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PREFACE TO KODAIRA'S ISSUE*

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Kunihiko Kodaira was born on March 16, 1915, the son of Gonichi Kodaira, a distinguished Japanese agriculturist who at one time held a cabinet post as vice-Minister of Agriculture and Forestry in the Japanese government. He received advanced degrees from the Department of Mathematics (1938) and Department of Physics (1941) of the University of Tokyo, and was an associate professor at the University of Tokyo from 1944 until 1951, where he received his Ph.D. in mathematics in 1949 by submitting a thesis on the subject of harmonic forms on Riemannian manifolds. In 1949, he came to the United States at the invitation of the Insitute for Advanced Study in Princeton, N.J., U.S.A. Following two years as a member of the Institute and a year as visiting professor at The Johns Hopkins University, he joined the Department of Mathematics of Princeton University in the autumn of 1952 (It was at exactly this time when the author of this preface, then a first-year graduate student at Princeton, began to follow Professor Kodaira's lectures.) Thereafter Kodaira divided his time on a half-and-half basis between Princeton University and the Institute for Advanced Study until 1961 when he accepted an invitation to visit Harvard University for one year. In the autumn of 1962, Kodaira joined the faculty of The Johns Hopkins University where he remained until 1965 when the Kodairas moved to Stanford University. They remained there until 1967 when Prof. Kodaira accepted a call to rejoin the faculty of the University of Tokyo, and went with his family to make a new home in the Nakaochiai section of Shinjuku-ku, Tokyo, where he resided for the rest of his life.

During his 18 year stay in the United States, Kodaira interacted with many other distinguished mathematicians to produce a prodigious quantity of mathematics of great beauty and profundity. Among these were W.L. Chow, F. Hirzebruch, L. Nirenberg, G. de Rham, and D.C. Spencer. Together Kodaira and Spencer produced at least twelve pioneering papers of the highest quality. Their relationship must surely be ranked as one of the most fruitful mathematical collaborations of all time.

In 1954, Kodaira shared the Fields Medals with J.P. Serre at the International Congress of Mathematicians held in Amsterdam that year. In 1957 Kodaira received the Japan Academy Prize and in the same year, the Cultural Medal, bestowed by the Emperor of Japan on only a very few Japanese citizens each year. Kodaira became a member of the Japan Academy in 1965.

Kodaira's earliest papers reflect a wide range of interests and were co-authored with several Japanese mathematicians of interests quite different from each other, including M. Abe, S. Iyanaga, and S. Kakutani.

After this initial period, Kodaira's work may be subdivided into several fields. Undoubtedly one of the most important influences on Kodaira's work was that of Hermann Weyl, whose work on solutions of partial differential equations using Hilbert space methods and the method of orthogonal projection which appear in Weyl's famous book, "Idee der Riemannschen Flächen" profoundly influenced Kodaira's work on harmonic differential forms on Riemannian manifolds, including as an important special case that of algebraic varieties. These ideas appear quite clearly in his earlier

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joint work with Spencer on the application of sheaves and their associated cohomology groups on algebraic varieties to algebraic geometry.

In fact, the subject of algebraic geometry is at the center of much of Kodaira's work, including joint work with W.L. Chow and with D.C. Spencer. But what makes this work so powerful and fruitful is its effective use of the methods of differential equations and regularity theorems, and the use of the parametrix to prove regularity theorems for solutions of partial differential equations, as well as the cohomology of complex manifolds with coefficients in coherent analytic sheaves. These tools from analysis had not been used effectively in most of the earlier work, for example of the Italians, in algebraic geometry. Their applications provided a real break- through in advancing this subject. Kodaira and Spencer and some of their collaborators and colleagues in the same field, such as M. Kuranishi and L. Nirenberg applied the same kinds of ideas in connection with other structures that could be superimposed on Riemannian manifolds, complex manifolds, and algebraic varieties. But the particular case of compact complex manifolds, and especially that of smooth complete complex algebraic varieties, with their richer structures and having the added advantage that the cohomology groups with coefficients in coherent analytic sheaves are finite dimensional, afforded a fertile field for developing sharp and clear results in this area.

Kodaira's work "On a differential-geometric method in the theory of analytic stacks", inspired by work of S. Bochner, has proved, with its application to vanishing theorems, to be an extremely useful criterion for the existence of non-constant meromorphic functions (when they exist) on compact complex manifolds, as Kodaira himself has remarked. This circle of ideas has deep implications for the work on deformations of complex structures.

The work on deformations of complex structures done by Kodaira and Spencer is perhaps their most profound in this area and suggests some most interesting questions that have yet to be explored. In the preface to his book mentioned below, Kodaira ascribes the inspiration for this joint work to results obtained by Fröhlicher and Nijenhuis in "A theorem on stability of complex structures" (Proc. Nat. Acad. Sci., U.S.A., 43(1957), 239-241), which Kodaira and Spencer also call the starting point of their large joint work. This truly pioneering work, "On deformations of complex structures, I-III", remains as a cornerstone of work accomplished up to now, and as the likely basis of some truly interesting progress in the future. It starts from the concept of variation of complex structure on a complex manifold V as measured by the first cohomology group $H^1(V, \Theta_V)$, where Θ_V is the sheaf of germs of holomorphic vector fields on V. This cohomology group is in many cases to be viewed as the tangent space to the moduli space of deformations of V, and its dimension in such cases is the number of moduli or effective parameters on which depend the complex structures on the underlying C^∞ manifold. These authors have provided criteria for these conclusions to be valid, for example the vanishing of the corresponding 0-th and 2-nd cohomology groups of V with coefficients in T_V. An excellent treatment of these topics in essentially textbook form is Kodaira's own book on the subject, published in 1986, "Complex Manifolds and Deformation of Complex Structures", which holds a virtually self-contained treatment of the subject of differential forms, harmonic forms, Hodge theory, etc.,as well as of the necessary regularity theorems in an appendix by Daisuke Fujiwara. This book appeared in the very year (1986) in which it was decided that the 1990 sessions of the International Congress of Mathematicians would be held in Kyoto, Japan.

A striking characteristic of Kodaira's work in general, as particularly exemplified in this book, is his use of explicit, down-to-earth, concrete calculations and constructions. The author of this preface was very much impressed by this feature of Kodaira's style upon attending Kodaira's lectures in Princeton, as alluded to earlier.

One must not at all think of Kodaira's time in Tokyo as a retirement from active mathematics. Indeed, the last 30 years of his life were very active. He organized a large group of advanced students and faculty of the University of Tokyo and other universities into a seminar which met regularly and weekly, usually on Saturdays, to have talks presented by members of the seminar and others on subjects of current interest, mostly in algebraic geometry in areas close to Kodaira's work. In 1975, because of retirement-age rules, Kodaira had to leave Tokyo University, after which he joined the faculty of Gakushuin University, the so-called "Peers' School" where many persons of high social standing, including the Royal Family of Japan, went for a higher education. Nevertheless, Kodaira was still active in mathematics and received frequent visits from students and friends, including S. Iitaka, T. Shioda, the author of this preface, and others. He continued very lively and maintained an active interest in mathematics and in other things. And in 1986 there was published Kodaira's book mentioned above.

Indeed, when in 1986, Kyoto, Japan, was selected as the site for the 1990 meeting of the International Congress of Mathematicians, Prof. Kodaira accepted appointment as chairman of the organizing committee for the Congress. In that capacity, he was very active in soliciting financial support for the Congress from prominent business men, many of whom were former university classmates of Kodaira. However, it seemed to the author of this preface that the very intensity of Kodaira's work and activity at this period may well have precipitated a decline in his health and well-being. He had never relished official and bureaucratic duties, perhaps because of his retiring and modest nature, as evidenced by his unhappiness with his duties as Dean of Sciences at the University of Tokyo. At any rate, from 1986 to 1990 his health underwent a decline, so that even as chairman of its organizing committee he was unable to be present at the meeting of the ICM in Kyoto in 1990. And this general deterioration in his health progressed until his death on July 26, 1997. The author of this preface was most pleased and honored to have enjoyed the privilege of Professor Kodaira's acquaintance and friendship over a period of nearly 45 years.

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STUDENT REMINICENSES OF KODAIRA AT STANFORD*

REESE HARVEY† AND BLAINE LAWSON‡

Kunihiko Kodaira was a shy, unassuming man. Nevertheless, for us, and for most of the graduate students at Stanford at that time, Kodaira was an immense presence. The sheer majesty of his work spoke for him. We were all more than a little proud just to share his corridors.

Kodaira's courses and seminars were idiosyncratic. For lectures he would prepare each word carefully in advance, and then write everything out in beautiful script on the blackboard. The material would be crafted in his clear and elegant style and always bore the hallmark of his penetrating vision. What had appeared to be a difficult and technical subject seemed quite natural when he finished explaining it.

One of the common sights for a Stanford graduate student in those days was that of Kodaira and Spencer walking together and talking – the tall Spencer bent attentively over Kodaira, listening and responding. It was a symbol of our time.

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KUNIHIKO KODAIRA: MATHEMATICIAN, FRIEND, AND TEACHER*

F. HIRZEBRUCH

Kunihiko Kodaira was friend and teacher for me. My wife and I remember our last visit to the Kodairas' house in Tokyo. He was working at the kitchen table on textbooks for secondary schools. Seiko Kodaira had to push the papers away when preparing the meal. In 1995 I congratulated him on his eightieth birthday. He answered in his charming way. But when we came to Tokyo in 1996, he was already in the hospital. We could not talk to him anymore.

I have read the obituaries by D. C. Spencer in the *Notices of the AMS* (March 1998) and by M. F. Atiyah for the London Mathematical Society. Both say much about our mutual friend; I do not have to repeat it.

I want to report about the influence which Kodaira had on my mathematical work. I shall emphasize the period from 1952 to 1954 when I was a member of the Institute for Advanced Study in Princeton. On Monday, August 18, 1952, I arrived in Hoboken, New Jersey, on the *Ryndam* of the Holland America Line. D. C. Spencer and Newton Hawley picked me up. On Saturday, August 23, I wrote to my parents that I had worked every day in the Institute with Kodaira, Spencer, and Hawley. When I read this letter again after forty-six years, I was surprised to see that my mathematical training in Princeton under Kodaira and Spencer started immediately after my arrival in spite of the Princeton summer.

When I arrived, I knew nothing about sheaves and very little about algebraic geometry and characteristic classes. This improved fast. Our heavy work was made easier by a fine picnic given by Kunihiko and Seiko Kodaira.

In 1975 Kodaira's *Collected Works* appeared in three volumes (Iwanami Shoten, Publishers, and Princeton University Press) with a preface by his student Walter L. Baily Jr. giving a survey and appreciation of Kodaira's work until then.

At the end of this paper I shall reproduce twenty-six entries from the table of contents of the *Collected Works* using the numbering there. These are mostly the papers quoted in my book *Topological Methods in Algebraic Geometry* (Translation and Appendix One by R. L. E. Schwarzenberger, Appendix Two by A. Borel), which was published by Springer-Verlag in 1966 as the English version of *Neue topologische Methoden in der algebraischen Geometrie* (Springer-Verlag, 1956). I added reference [28] ("Work done at Princeton University, 1952"). These are the lecture notes of his course at Princeton University which I attended, at least partially, in the winter 1952-53. I do not know how much of [28] he covered in his course, but this rich material certainly occurred in the many conversations and private seminars of Kodaira, Spencer, and me. In September 1952 Spencer picked me up by car quite regularly at 9 a.m. and drove me to the Institute, where we worked until 5 p.m., mostly with Kodaira, whose course began at the end of September. I also added [37], which is the announcement of his great result that the Hodge manifolds are all projective algebraic, which is fully presented in [38]. I added [63, 66, 68], which together with [60] are the four papers of the famous series "On the structure of compact complex

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analytic surfaces" , which I quoted in my paper *Hilbert modular surfaces* (Enseign. Math. **19** (1973), 183-281) and which I used so much in teaching and research. This paper on Hilbert modular surfaces grew out of my International Mathematical Union lectures, Tokyo, February March 1972. I remember vividly that Kodaira and Kawada picked us up at the airport. This was the first journey to Japan by my wife and me. Kodaira, having returned to Japan in 1967, was in full action as dean at the University of Tokyo. He introduced me to many of his brilliant students, who later became research visitors in Bonn. For the first time we enjoyed Kunihiko's and Seiko's hospitality in Japan. Finally, I added [64] to the list because it gave rise to my paper *The signature of ramified coverings*, published in the volume *Global Analysis* (University of Tokyo Press, Princeton University Press, 1969), dedicated to Kodaira at the time when he left the United States for Japan.

I have explained how my selection of papers from Kodaira's *Collected Works* was motivated. Among them are eight joint papers by Kodaira and Spencer. Atiyah characterized the collaboration between the two, from which I profited so much: "The Kodaira-Spencer collaboration was more than just a working relationship. The two had very different personalities which were complementary. Kodaira's shyness and reticence were balanced by Spencer's dynamism. In the world of university politics Spencer was able to exercise his talents on Kodaira's behalf, providing a protective environment in which Kodaira's mathematical talents could flourish."

I now begin to go into more detail concerning some of the selected papers. In [28] results of earlier papers are incorporated. Sheaves do not occur yet. The theory of harmonic integrals is used to study the vector space of all meromorphic differentials W of degree n on an n-dimensional Kähler manifold V_n of dimension n which satisfy $(W)+S \geq 0$, where S is a given divisor of V_n. The dimension of this space equals dim $|K+S|+1$ if K is a canonical divisor, where $|K+S|$ is the complete linear system of divisors linearly equivalent to $K+S$ and is called the *adjoint system* of S. Several Riemann-Roch type formulas for dim $|K+S|$ are proved. Following Severi, Kodaira introduces the numerical characteristic $a(V_n)$ by the formula

$$(1) \qquad a(V_n) = g_n(V_n) - g_{n-1}(V_n) + \dots + (-1)^{n-1} g_1(V_n),$$

where $g_i(V_n)$ is the dimension of the space of holomorphic differentials of degree i. He formulates a Riemann-Roch theorem for adjoint systems (assuming now that V_n is projective algebraic and E is a smooth hyperplane section for some embedding). He proves (Theorem 2.3.1 in [28])

$$(2) \qquad \dim |K+E| = a(V_n) + a(E) - 1.$$

Not much later we would say that it is better to consider the holomorphic Euler number

$$(3) \qquad \chi(V_n) = \sum_{i=0}^{n} (-1)^i g_i(V_n)$$

($g_0 = 1$ if V_n is connected) and for a divisor D the number

$$(4) \qquad \chi(V_n, D) = \sum_{i=0}^{n} (-1)^i \dim H^i(V_n, \Omega(D)),$$

where $\Omega(D)$ is the sheaf of local meromorphic functions f with $(f)+D \geq 0$. Then dim $H^0(V_n, \Omega(D)) = \dim |D| + 1$. By the Kodaira vanishing theorem [35] the spaces

$H^i(V_n, \Omega(K+E))$ are zero for $i > 0$. Hence dim $|K+E| + 1 = \chi(V_n, K+E)$, but by Serre duality $\chi(Vn, K+E) = (-1)^n \chi(V_n, -E)$ (true for any divisor E and for the individual terms in the alternating sum). Serre duality is mentioned in [34] and [35]. Therefore (2) becomes

$$\chi(V_n, -E) = \chi(V_n) - \chi(E), \tag{5}$$

which follows from an easy exact sequence of sheaves.

But let us go back to [28]. Kodaira proves that dim $|D + hE|$ is a polynomial $v(h, D)$ in h for large h (often called a Hilbert polynomial) and that $v(0, D)$ depends only on D. It is called the virtual dimension of $|D|$. Now we follow [31]. There are two distinct ways in which arithmetic genera may be defined. In the first place we may define the *arithmetic genus* $P_a(V_n)$ to be the virtual dimension $v(0, K)$ of $|K|$ increased by $1 - (-1)^n$ and alternatively the *arithmetic genus* $p_a(V_n)$ by $(-1)^n v(0,0)$. In [31] the authors point out that $p_a(V_n) = P_a(V_n)$ has not been established before for $n \geq 5$. They prove it using sheaves. In [28] Kodaira showed $P_a(V_n) = a(V_n)$ in general and $P_a(V_n) = p_a(V_n)$ for a special class of varieties, including complete intersections in projective spaces. A little later we would say

$$\dim |D + hE| + 1 = \chi(V_n, D + hE)$$

for h large by the Kodaira vanishing theorem. But $\chi(V_n, D + hE)$ is a polynomial for all h. Therefore $v(0, D) + 1 = \chi(Vn, D)$ and

$$v(0,0) + 1 = (-1)^n (v(0, K) + 1)$$

by Serre duality. But this is the equality $p_a = P_a$. I pointed out "A little later we would say...." Here I must mention the papers [31]–[36] from which I could learn so much (of course, before the papers were written). These papers were all communicated to the *Proceedings of the National Academy of Sciences* by S. Lefschetz. Let us look briefly at [32] and [35].

In [32] Kodaira works with a compact complex analytic variety V of complex dimension n and a holomorphic line bundle F over V and studies the sheaf (faisceau, stack) $\Omega^p(F)$ over V of germs of holomorphic p forms with coefficients in F. I quote from [32]: "The faisceau $\Omega^p(F)$ introduced recently by D. C. Spencer and, independently, by J.-P. Serre turned out to be of importance to applications of faisceaux to the theory of compact analytic varieties. However, for these applications, we need a basic theorem to the effect that the cohomology groups $H^q(V; \Omega^p(F))$ of V with coefficients in $\Omega^p(F)$ have finite dimension. The purpose of the present short note is to give an outline of a proof of this basic theorem." Kodaira uses a Hermitian metric and has to generalize Hodge theory on Kähler manifolds to this more general case using the complex Laplace-Beltrami operator studied earlier by Garabedian and Spencer in the case where F is trivial. The Laplace-Beltrami operator is elliptic. Solution spaces are finite dimensional. With respect to this operator $H^q(V; \Omega^p(F))$ can be identified with the vector space $H^{p,q}(F)$ of all harmonic forms of type (p, q) on V with coefficients in F. In [35] Kodaira writes, "In the present note we shall prove by a differential-geometric method due to Bochner some sufficient conditions for the vanishing of $H^q(V; \Omega^p(F))$ in terms of the characteristic class of the bundle F." In particular he proves: If the characteristic class of F is positive in the sense of Kodaira (representable by a Kähler form), then $H^q(V; \Omega^n(F))$ and $H^{n-q}(V, \Omega^0(-F))$ (Serre duality) both vanish for $1 \leq q \leq n$. Bochner's papers (*Curvature and Betti numbers, I and II*) appeared in the *Annals of Mathematics* in 1948 and 1949. In [36] Kodaira and Spencer study the holomorphic Euler numbers

$$\chi_V^p(F) = \sum_q (-1)^q \dim H^q(V; \Omega^p(F)) \tag{6}$$

and prove the "form term formula"

$$\chi_V^p(F) = \chi_V^p(F - \{S\}) + \chi_S^p(F_S) + \chi_S^{p-1}(F_S - \{S\}_S) \tag{7}$$

(for a line bundle or divisor F and a smooth hypersurface S), which was very important for me when I studied the polynomial

$$\chi_y(V, F) = \sum_{p=0}^{n} \chi_v^p(F) y^p \tag{8}$$

(also for a holomorphic vector bundle F), where the χ_y-genus $\chi_y(V)$ is the polynomial obtained if F is the trivial line bundle. These polynomials occur in the proof of my Riemann-Roch theorem.

The χ_y-genus is a generalization of the holomorphic Euler number $\chi(V)$ (for $y = 0$) and $\chi(V)$ equals $(-1)^n a(V) + 1$ if $n = \dim V$ (see (3)).

At this point let me emphasize the Paris-Princeton relations of the early 1950s. I recommend reading the letter of Serre to Borel of April 16, 1953 (published in Serre's *Collected Papers*, Vol. 1, No. 20, Springer-Verlag, 1986), and Serre's comments (Vol. 1, p. 588).

In my recent lecture "Learning Complex Analysis in Münster-Paris, Zürich and Princeton from 1945 to 1953" (Journée en l'Honneur d'Henri Cartan, June 14, 1997; *Gazette des Mathématiciens* **74** (1997), 27-39) I talk about Paris-Princeton on pp. 35-36.

In the introduction of my book I speak of four definitions of the arithmetic genus

$$p_a(V), P_a(V), a(V) = g_n - g_{n-1} + \dots + (-1)^{n-1} g_1$$

and the Todd genus. The basic reference is J. A. Todd, *The arithmetical invariants of algebraic loci*, Proc. London Math. Soc. **43** (1937), 190-225, where Todd uses the characteristic classes K_i of Eger and Todd, which are $(2n - 2i)$-dimensional cycles $(K_1 = K)$, to express $(-1)^n P_a(V) + 1$ as a polynomial in the K_i. The proof relies on an unproved lemma of Severi from which Todd concludes that such polynomials must exist. He characterizes them by requesting that they give the correct values on the complete intersection of smooth hypersurfaces of degrees $n_1, n_2, \dots, n_d$ in the projective space of dimension $2d$. Todd's formalism of his polynomials is very difficult to read. Kodaira ([28], (6.1.1)) gave a formula for the Todd polynomial which is close to my multiplicative sequences and stems from his careful analysis of Todd's paper. However, I do not remember whether I realized this in the old days. Clearly, the power series $(e^\chi - 1)/\chi$ is recognizable in his formula, as it is in Todd's formula (22),which can be interpreted as a formula for the arithmetic genus of the complete intersection V of smooth hypersurfaces of degrees $n_1, \dots, n_d$ in $P_{2d}(\mathbb{C})$ involving the total Todd class of the normal bundle of V in $P_{2d}(\mathbb{C})$ and the total Todd class of $P_{2d}(\mathbb{C})$. Staying close to Todd's formalism, Kodaira proves that the Todd polynomial gives the arithmetic genus $P_a(V)$ for a class of varieties including complete intersections in projective spaces. At one point, relating the characteristic classes of the tangent bundle of a hypersurface to those of the ambient variety and of the normal bundle, he needs the help of S. S. Chern, who had just proved his "duality theorem" for Chern classes. Kodaira was aware of the fact that the K_i of Eger and Todd coincide up to the factor $(-1)^i$ with the Chern classes $c_i \in H^{2i}(V, \mathbb{Z})$. With the use of multiplicative sequences, the inductive proof for Kodaira's result that the Todd polynomials give the arithmetic genus on complete intersections became very simple. At a time when I had formulated the Riemann-Roch theorem but could not yet prove it, I also conjectured as a special case of the Riemann-Roch theorem that the polynomial $\chi_y(V)$ is the genus belonging to the power series

$$\frac{\chi(y+1)}{1-e^{-\chi(y+1)}} - \chi y = \frac{\chi}{f_y(\chi)}, \tag{9}$$

and I proved it for complete intersections. This leads me to the following story. (Compare my *Collected Papers*, Vol. 1, Commentaries, Springer-Verlag, 1987, p. 785.)

A. Weil wrote to Kodaira on October 22, 1953, asking in particular for the Hodge numbers of the complete intersection of two quadrics. Kodaira answered on November 4, 1953, explaining my result on the χ_y-genus of complete intersections by which all Hodge numbers of complete intersections are known. At the end of this letter Kodaira writes:

"Recently I could prove that every Hodge variety (i.e. a Kähler variety whose fundamental form $i \sum g_{\alpha\bar{\beta}} dz^\alpha \wedge dz^\beta$ is homologous to an integral co-cycle) is an algebraic variety imbedded in a projective space. I believe that my proof is correct; however, I am afraid that my result is too good. I would appreciate very much your comment on this result."

Here Kodaira announces one of his most famous and deep results ([37], communicated by S. Bochner on February 23, 1954, and [38]). I do not know Weil's answer. He must have reacted, because Kodaira wrote him on November 18, 1953, thanking him for a letter and explaining to him in all detail my formulas, which make the calculation of the $h^{p,q}$ of complete intersections more explicit and which are published in the *Proceedings of the International Congress of Mathematicians, Amsterdam, 1954* (my *Collected Papers*, Vol. 1, No. 12, formula (1)).

In the introduction to [38] Kodaira recalls that Hodge had introduced in 1951 the Kähler manifolds of special type and A. Weil had called them Hodge manifolds (A. Weil, *On Picard varieties*, Amer. J. Math. **74** (1952), 865-894). A. Weil proves theorems on Hodge manifolds and recalls Hodge's result that the Picard variety of a Hodge manifold is projective algebraic.

For the proof of his fundamental result Kodaira has to use results of earlier papers, for example, his vanishing theorem. He proves that for a Hodge variety V there exists a real (1,1)-form β such that, for any complex line bundle F whose characteristic class contains a closed real (1,1)-form $\gamma > \beta$, the holomorphic sections of F define a bi-regular mapping of V into a projective space (Theorem 3 in [38]).

Kodaira's fundamental theorem generalizes classical results characterizing those complex tori which are projective algebraic. He gives several applications. One was especially important for me. Section 18 of my book *Topological Methods in Algebraic Geometry* carries the title "Some fundamental theorems of Kodaira". I quote Theorem

18.3.1 (Kodaira):

"A complex analytic fiber bundle L over the projective algebraic manifold V with the projective space $P_r(\mathbb{C})$ as fiber and $\mathrm{PGL}(r+1, \mathbb{C})$ as structure group is itself a projective algebraic manifold."

This is used for the proof of my Riemann-Roch theorem, which was completed on December 10, 1953, and announced in the *Proceedings of the National Academy of Sciences* (communicated by S. Lefschetz on December 21, 1953). I had to reduce everything to complex split manifolds where the structural group is the triangular group contained in the general linear group. Then the arithmetic genus can be expressed by virtual signatures which (by the signature theorem as a consequence of Thom's cobordism theory) can be expressed by characteristic classes. But for certain inductive processes I had to stay in the category of projective algebraic manifolds. For a projective algebraic manifold the total space of the flag manifold bundle associated to the tangent bundle is a split manifold. It is projective algebraic by repeated applications of Kodaira's theorem 18.3.1. In my announcement I refer to Kodaira in footnote 9 ("Kodaira, K., not yet published"). I also needed results on the behavior of genera in fiber bundles. The best result is in Appendix Two (by A. Borel) of my book :

"Let $\xi = (E, B, F, \pi)$ be a complex analytic fiber bundle with connected structure group, where E, B, F are compact connected, and F is Kählerian. Let W be a complex analytic fiber bundle over B. Then $\chi_y(E, \pi^* W) = \chi_y(B, W)\chi_y(F)$."

The cooperation with A. Borel in Princeton was of great importance for Kodaira, Spencer, and me in learning characteristic classes and in many other ways, as can be seen, for example, by remarks of Kodaira in [38].

Of course, I am very proud to have one joint paper with Kodaira [41], which was published only in 1957, though I had announced the result already in my talk in Amsterdam in 1954 (loc. cit.). One of my main discoveries (standard joke) is the formula

$$\frac{\chi}{1-e^{-\chi}} = e^{\chi/2} \cdot \frac{\chi/2}{\sinh \chi/2},$$

which showed that the Todd genus is expressible by the first Chern class c_1 and the Pontryagin classes. The latter ones do not depend on the complex analytic structure. For a divisor D the number $\chi(V, D)$ depends on the cohomology class $d + c_1/2$ where d is the cohomology class of D and otherwise only on the oriented differentiable manifold V. This we used in [41]. This remark led to the introduction of the $\hat{A}$-genus which is defined for oriented differentiable manifolds. It equals $\chi(V, D)$ if $2d + c_1 = 0$. From here a new development started whose beginning for me was Atiyah's lecture at the Bonn *Arbeitstagung* in 1962, where it was conjectured that for a spin-manifold the $\hat{A}$-genus is the index of the Dirac operator. This was proved a little later by Atiyah and Singer as a special case of their general index theorem for linear elliptic operators. The index theorem also included my Riemann-Roch theorem as a special case even for complex manifolds (used by Kodaira in [60]). The paper [41] was for me a sign of the importance of the $\hat{A}$-genus.

One more word about the χ_y-genus. If S and T are smooth hypersurfaces in the projective algebraic manifold V and if the divisor $S+T$ is also represented by a smooth hypersurface such that the intersections $S \cdot T$ and $S \cdot T \cdot (S+T)$ are transversal and hence smooth, then

$$(10)\quad \chi_y(S+T) = \chi_y(S) + \chi_y(T) + (y-1)\chi_y(S\cdot T) - y\chi_y(S\cdot T\cdot (S+T)),$$

which I deduced from the four-term formula (7). The functional equation (10) is also true for the χ_y-genus in terms of characteristic classes using the power series (9). It follows from a corresponding elementary functional equation of $f_y(\chi)$. Kodaira often proved and used (10) for $y = 0$. (See his concept of A-functional in [28], Sections 2.7 and 6.3). It is clear that (10) is useful for the study of complete intersections (inductive proofs).

Kodaira's and Spencer's joint work on deformations of complex analytic structures ([43], [48], and several other papers) is perhaps the greatest achievement of their cooperation. It is an enlightment to read in the introduction of [43], "... we *define* a differentiable family of compact complex structures (manifolds) as a fiber space $\mathcal{V}$ over a connected differentiable manifold M whose structure is a mixture of differentiable and complex structures." Kodaira and Spencer introduce the sheaf Θ on $\mathcal{V}$, the corresponding sheaf of cohomology $\mathcal{H}^1(\Theta)$ on M, and a homomorphism (the Kodaira-Spencer map) $\rho : T_M \to \mathcal{H}^1(\Theta)$ where T_M is the sheaf of germs of differentiable vector fields of M. The complex structure V_t, $t \in M$, is independent of t if and only if ρ vanishes. By restricting Θ to V_t (fixed fiber over $t \in M$) they obtain the homomorphism $\rho_t : (T_M)_t \to H^1(V_t, \Theta_t)$ of Frölicher and Nijenhuis, where $(T_M)_t$ is the tangent space of M at t and Θ_t is the sheaf of germs of holomorphic vector fields on V_t . The vanishing of ρ_t for all t does not imply the vanishing of ρ, as "jumps" show, for example, from the smooth quadric surface to the singular quadric with a node and the node blown up (Atiyah, Brieskorn). Now I quote again from the introduction of [43]: "Next we extend Riemann's concept of number of moduli to higher dimensional complex manifolds (Section 11). The main point here is to avoid the use of the concept of the space of moduli of complex manifolds which cannot be defined in general for higher dimensional manifolds (Section 14, (γ)). Moreover, a necessary condition for the existence of a number $m(V_0)$ of moduli of a complex manifold V_0 is that $H^1(V_0, \Theta_0)$ contain only one deformation space; hence $m(V_0)$ is not defined for all compact complex manifolds...." Kodaira and Spencer find it surprising that $m(V_0) = \dim H^1(V_0, \Theta_0)$ for so many examples and consider a better understanding of this fact as the main problem in deformation theory. I do not want to say more about their deformation theory. Surveys are in Baily's preface to Kodaira's *Collected Works* and in the introduction by K. Ueno and T. Shioda to the volume *Complex Analysis and Algebraic Geometry* (Iwanami Shoten, Publishers, and Cambridge University Press, 1977), dedicated to Kodaira on the occasion of his sixtieth birthday. Anyhow, this report is personal and concerns those aspects of Kodaira's work related to my own. Hence, for lovers of Riemann-Roch, I write what this theorem gives for Θ_0 in dimension $n = 1$ (Riemann) and $n = 2$ (Max Noether).

$n = 1:$

$$\dim H^0(V_0, \Theta_0) - \dim H^1(V_0, \Theta_0) = 3 - 3g.$$

The number of moduli equals $3g - 3+$dimension of the group of automorphisms of V_0.

$n = 2:$

$$\dim H^0(V_0, \Theta_0) - \dim H^1(V_0, \Theta_0) + \dim H^2(V_0, \Theta_0) = -10\chi(V_0) + 2c_1^2,$$

where χ is the holomorphic Euler number.

Let us remark that by Serre duality

$$H^i(V_0, \Theta_0) \simeq H^{n-i}(V_0, \Omega^1(K)),$$

which, for $n = 1$ and $i = 1$, is the isomorphism to the space of holomorphic quadratic differentials (see the obituary by Spencer). We have

$$\chi(V_0, \Theta_0) = (-1)^n \chi^1_{V_0}(K).$$

These numbers can be calculated by the Riemann-Roch theorem as linear combinations of Chern numbers. For a Kähler manifold with trivial canonical bundle, $\dim H^i(V_0, \Theta_0)$ equals the Hodge number $h^{1,n-i}$. For a $K3$-surface we have $h^{1,1} = 20$. Kodaira and Spencer discuss many more examples. For the complex projective space $P_n(\mathbb{C})$ we have $\dim H^1(P_n(\mathbb{C}), \Theta_0) = 0$ in agreement with the result in [41].

With the exception of three papers, the whole Volume III of Kodaira's *Collected Works* is concerned with complex analytic surfaces. His work in this area is overwhelming. He can use his earlier papers on complex manifolds and on deformations. I have used the papers in Volume III very often. Looking, for example, at my joint paper with A. Van de Ven, *Hilbert modular surfaces and the classification of algebraic surfaces* (Invent. Math. **23**(1974), 1 - 29), I find that we used the following:

1. *Rough classification of surfaces, Kodaira dimension* ([68], Theorem 55). Kodaira proves that the compact complex surfaces free from exceptional curves can be divided into seven classes. Class 5 comprises the minimal algebraic surfaces of general type. Class 7 surfaces are mysterious surfaces with first Betti number equal to 1. Van de Ven and I specialize Kodaira's classification to algebraic surfaces, where this classification in broad outline was known to the Italian school, but many of the proofs are due to Kodaira.
2. *Kodaira's proof of Castelnuovo's criterion for the rationality of algebraic surfaces.*
3. *Study of elliptic surfaces, their multiple fibers, and a formula for the canonical divisor.*
4. *Classification of the exceptional fibers in elliptic surfaces.*
5. *The fact that all K3-surfaces are homeomorphic and hence simply connected.* Kodaira proves more ([60], Theorem 13): Every *K3*-surface is a deformation of a nonsingular quartic surface in a projective 3-space.

The surfaces in Class 7 are also called VII_0-surfaces ([60], Theorem 21). Masahisa Inoue (*New surfaces with no meromorphic functions II*, in the volume dedicated to Kodaira's sixtieth birthday) has constructed such surfaces using my resolution of the cusp singularities of Hilbert modular surfaces. Such a surface has only finitely many curves on it. They are rational and arranged in two disjoint cycles.

Now a last case where a paper of Kodaira was especially close to my interest. In [64] he constructed algebraic surfaces with positive signature whose total spaces are differentiable fiber bundles with compact Riemann surfaces as base and fiber. In the early 1950s we did not know a single surface with signature greater than 1 and often talked about it at Princeton. How the situation developed over the years can be seen, for example, in the book *Geradenkonfigurationen und Algebraische Flächen* (by Gottfried Barthel, Thomas Höfer, and me, Vieweg, 1987). Also, Kodaira's surfaces give examples in which the signature of the total space of a fibration is not equal to the product of the signatures of base and fiber. The multiplicativity of the signature in fiber bundles (of oriented manifolds) was proved by S. S. Chern, J.-P. Serre, and me under the assumption that the fundamental group of the base operates trivially on the real cohomology of the fiber (*On the index of a fibered manifold*, Proc. Amer. Math. Soc. **8** (1957), 587 - 596).

Far from attempting to give a thorough appreciation of Kodaira's great mathematical work, I wanted to show how much I am indebted to him and where our mathematical lives crossed.

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KUNIHIKO KODAIRA AS I HAVE SEEN HIM*

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For those who are interested in the life and works of Kunihiko Kodaira (1915-1997), it is fortunate that he has left his autobiography [1] as well as his Collected Works [2]. The latter is provided with an excellent preface by Professor W. L.. Baily, Jr. at the University of Chicago, one of Kodaira's students at Princeton University in the early 1950's, giving a thorough and careful explanation of its contents. Kodaira has left, moreover, a large number of publications in Japanese, including many text-books in various fields of mathematics and also essays on non-mathematical subjects, the latter of which were gathered in his book [3]. Thus we have an abundance of first hand material on him.

As I shall recount in the following lines, I made his acquaintance in 1935 and shared the life on this earth for more than 60 years since then in a relatively close relationship with him. I should be happy, if this article based largely on the above material, including, however, some of my personal reminiscences on him could interest the reader of this journal.

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Let me permit myself to begin this article by speaking for a while of my own career. I studied in the Department of Mathematics of Tokyo University for 1926-29 in the undergraduate course, then one and half more years in the graduate course under the guidance of Professor Teiji Takagi, founder of the class field theory in which I could begin some research works. In 1931 I left Japan to study further in Europe. First I went to Hamburg to study with Artin, who had got an important result to complete the class field theory. I was very fortunate to make there the acquaintance of Claude Chevalley who had come from Paris just for the same period. In 1932, I participated in the International Congress of Mathematicians at Zurich where I was happy to again see Professor Takagi, invited there as one of the vice-presidents together with such mathematicians as Hilbert and Hadamard. I stayed two more years in Europe, principally in Paris, where I had a chance to meet Henri Cartan, André Weil, Jean Dieudonné and others, who began the well known work of innovation of entire mathematics under the collective pseudonym of Bourbaki (since 1936, after my return to Japan.) I could learn from them, among other things, the "unity of mathematics," i.e. that the entire mathematics has a common ground inspite of its different branches; arithmetic, algebra, geometry, analysis etc. I returned to Tokyo in 1934 and in the following year, I was admitted as teaching staff (associate professor) of Tokyo Univesity. Incidentally, Professor Takagi attained the retirement age of 60 years in 1935 and he was to give his last lectures at this Department in 1935-36. The subject of these lectures was an Introduction to Analysis for the first year students (the infinitesimal calculus plus the beginning of the complex analysis of elementary functions) and I had the honor of being charged with doing the exercises of these

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lectures.

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It happened that Kodaira entered this Department in this year 1935 and I met him in a class room just a few weeks later in an hour of doing exercises. I remember that I had given as an exercise problem to prove that the base e of the natural exponential function is not an irrational of the second degree (after it had been proved in a lecture that e is irrational.) Kodaira came to the black-board and wrote his proof in a few lines without speaking any word. In reading these lines with other students, we admired his perfect proof, where every word was to the point! I heard later that his lectures in Princeton and elsewhere in the United States were enjoyed rather by seeing than by hearing, i.e. he used to speak few words in low voice, but wrote on the black-board in clear English what he had to say.

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In his autobiography [1], he says that he was a poor pupil in primary and middle schools, being good only in math and in no other subjects, as expressed in its title. I suspect that he was overly modest in entitling thus his autobiography; I believe he certainly did well at least in English in the middle school and in German in the First High School; he entered, by the way, at the top of the class this last school which was known as the best school of its kind at that period. It is true that he was physically of rather small stature and apt to falter in speaking so that he was very shy.

According to [1], he was interested in numbers and played with beans when he was very young. Since he was the third grader in the middle school, he began to attack the thick book "Algebra" by Professor Fujiwara at the Tohoku University. One can imagine that this was hard reading for him, as this book was written for university students and researchers and not for middle school students. Kodaira made the effort, however, to understand it and enjoyed it.

In the First High School, he saw the teachers of mathematics enjoying mathematics so that he wished to become a teacher of mathematics in such a school. I believe that he meant in particular Dr. Aramata, who got an interesting result on the "divisibility of zeta-functions" toward that time and was a good friend of Professor Suetuna at our Department which he often frequented. I had heard also from him of the "exceptionally bright student Kodaira" before he entered our Department.

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As said above, I was in charge of the exercises of Professor Takagi's lectures in 1935-36. I was very lucky to encounter in this class students like Kunihiko Kodaira, Kiyosi Ito, Yukiyosi Kawada, Shigeru Furuya who all became later well known mathematicians.

After the retirement of Professor Takagi, Professor Suetuna succeeded to his chair of arithmetic and algebra. Professor Nakagawa who had been in charge of the geometry course since longtime, had to retire in the following year and I was to give lectures of geometry beginning 1937. In 1936-37, I lectured on modern analysis along the lines of J. von Neumann. I had been impressed by his articles on the theories of Hilbert spaces and almost periodic functions on groups and his book on the foundations of quantum mechanics. These lectures were attended by Kodaira.

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Kodaira says in [1] that he went to visit Professor Suetuna together with Kawada toward the end of 1937 and asked him to admit them to his seminar in the next year. Professor Suetune seemed to accept their proposal and Kawada was admitted in fact, but wrote afterward to Kodaira that he had better study geometry in my seminar. He came then to visit me with the letter of Professor Suetuna and expressed his wish to join my seminar. I have now completely forgotten about the letter of Professor Suetuna, but I was very much pleased to welcome him. Since that time, Kodaira came quite frequently to my house together with Furuya, who was a good friend of his.

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Kodaira was born on March 16, 1915 as the eldest son of Mr. Gon'ichi Kodaira and Mrs. Ichi Kodaira, née Kanai. Both parents originated from the Suwa region of Nagano prefecture, in central Japan. Mr. Gon'ichi Kodaira was a vice-minister of agriculture, a competent administrator and at the same time a scholarly person. The family of the mother of Kodaira was also well known in the Suwa region. Mrs. Kodaira was associated to the so-called no-church sect of Christianism. Thus Kodaira was born into a diligent and intelligent family with a general cultural interest. His father brought back a piano from his official trip to Germany.

Kodaira began to learn playing it since the time he was a third grader of middle school with a student of the Tokyo University who was a good pianist but after a few years he graduated from the University and had to move to another city. Kodaira was left with the student's sister, Miss Tazuko Nakajima, who was also a good musician, but a violinist rather than a pianist. Kodaira was an especially talented pianist, good in particular in sight-reading.

I had, by the way, a sister who played piano and we had also a piano in my house. Once when Kodaira came to visit us, we asked him to play music of Albeniz, Spanish composer, and were amazed in hearing him play it full of sentiment. Miss Nakajima had a number of followers and organized concerts for them once or twice a year and Kodaira used to accompany them. It happened that my sister Seiko was among her followers and Kodaira accompanied her frequently. I do not remember exactly the period when this took place.Anyway, Kodaira and Seiko got married in 1943.

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I said that Kodaira joined my seminar in 1938. I have to tell now what happened in five years 1938-1943. In my seminar, Kodaira studied topology from the well known book of Alexandroff–Hopf together with Makoto Abe, who joined my seminar in the next year. After Kodaira finished the undergraduate course in 1939, he wished to study theoretical physics in the Physics Department of the same Faculty, which he did in 1939-42. He was charged with giving lectures in the Physics Department soon after graduation and one year later he was named lecturer of mathematics at the Tokyo Bunrika University, so that he was secured in a living when he got married. He began to publish his scientific papers from 1937 when he was a student of the Department of Mathematics and continued to publish regularly since then. It was very unfortunate, however, that our country had engaged in World War II since December 1941, which was already turning worse in 1943!

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Kodaira writes in the Introduction to [1]: "I had thought to live always in Japan, enjoying mathematics and music. This thought was completely destroyed by the War."

In fact, the situation of Japan became worse and worse toward 1944-45 and it became difficult to continue to live in Tokyo. The government permitted and even encouraged those of us working in national universities that each Department took refuge in an appropriate place in the countryside, which was, on the other hand, not so easy because people in the countryside did not like to receive a large number of Tokyoites into their regions. For the Departments of Mathematics and Physics of the University of Tokyo, this was made possible by the influence of Kodaira's father: the students and part of the teaching staff of these Departments (as some of the teaching staff preferred to stay in Tokyo) were received in the Suwa region.

I had a chance to visit the native house of Kodaira's father which was a simple farmer's house. Kodaira's family as well as my family, the families of my father and of Makoto Abe, who got married to Taeko, another sister of mine in the meanwhile, all took refuge in this region, where we had to experience, however, the following sad events.

The Department of Mathematics of Tokyo Bunrika University, where Kodaira worked together with Kawada and Abe, took refuge in Shigeno, another place in the Nagano prefecture. Abe, who was not of strong constitution, had to work hard in Shigeno, and came to join Taeko in Suwa to take rest. But it was unfortunately too late and he died there in early 1945.

Out of the marriage of Kunihiko and Seiko, they got a lovely boy Kazuhiko, who developed a kidney illness, very unfortunately, and also died in Suwa in 1946, some months after the end of the War.

The house of Kodaira, as well as my house, in Tokyo was burnt down by American bombardment. In the beginning of August, we heard what happened in Hiroshima and Nagasaki. I was deeply relieved by the broadcast of the Emperor on August 15, 1945 announcing the end of the War, and a brief additional statement of Prime Minister Suzuki which seemed to give us hope for our country to develop in doing other things than a war.

I shall not give a detailed explanation of the scientific works of Kodaira leaving it to the preface of [2], but I shall mention here that since 1944 he was deeply concerned with the generalization of the theory of Riemann surfaces, i.e. the theory of algebraic functions of one complex variable as described in the famous book of Hermann Weyl to the case of many variables. He noticed a result of Hodge whose proof was not perfect but could be rectified by Weyl's "method of orthogonal projection." He noticed also that the cases which were not treated by Hodge can also be dealt with in the same way. He wrote up these ideas in three notes in the Proceedings of the Japan Academy communicated by Professor Takagi in 1944. Meanwhile, the American bombardment became more effective, and the publication of scientific journals in Japan became impossible. I encouraged Kodaira, however, to write a paper giving a more detailed exposition of his result. (Kodaira says in [1] that he did not understand why he wrote such a long paper without knowing whether its publication was possible, but I believe that I encouraged him in the atmosphere of the immediate post-war time. He continued to write this paper even at the bedside of Kazuhiko. There was, by the way, another paper by Iwasawa on topological groups, whose completion I encouraged at that period.) I should recall here, the unusual situation of Japan at that time in which we were not permitted to communicate freely with foreign countries and the kind intervention of Kakutani to help us in this situation. (Kakutani, now professor emeritus at Yale University had collaborated with Kodaira in 1943.) He had been invited to the Institute for Advanced Studies in Princeton before the War and repatriated by the special American ship just after the Japanese declaration of War. After the end of the War, he acquired a number of friends among the Occupation Army. As I told Kakutani of our desire to send the excellent papers of Kodaira and Iwasawa to an American journal, Kakutani asked one of his friends to kindly take care of sending these papers to the Annals of Mathematics, where these paper were accepted and soon published. They appeared in fact in this famous journal and Kodaira's paper attracted the attention of Hermann Weyl. Kodaira obtained thus an invitation letter from Hermann Weyl to the Institute in Princeton in 1949 (though he could not take his family.)

His book [3] contains a number of his letters to Seiko written in the period August 1949 through September 1950, showing how he was amazed in arriving in the U.S. from Japan which was in ruins. I had a chance to see him again in August 1950 when participating in the first International Congress of Mathematicians after the War which took place in Harvard University.

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In 1949, Kodaira had been kindly welcomed by smiling Hermann Weyl, who seemed, however, a little surprised to see him hardly speaking English whereas his papers were written in good English. He proposed to organize his seminar later. A seminar by Siegel and de Rham took place soon, however, which Kodaira was asked to join. Thus a friendship was born between de Rham and Kodaira. Professor Spencer at Princeton University, who came there in just the same year from Stanford, asked him also to speak in his seminar, and the close friendship between them developed further, to a very successful collaboration between them through the whole 1950's.

Kodaira had first thought to stay in the States just for one year, but he obtained offer first by W. L. Chow at Johns Hopkins University followed by offers from other places. His family (Seiko and two daughters Yasuko and Mariko) could join him in 1950 and finally he stayed 18 years in the United States, occupying himself with his scientific works (and the music together with his family.) In the International Congress of Mathematicians at Amsterdam in 1954, he was a co-recipient of the Fields medal together with Jean-Pierre Serre. I had a chance to attend the ceremony where he received a medal from Professor Hermann Weyl and could imagine his heart-felt joy. He was the first recipient of this medal originating from other regions than Europe and North America. After that he received a number of Japanese and international prizes, which I shall not enumerate here.

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I shall conclude this article in recounting a little more about his life after his return to Japan in 1967. He taught at Tokyo University until 1975, then ten more years at the Gakushuin University. He was named dean of the Faculty of Science of Tokyo University in 1972-73 which did not please him, particularly because of students' unrest at that time, but more fundamentally, he estimated himself entirely inappropriate to this kind of rôle. (It seems to me, however, that in fact, he shared the ability of his father as an administrator, but he did not like administrative works necessitating attentions to multitude of affairs. He preferred to concentrate himself rather in deeper questions. The title of his book [3] meaning: Notes of an idle mathematician, may surprise as a title of a book written by such a diligent mathematician. This comes principally, I believe, from his experience as a dean: he seemed idle to himself as he could not fulfil his duty sufficiently well.) In the University of Tokyo, he had such brilliant students as Iitaka and others who are continuing and developing his works.

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His elder daughter Yasuko was married to Mr. Keitaro Hashimoto, Professor of physiology at the Yamanashi Medical University, Yamanashi being a prefecture placed between Nagano and Tokyo; and his younger daughter Mariko to Dr. Mutsuo Oka, Professor of Geometry at the Municipal University of Tokyo.

Toward the end of the 1980's, the health of Kunihiko Kodaira began to decline. He had first troubles in his respiratory system, then in his hearing organ. It was sad to see him having difficulties in hearing and losing the pleasure in hearing music. He was hospitalized in a hospital in the Yamanashi prefecture and passed his last days in a room from the windows of which one can enjoy the view of Mt. Fuji.

After his death in this hospital on July 26, 1997, Seiko has continued to live in the same house in Nakaochiai, Tokyo, together with the Oka family, at the same place where Kunihiko Kodaira had lived with his parents, until the beginning of the January of 2000 when she suddenly died, which was another sad event for me.

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REMEMBERING PROFESSOR K. KODAIRA*

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I first met Professor Kodaira in September of 1953 when I started graduate work at Princeton. It was in the organization meeting of the famous "nothing seminar" which was run by Kodaira and Spencer, to discuss a variety of recent developments in complex analysis and in geometry. There was a sense of excitement at this seminar. The main topic of discussion was the theory of deformations which was then being developed by them; it is hard for me to do justice to the frontier spirit engendered by their work. This spirit is described by Kodaira in the preface to his book on deformations of complex structures (Springer-Verlag, 1981):

"In order to clarify this mystery , Spencer and I developed the theory of deformations of compact complex manifolds. The process of the development was the most interesting experience in my whole mathematical life. It was similar to an experimental science developed by the interaction between experiments (examination of examples) and theory. In this book I have tried to reproduce this interesting experience; however I could not fully convey it. Such an experience may be a passing phenomenon which cannot be reproduced."

I also was fortunate to attend Kodaira's course on the subject. Even though the course was way above my head it served as a wonderful introduction to: the theory of several complex variables, differential geometry, algebraic geometry, and the theory of elliptic partial differential equations. Kodaira's lectures were marvelous. Each was like a work of art. The theory was perfectly balanced with examples, digressions and historical remarks. I have never seen such remarkable blackboard technique. The formulas, diagrams and writing in perfect harmony every symbol and index of just the right shape and size. Somehow the beauty and depth of the content of the lectures was matched by the esthetics of the presentation. After moving to Japan Kodaira wrote a series of texts which are known for their extraordinary exposition.

In the summer of 1966 I participated in the Taniguchi Symposium on Lake Biwa and also in a conference in Kyoto. This was the first time in seventeen years that Professor Kodaira had returned to Japan. He was well known by the general public in Japan and newspaper reporters covered his return. D. C. Spencer and I had the privilege to accompany Kodaira when he was welcomed in Japan. Professor Akizuki was in charge and Kodaira and (by association) we were given the royal treatment. There were banquets, tea ceremonies, temples, theater, music etc. It was very moving to see a culture that puts such a high value on intellectual achievements.

To conclude these remarks I will present a few "snapshots" that are engraved in my memory of Professor Kodaira.

On several occasions I sat next to Kodaira in the back seat of a car while Spencer was driving and an out of town guest sat next to him in the front. Invariably a mathematical discussion would start between Kodaira and Spencer. As it got more involved Spencer would turn around to face Kodaira while driving full speed ahead.

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The guest and I would become very apprehensive, to say the least, while both Kodaira and Spencer were oblivious to any danger. Miraculously nothing ever happened so, as usual, Kodaira and Spencer were right there was nothing to worry about. Once I asked Professor Kodaira whether he knew how to drive, he thought for a moment and then answered: "Yes, but only in theory."

Professor Kodaira loved classical western music and played it beautifully on the piano. I believe that I heard him play only once but both the music and Kodaira's demeanor left an unforgettable impression.

Whenever someone would ask Professor Kodaira a question, which showed that the questioner did not understand the subject, Kodaira would concentrate very hard. Then, with infinite patience, he would formulate an answer which would clear up the questioner's difficulties. I was fortunate to be the enlightened questioner several occasions. I remember once at a conference (I think in Williamstown) Kodaira's daughters came running up to him. They were saying: "Oh come on daddy" sounding like typical American teenagers. Evidently Mrs. Kodaira had told them that they were not allowed to go out and they were appealing this decision. Kodaira sat down and had the same look of hard concentration as when dealing with the uninformed questioner.

In conclusion, to summarize my memories of Kodaira. He was passionately dedicated, in a very quiet, way to: research, teaching and his family. He enjoyed classical music, good food and especially the beauty of mathematics. The combination of his special human qualities with his stellar mathematical talent and production is very rare.

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PERSONAL REMINISCENCES OF PROFESSOR KODAIRA*

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It was in the late summer of 1958 in Princeton that I met Prof. Kunihiko Kodaira for the first time. We (my wife and I) had just arrived from Paris to spend one (actually two) academic year(s) at the Institute for Advanced Study. Since I was a student at Tokyo University in 1947-50 and he was an Assistant Professor of the Physics Department there in 1944-49, we must have had a chance to see each other, but I knew him then only by name.

Prof. and Mrs. Kodaira came to our house (the project house of the Institute) to say hello and kindly gave us a necessary guidance to live in the States and in Princeton. Prof. Kodaira told me that he had already started his course on the deformation theory of complex manifolds, in which he needed some details on exceptional Lie groups. But apparently my knowledge in Lie groups was not adquate to answer his questions so that I could only give him some relevant references.

At the Institute Prof. Kodaira and Don Spencer were running a seminar, called "Nothing Seminar", meaning that there is no restrictin on the topics. I gave a couple of talks in this seminar, but did not attend it so regularly; Grauert and three Italian mathematicians, Andreotti, Calabi and Vesentini were among the regular attendants. I also had opportunities to attend some lectures of Kodaira at Princeton University and also later at Johns Hopkins University. His lecture was a kind of magic; it was so clearly organized that he needed almost no word of explanation. Once, in a party at Igusas, I heard Mumford raise a question asking how was it possible for Kodaira and Spencer to communicate to each other on such a complicated subject with such little conversation.

In Princeton, we often accompanied Kodairas to the weekend drivings in the suburbs. We had very enjoyable afternoons visiting Newhope, Washington Crossings, ... and very pleasant evenings in the concerts at the chapel of Princeton University. Also at many occasions they were so kind to invite us to dinner in their home. When we were relaxed, Prof. Kodaira liked to tell us many interesting stories (mostly anecdotes) of mathematicians and musicians. As is well known, he loved music, being himself a very good player of piano. He amused himself by saying there was some score of Bartok which was physically impossible to play. He was also interested in animals; sometimes he showed us a picture of a funny face of an animal or a fish saying it must remind us of a face of a famous mathematician.

But, when visiting his house, I was very impressed to see that he was ready to do mathematics at all times. In the living room and everywhere, beside each chair he had a chance to sit, I noticed there were some scratch papers with a trace of calculations. It is famous that he worked in the dining room rather than in his study. Many mathematicians would like to work in quiet isolated circumstances. But, Prof. Kodaira certainly preferred to concentrate in mathematics in the warm family

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atmosphere around Mrs. Kodaira.

I knew also Prof. Kenkichi Iwasawa very well, who was a teacher of mine at Tokyo University and was two and half years younger than Prof. Kodaira. It was a very interesting experiment in my mind to make a comparison of these two distinguished mathematicians. Contrary to Kodaira, Prof. Iwasawa showed little interest in music or biology, but was more interested in literature. While Kodaira had a large collection of records of classic music, we found a series of collected works of the novelists such as Ibuse Masuji and Tanizaki Jyun'ichiro on the book shelves of Prof. Iwasawa. Though they were so different in their character, they had also some habits in common. Both of them did not drive car at all, perhaps because their wives were exceptionally good drivers (among Japanese wives). Both of them were indolent for traveling. — Once, in Woodshole, while I was with Prof. Kodaira, Andreotti approached him to make an offer; he wanted to invite Kodaira to Italy. He (Andreotti) said he would make all arrangements for the travel and pay all necessary expenses for Kodairas if only he would just come to Italy and give few lectures. I thought that was an extremely generous offer. But, Prof. Kodaira never said yes.

One of very few exceptions of his indolence for traveling was his trip to Israel in 1985 to receive a Wolf prize. Incidentally, around the same time I was also invited to Tel Aviv by Piatetskii Shapiro, who kindly made an arrangement for us to be permitted to the ceremony. It was an unforgettable experience for us, too. But, Prof. Kodaira, accompanied with his daughter Yasuko in Japanese kimono, looked a little tired after the long journey, many receptions, and lecturing in few universities.

Prof. Kodaira and his family had been back to Japan since 1967, around the same time I moved to Berkeley from Chicago. After we came back to Sendai in 1980, we occasionally had chance to meet Kodairas in Tokyo, sometimes in their home near Ikebukuro, though we could have less chance to meet in Japan than in the United States. I remember that he had a very good stereo video system in the dining room, by which he was enjoying the "rakugo" (Japanese comic stories) as well as music. Once, when we were visiting his house, Prof. Kodaira excused himself in a cheerful manner to go out to meet an electrician. Mrs. Kodaira commented to us that her husband was a very good customer of a nearby shop selling electronic equipments.

In 1986, when the Mathematical Society of Japan began organizing the ICM 1990 in Kyoto, he was designated to be the honorary president, and in the first few meetings he was serving as chairman of the organizing committee. By a telephone call from him, I was asked to be the chief editor of the Proceedings of the ICM, which I did with Springer-Verlag Tokyo. But, unfortunately, Prof. Kodaira had been suffering from asthma, which soon got worse to the extent that he had to give up all his duties in the ICM. Besides he had a trouble in hearing and could not enjoy music any more, which must have also affected his health. Few years after the Congress in Kyoto, we were told that he was hospitalized in Yamanashi, where a son-in-law of his (Yasuko's husband) was a doctor. In the summer of 1997 we were very saddened to hear that he finally passed away there. However, Prof. Kodaira will be remembered by us all for a long time to come, not only because of his great achievement in mathematics, but also for his warm and amiable personality which enchanted everybody who knew him.