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ERRATUM TO: REGULARITY OF $\overline{\partial}$ ON PSEUDOCONCAVE COMPACTS AND APPLICATIONS BY G. M. HENKIN AND A. IORDAN, ASIAN J. MATH., 4, 855-884, 2000 *

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We would like to correct some significant inexactitudes in the formulation and the proof of Lemma 6.5 from our above cited paper and we appologize to the reader for this inconvenience. The theorems of this paper use in fact the following version of Lemma 6.5:

LEMMA 6.5 *Let Ω be a domain with Lipschitz boundary of a compact Kähler n-dimensional manifold X, E a holomorphic hermitian vector bundle on X and $v \in L^2_{(p,q)}(X;E)$, $q \geq 1$, such that $v = 0$ on $\Omega_- = X \backslash \overline{\Omega}$ and $v \in L^2_{(p,q)}(\Omega; d^{-k+1}; E)$ where k is a positive integer. We denote by G the Green operator on X and $\overline{\partial}^*$ the L^2-adjoint of $\overline{\partial}$. Then $\overline{\partial}^* Gv \in W^k_{(p,q-1)}(\Omega_-; E)$ and $\left\|\overline{\partial}^* Gv\right\|_{k,\Omega_-} \leq CN_{-k+1,\Omega}(v)$, where C is a constant independent of v.*

Proof. G is an integral operator which has a singularity of order $2n-2$ on the diagonal so

$$\nabla^i_x \overline{\partial}^* Gv(x) = \int_\Omega \nabla^i_x \left[\overline{\partial}^* G(x,y)\right] d^{k-1}(y) \wedge v(y) d^{-k+1}(y) dy$$

where ∇^i_x is a covariant derivation of order i with respect to x.

Since $d(x,y) \geqslant d(y)$ for every $x \in \Omega_-$ and $y \in \Omega$ it follows that $\nabla^{i+1}_x G(x,y) d^{k-1}(y)$ has a singularity of order $2n+i-k$ on the diagonal for every $0 \leq i \leq k$. The trivial extension to X of vd^{-k+1} belongs to $L^2_{(p,q)}(X;E)$ and it follows that $\nabla^i_x \overline{\partial}^* Gv \in L^2_{(p,q)}(\Omega_-; E)$ for every $0 \leq i < k$ and

$$\left\|\overline{\partial}^* Gv\right\|_{W^{k-1}_{(p,q-1)}(\Omega_-;E)} \leq CN_{-k+1,\Omega}(v). \tag{1}$$

In order to prove that $\nabla^k_x \overline{\partial}^* Gv \in L^2_{(p,q-1)}(\Omega_-; E)$ it is enough to prove that

$$\nabla^{k+1}_y \int_{\Omega_-} G(x,\cdot) \wedge \psi(x)\, dx \in L^2_{(n-p,n-q)}(\Omega; d^{k-1}; E^*) \tag{2}$$

for every $\psi \in L^2_{(n-p,n-q)}(\Omega_-; E^*)$. Indeed, from (2) we obtain

$$\infty > \int_\Omega \left(\left(v(y) d^{-k+1}(y)\right) d^{k-1}(y) \wedge \nabla^{k+1}_y \int_{\Omega_-} G(x,y) \wedge \psi(x)\, dx \right) dy$$
$$= \int_{\Omega_-} \left(\psi(x) \wedge \int_\Omega \nabla^{k+1}_y G(x,y) \wedge v(y) dy \right) dx.$$

But

$$\nabla^{k+1}_y G(x,y) = (-1)^{k+1} \nabla^{k+1}_x G(x,y) + H(x,y) \tag{3}$$

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where $H(x,y)$ has a singularity of order $2n+k-2$ on the diagonal. It follows as before that

$$\int_{\Omega} H(\cdot,y)\, d^{k-1}(y) \wedge v(y) d^{-k+1}(y) \in L^2_{(p,q)}(\Omega_-;E).$$

So

$$\int_{\Omega_-} \left(\psi(x) \nabla_x^{k+1} \int_{\Omega} G(x,y) \wedge v(y) dy \right) dx < \infty$$

for every $\psi \in L^2_{(n-p,n-q)}(\Omega_-;E^*)$ and therefore $\nabla_x^k \overline{\partial}^* Gv \in L^2_{(p,q)}(\Omega_-;E)$.

Let $\psi \in L^2_{(n-p,n-q)}(\Omega_-;E^*)$. Since the form $\int_{\Omega_-} G(x,\cdot) \wedge \psi(x)\, dx$ belongs to $W^2_{(n-p,n-q)}(X;E^*)$ and it is harmonic on Ω, the form $\nabla_y^{k+1} \left(\int_{\Omega_-} G(x,\cdot) \wedge \psi(x)\, dx \right)$ belongs to $W^{-k+1}_{(n-p,n-q)}(\Omega;E^*)$ and (2) follows by known results (see for example [1], [2]).

Since (2) implies that

$$\left\| \nabla_y^{k+1} G(\cdot,y) v(y) \right\|_{L^2_{(p,q)}(\Omega_-;E)} \leqslant C N_{-k+1,\Omega}(v),$$

by (3) we have also

$$\left\| \nabla_x^k \overline{\partial}^* Gv \right\|_{L^2_{(p,q)}(\Omega_-;E)} \leqslant C N_{-k+1,\Omega}(v).$$

This inequality and (1) completes the proof of the Lemma. □

We use this opportunity to correct some other inexactitudes from our paper:

-page 856, row 28: read "$(n,n-1)$-forms on $\mathbb{CP}_n \backslash \overline{\Omega}$ with coefficients in the Sobolev space $W^{k+1}\left(\mathbb{CP}_n \backslash \overline{\Omega}\right)$" instead of "$(n,n-1)$-forms on Ω with coefficients in the Sobolev space $W^k(\Omega)$";

-page 864, row 6: read "If $1 \leq q \leq n-1$ and α is big enough..." instead "If $1 \leq q \leq n-1$...";

-page 868, last row: read "using the beginning of the proof of Lemma 6.5, we obtain $F_k = \overline{\partial} \widetilde{u}_k$ with $\widetilde{u}_k = \overline{\partial}^* G F_k \in C^k_{(p,q-1)}(L;E)$" instead of "using Lemma 6.5, we obtain $F_k = \overline{\partial} \widetilde{u}_k$ with $\widetilde{u}_k = \overline{\partial}^* G F_k \in W^k_{(p,q-1)}(X;E)$".

-page 870, row 14 read "$W^k_{(p,q-1)}(\Omega_-;E)$" instead of "$W^k_{(p,q-1)}(X;E)$";

-page 872, row 8: read "...a solution $v_k \in L^2_{(p,q)}(\Omega;\delta^{-k+2};E)$ smooth up to the boundary..." instead of "...a solution $v_k \in L^2_{(p,q)}(\Omega;\delta^{-k+2};E)$...";

-page 877, row 24:read "$C_\varphi \|h\|_{-k,\Omega}$" instead of "$C_\varphi \|bv(h)\|_{-k,\Omega}$";

- page 878, row 13: read "$W^{k+1}_{(n,n-1)}(\Omega_-)$" instead of "$W^{k+1}_{(n,n-1)}(\mathbb{CP}_n)$".

REFERENCES

[1] S. Bell, *A Duality Theorem for Harmonic Functions*, Michigan Math. J., 29 (1982), pp. 123–128.

[2] D. Jerison and C. E. Kenig, *The Inhomogeneous Dirichlet Problem in Lipschitz Domains*, J. Funct. Anal., 130 (1995), pp. 161–219.