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Special issue in honor of Professor David Mumford, dedicated to the memory of Jennifer Mumford

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With this volume, on the occasion of his 80th birthday, we celebrate David Mumford's remarkable second career. Previously, David was known for his transformative work in algebraic geometry, which earned him the Fields Medal in 1974. Subsequently, David began work as an applied mathematician. Among his many awards, he received the National Medal of Science in 2009, recognizing the full scope of his work in both pure and applied mathematics: "For his contributions to the field of mathematics, which fundamentally changed algebraic geometry, and for connecting mathematics to other disciplines such as computer vision and neurobiology."

This volume is about David's work in applied mathematics. There are eleven papers, each relating to one of David's many successful forays into the business of constructing models, devising algorithms, and revealing mechanisms behind mysterious and complex phenomena. The paper by Wei et al. and the one by Debroux, Guyader, and Vese are both on the problem of detecting boundaries in images. These two papers are representative of the thousands of papers that trace back to the Mumford and Shah continuum formulation of the boundary-detection problem, and their introduction of variational methods for its solution. Wei et al. review some more recent developments in the Mumford-Shah formulation, propose the addition of a "force" with a probabilistic interpretation (absent in almost all existing formulations), devise an effective optimization strategy, and demonstrate excellent results. With an eye on some very challenging boundary and crack detection problems, Debroux et al. explore substantial generalizations of the Mumford-Shah variational formulation, provide a remarkably complete analysis of existence and uniqueness, as well as other analytic guarantees, and ultimately return to the motivating examples, on which they achieve remarkable performance.

Shah himself is one of our contributors, but writes on another of David's many interests—the history of mathematics. Shah uses a statistical method introduced by David for measuring the accuracy of ancient Chinese algorithms that were used to predict eclipses, and applies it more broadly to the much earlier efforts by Ptolemy and a later method from medieval Indian astronomy. Ptolemy, it seems, was more than a thousand years ahead of his time.

Two more papers are about texture modeling and synthesis. Raad et al. have put together a sweeping review, that includes sample results from a variety of approaches and brings us up-to-date on the state of the art. They challenge us by noting that there are as yet no general synthesis algorithms that can fool human beings; there is more to be done. Wu et al. start with a review of the highly influential FRAME model, devised by Mumford and Zhu, and then present a number of startling generalizations. One is a kind of marriage between deep discriminative networks and generative (Bayesian) models, which connects to yet another influential paper by Mumford and Zhu on compositional modeling and the Bayesian approach to image parsing. Along a similar vein, J. Wang et al. use a clever clustering trick to reveal the surprisingly explicit features that can be extracted from a fully trained deep network, and then demonstrate a direct path to using these features as the building blocks for a fully Bayesian (i.e. generative) model. Diaconis and G. Wang offer an excellent introduction to Bayesian modeling and Bayesian goodness of fit tests, with a wealth of deceptively simple examples. We are reminded that despite fast computers and a toolkit full of Monte Carlo algorithms, there is plenty we don't know, or at least don't know how to do. Bayesian modeling is a tricky business.

David has long been a student of Ulf Grenander's path-breaking work on abstract inference and Pattern Theory. Evidence of Grenander's influence on David's ideas and discoveries can be found throughout this volume. A good case in point is Grenander's formulation of "digital anatomy." Ulf's idea was to build a library of "normal" shapes of organs and other structures in the human body. Oftentimes, pathologies are discovered or diagnosed through medical imaging, and the shape of a structure, e.g. the hippocampus or corpus callosum of the brain or the left ventricle of the heart, carries information about the likelihood and severity of a disease. Ulf proposed various notions of distance between shapes—how different is the shape of a particular anatomical structure in a given patient from that of the prototypical (or average) structure? These distances are based on certain diffeomorphisms between pairs, which, among other things, then provide a principled foundation for statistical measures of normal and statistical characterizations of typical pathologies. In effect, a theory of shapes. There are many very beautiful and powerful variations on the theme, most involving algebraic and differential geometry, a number of which were introduced by Mumford and his co-authors.

Four of our papers belong to the category of applied algebraic or differential geometry. Two of these, one by Holm and the other by Younes, propose improvements to state-of-the-art formulations of shape theory. Younes shows how to add geometric considerations to the typical (so-called large deformation diffeomorphic—LDD) metrics used to measure distances in shape space. The combination produces a new formulation that is more expressive yet still computationally tractable. In a series of experiments on curves and surfaces, Younes shows that the resulting mappings between shapes are much more natural than those produced in the unmodified LDD framework. Holm builds on earlier work in which he and others formulate distances through geodesic paths, which are formally analogous to fluid flows. By replacing deterministic flows with stochastic flows, a statistical framework emerges in a natural manner—organically, as opposed to post hoc.

The remaining two papers are less directly connected to David's work on shape theory, but share with it some salient properties: they are rooted in algebraic and differential geometry, and, since they are about applications, computation plays a central role in their development. The paper by Alsing, Miller, and Yau is about the Ricci flow equation, well known for its role in Perelman's proof of the Poincaré conjecture, but also known for applications in physics, graphics, geometric modeling, and other areas in the computer and engineering sciences. Alsing et al. review and extend their earlier work on the discrete Ricci flow known as Simplicial Ricci Flow. They discuss a canonical representation in terms of a set of ordinary differential equations for the edge lengths of a piecewise flat simplicial geometry, devise a novel numerical scheme for integrating through singularities, and demonstrate the power of the approach by obtaining the correct geometrization in an illustrative example. Finally, Cai et al. study systems of linear equations under the surprisingly common condition that the solution would be underdetermined if not for the constraint that it lies on a known manifold (specifically, an algebraic variety). Motivating examples include compressed sensing, problems in phase retrieval, and low-rank matrix recovery, to name a few. Cai et al. provide general conditions for existence of a solution and explicit numerical methods for finding one. The computational feasibility of their approach is demonstrated on the matrix recovery and phase retrieval problems.

We hope that you find these eleven papers to be a fitting and informative celebration of David Mumford's wide-ranging contributions to applied mathematics.

Guest Editors: Stuart Geman, David Gu, Stanley Osher,
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