start with M-theory compactified on a torus T^2 with radii R_1 and R_2. The type IIB five-branes arise as M-theory Kaluza-Klein monopoles associated with one of the cycles which wrap the other cycle. For example, let us go from M-theory to type IIA by reducing on R_1, and consider Kaluza-Klein monopoles associated with R_1, so that their r parameter is R_1. These solitons are D6-branes in the type IIA string theory. T duality on R_2 maps us to the type IIB theory, and the Kaluza-Klein monopoles become D5-branes at points on a circle of radius $R_B = \frac{1}{M_{pl}^3 R_1 R_2}$. S-duality converts them to NS five-branes at points on the circle. Instead, we can start with Kaluza-Klein monopoles in the IIA theory associated with R_2, so their r parameter is R_2, and T dualize R_2 to find the NS five-branes of IIB at points on a circle of radius R_B.

The low energy theory on the NS five-branes is a $U(k)$ gauge theory with gauge coupling $\frac{1}{M_s^2}$ [5]. When R_B is finite, the moduli space of vacua of this theory is

$$\frac{(\mathbb{R}^3 \times S^1)^k}{\mathbf{S}_k}. \tag{2.9}$$

The radius P of the S^1 factors is easy to determine, e.g. by starting with the wrapped D6-brane description in the previous paragraph, and performing the duality transformations. We find:

$$P = \frac{M_s^2 R_B}{g_s^B} = \frac{M_s}{g_s^A}. \tag{2.10}$$

As in the previous subsection, we see that P has a factor of $\frac{1}{g_s}$ relative to the naive result. As we said there, this is unlike the case of D-branes. This factor of $\frac{1}{g_s}$ can be explained as in that case. The gauge fields are RR fields, which for the Kaluza-Klein monopole arise from the reduction of the three-form on the non-trivial two-cycle, while the four scalars are NS-NS fields. Rescaling the NS-NS scalars to have the same kinetic terms as the one-forms leads to (2.10).

The factor of $\frac{1}{g_s}$ also has consequences for the possible decoupling limits. Decoupling requires taking $g_s^A, g_s^B \to 0$. Once again, the period for the scalar decompactifies, and we are driven back to the theory of parallel type IIB five-branes in $\mathbb{R}^{10}$. Also, in analogous fashion to the case with $(2,0)$ supersymmetry, there is a parameter R_B, which changes the spacetime description, but does not alter the decoupled physics.

3 A Matrix Definition of M-Theory Five-Branes on Compact Spaces

A matrix model [9] for the M-theory five-brane on $\mathbb{R}^9 \times T^2$ follows naturally by extending the quantum mechanics describing the longitudinal five-brane [10] to $k > 1$ five-branes, and to a 2+1 dimensional field theory with eight supersymmetries. For related discussions see [7,25]. The theory has a $U(N)$ gauge symmetry where N is the number of zero-branes used to probe the longitudinal five-brane. The coupling to k parallel five-branes is represented by k hypermultiplets in the fundamental of the gauge group. There is also an adjoint hypermultiplet, which encodes motion of the zero-branes within the longitudinal five-brane.

The interaction of the five-branes with spacetime is encoded in the dynamics on the Coulomb branch of the theory. For $k > 1$, there is also a Higgs branch in the model that corresponds to physics localized within the brane. Points on the Higgs branch essentially describe the dynamics of zero-branes, which have fattened to instantons within the five-branes [26].

The parameters of the M(atrix)-theory are determined in terms of the radius of the longitudinal direction R, and the two radii R_1 and R_2 of the compact part of spacetime $T^2 \times \mathbb{R}^7$. In terms of these parameters [9, 27–29], the 2+1 dimensional theory is on a compact space with radii

$$\Sigma_i = \frac{1}{M_{pl}^3 R_i R}, \tag{3.1}$$

and the Yang-Mills gauge-coupling is

$$g_{YM}^2 = \frac{R}{R_1 R_2} = R^3 M_{pl}^6 \Sigma_1 \Sigma_2. \tag{3.2}$$

It is convenient to express the dimensions of the torus in terms of the string scale and string coupling

$$\begin{aligned} \Sigma_1 &= \frac{1}{M_s^2 R}, \\ \Sigma_2 &= \frac{1}{M_s^2 R} g_s^B . \end{aligned} \tag{3.3}$$

Consider first the 2+1 dimensional theory on $\mathbb{R}^3$. Both the Higgs and the Coulomb branches are described by hyperKähler manifolds. A non-renormalization theorem guarantees that the Higgs branch is immune to quantum corrections [30]. In terms of the fields in the Lagrangian, it provides the ADHM hyperKähler quotient construction of the moduli space of N instantons in $SU(k)$ gauge theory in four dimensions. The Coulomb branch of the theory for $N = 1$ was analyzed in [31, 32]. Its metric is a Taub-NUT metric. For higher N the metric appears to be a symmetric product of Taub-NUT metrics. The Coulomb and Higgs branches touch at a singular point where the theory flows to a non-trivial interacting three dimensional fixed point. The infrared limit is the same as taking the dimensionful coupling constant $g_{YM}^2 \to \infty$. It is a property of this fixed point that the Higgs branch and the Coulomb branch both emanate from it. Therefore, these two branches are not decoupled here.

Now we consider the theory with finite $\Sigma_{1,2}$. There can be Wilson lines on T^2, but we will ignore them. We are going to explore this theory for fixed Σ_1 as a function of $\Sigma_2 \ll \Sigma_1$ and $g_{YM}^2 \gg 1/\Sigma_1$. For $N = 1$, this problem was analyzed in [33]. The relevant dimensionless quantity which controls the dynamics is

$$\gamma = g_{YM}^2 \Sigma_2 = \frac{1}{(R_2 M_s)^2}. \tag{3.4}$$

Consider first the limit $\gamma \gg 1$. At energies larger than $1/\Sigma_2$ the theory is three dimensional and its Coulomb branch becomes a symmetric product of

Taub-NUT spaces. At energies of order $\frac{1}{\Sigma_2}$ the theory becomes two dimensional. The two dimensional sigma model based on the Taub-NUT metric is conformally invariant, and therefore this metric does not change as we flow to the infrared. In the opposite limit, $\gamma \ll 1$, the theory becomes two dimensional at the scale $\frac{1}{\Sigma_2}$ before the gauge interactions become strong. Therefore, here the dynamics is that of the two dimensional gauge theory. The result of this dynamics is a metric with an infinite tube [34]. For $N = 1$, the explicit answer which interpolates between the $1/|x|$ behavior for $\gamma \gg 1$ and the $1/|x|^2$ behavior for $\gamma \ll 1$ was found in [33].

These results are consistent with the spacetime picture. The parameter $\gamma = 1/(R_2 M_s)^2$ interpolates between the two and three dimensional theories, which are appropriate to type IIA and type IIB, respectively. For $R_2 \gg 1/M_s$ the metric we expect is the tube metric of the NS five-brane of the IIA theory [34], while for $R_2 \ll 1/M_s$ we expect the Taub-NUT metric of the Kaluza-Klein monopoles in type IIB theory. It is satisfying to see how the matrix model reproduces these answers.

We can now consider the decoupling limit described in the previous section in the context of this matrix model. In this limit $\Sigma_2 \to 0$, $g^2_{YM} \to \infty$ while Σ_1 and γ are fixed. The 2+1 dimensional theory becomes 1+1 dimensional. Now we can use the arguments of [2, 11, 12] to argue for the decoupling of the Higgs branch and the Coulomb branch in this limit. For this decoupling it is crucial that we consider the two limits $\Sigma_2 \to 0$ and $g^2_{YM} \to \infty$. In particular, without the $\Sigma_2 \to 0$ limit, the theory is 2+1 dimensional where no such decoupling happens.

It is interesting to examine the R_2 or the γ dependence in this limit. The physics of the Higgs branch is independent of these parameters. This follows from the non-renormalization theorem mentioned above, as well as from the fact that the Higgs branch metric is independent of Σ_2 [35]. This independence is in accord with our statements in the previous section about the decoupled physics on the five-brane being independent of R_2. On the other hand, as mentioned above, the Coulomb branch depends on R_2 corresponding to the fact that the spacetime metric depends on R_2.

There is a subtlety that is worth mentioning. In the case without the longitudinal five-brane, the 2+1 dimensional theory has sixteen supersymmetries. This theory has an interacting fixed-point with $Spin(8)$ global symmetry [4, 28, 29]. In this case, there are two inequivalent limits in which the field theory becomes 1+1 dimensional. The first is dimensional reduction to Yang-Mills in two dimensions. Flow to the infrared gives an orbifold conformal field theory which describes the type IIA string theory [29, 36, 37]. The second is obtained by first flowing to the 2+1 dimensional fixed point, and then reducing to 1+1 dimensions. In this case, the resulting 1+1 dimensional conformal field theory describes the type IIB string. The difference is essentially in the way that the extra spacetime dimension is acquired: In the first case by dimensional reduction, while in the second, by dualizing the gauge-field in the Abelian case, or flowing to the interacting fixed point for the non-Abelian case. By contrast, in the situation with the longitudinal five-brane, we are interested in the Higgs branch of the theory, and then the two limits commute. There is only one decoupled 1+1 dimensional conformal field theory: the theory which describes parallel type IIA five-branes.

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